On the Stability, Asymptotically Stability, Integrability, Uniformly Stability and Boundedness of Solutions for a Class of Non-Linear Volterra Integro - Differential Equations

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Volterra integral and integro-differential equations, integral equations and integro-differential equations have many applications in sciences and engineering (see Burton [2], Rahman [7], Wazwaz [18] and the cited references therein). Due to these facts, in the last years, stability, asymptotic stability, uniform stability, boundedness, exponentially stability, etc., of linear and non-linear Volterra integro-differential equations, Volterra integral equations, integral equations and integro-differential equations have been discussed by many researches. In particular, as a brief information, the reader can referee to the articles of Becker [1], Furumochi and Matsuoka [3], Graef *et al.* [4], Mahfoud [5], Raffoul [6], Rama Mohana Rao and Srinivas [8], Tunç ([9], [10], [11], [12]), Tunç and Mohammed [13], Tunç and Tunç ([14], [15]), Wang ([16], [17]) and the works mentioned in that sources for the former scientific results that can be found in the literature on the diverse qualitative behaviors of various of Volterra integro-differential equations, Volterra integral equations, integral equations and integro-differential equations.As a distinguished information from this line, the following article is notable. In 2000, Wang [17] considers the following Volterra integro-differential equation

$$\frac{dx}{dt} = A(t)x(t) + \int_0^t C(t,s)x(s)ds, \tag{1}$$

in which t is non-negative and real variable, $x \in \Re^n$, $n \geq 1$, $A(.)$ and $C(.)$ are $n \times n-$ matrices, which are continuous for $0 \leq t < \infty$ and $0 \leq s \leq t < \infty$, respectively.

Wang [17] proves three theorems related to the stability, uniform stability and asymptotic stability of solutions of Volterra integro-differential equation (1). The author gives an example verifying the established assumptions. The results obtained in [17] are variants of the results that can be found throughout Burton [2].

In this article, motivated by the results of Wang [17], we take into consideration the nonlinear Volterra integro-differential equation

$$\frac{dx}{dt} = -A(t)x + \int_0^t C(t,s)g(s,x(s))ds + h(t,x), \tag{2}$$

where t is non-negative and real variable, $x \in \Re^n$, $A(.)$ and $C(.$ have the same properties as in the Volterra integro-differential equation (1), $g: \Re^+ \times \Re^n \to \Re^n$, $h: \Re^+ \times \Re^n \to \Re^n$ are continuous functions with $\Re^+ = [0, \infty)$, and $g(s,0) = 0$.

We will discuss the stability, asymptotic stability, uniform stability of trivial solution, integrability and boundedness of solutions of Volterra integro-differential equation (2) by help of appropriate Lyapunov functionals for the cases of $h(.)\equiv$ and $h(.) \neq 0$, respectively.

Briefly, in this wok, new Lyapunov functionals are defined. We apply that functionals to get sufficient conditions guaranteeing the stability, asymptotic stability, integrability, uniform stability and boundedness of solutions of certain non-linear Volterra integro-differential equations of first order. The results obtained have improvements and extensions of the former the results that can found in literature. We give examples to show applicability of the results obtained and for illustrations. In the particular cases, using MATLAB-Simulink, it is clearly shown the behaviors of the orbits of the Volterra integro-differential equations considered.

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