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INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION (ICMME - 2018) BOOK OF ABSTRACTS

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(ICMME - 2018)

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PREFACE

The International Conference on Mathematics and Mathematics Education "Mathematics in ORDU" (ICMME-2018) will be held on June 27-29, 2018 in Ordu, Turkey.

MATDER-Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now 14 national and 2 international mathematics symposium were organized by MATDER.

These meetings have been one of the main national symposiums. Since the talks in the meetings covers almost all areas of mathematics, mathematics education and engineering mathematics, the conferences have been well attended by mathematicians from academia, Ministry of Education and engineers as well. The last five conferences have been held in Şanlıurfa (ICMME-2017), Elazig (ICMME-2016), Niğde (2015), Karabük (2014) and Ankara (2013). This year ICMME-2018 has been held at Ordu University in Ordu/Turkey on 27-29 June 2018 as an international conference.

The main aim of this conference is to contribute to the development of mathematical sciences, mathematical education, and their applications and to bring together the members of the mathematics community, interdisciplinary researchers, educators, mathematicians, and statisticians from all over the world. The conference will present new results and future challenges, in series of invited and short talks, poster presentations, workshops, and exhibitions. All presented paper's abstracts will be published in the conference proceeding. Moreover, selected and peer review articles will be published in the following journals:

- Turkish Journal of Mathematics & Computer Science (TJMCS)
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INVITED SPEAKERS

On the Stability, Asymptotically Stability, Integrability, Uniformly Stability and Boundedness of Solutions for a Class of Non-Linear Volterra Integro - Differential Equations

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ABSTRACT

Volterra integral and integro-differential equations, integral equations and integro-differential equations have many applications in sciences and engineering (see Burton [2], Rahman [7], Wazwaz [18] and the cited references therein). Due to these facts, in the last years, stability, asymptotic stability, uniform stability, boundedness, exponentially stability, etc., of linear and non-linear Volterra integro-differential equations, Volterra integral equations, integral equations and integro-differential equations have been discussed by many researches. In particular, as a brief information, the reader can referee to the articles of Becker [1], Furumochi and Matsuoka [3], Graef *et al.* [4], Mahfoud [5], Raffoul [6], Rama Mohana Rao and Srinivas [8], Tunç ([9], [10], [11], [12]), Tunç and Mohammed [13], Tunç and Tunç ([14], [15]), Wang ([16], [17]) and the works mentioned in that sources for the former scientific results that can be found in the literature on the diverse qualitative behaviors of various of Volterra integro-differential equations, Volterra integral equations, integral equations and integro-differential equations.As a distinguished information from this line, the following article is notable. In 2000, Wang [17] considers the following Volterra integro-differential equation

$$\frac{dx}{dt} = A(t)x(t) + \int_0^t C(t,s)x(s)ds, \tag{1}$$

in which t is non-negative and real variable, $x \in \Re^n$, $n \geq 1$, $A(.)$ and $C(.)$ are $n \times n-$ matrices, which are continuous for $0 \leq t < \infty$ and $0 \leq s \leq t < \infty$, respectively.

Wang [17] proves three theorems related to the stability, uniform stability and asymptotic stability of solutions of Volterra integro-differential equation (1). The author gives an example verifying the established assumptions. The results obtained in [17] are variants of the results that can be found throughout Burton [2].

In this article, motivated by the results of Wang [17], we take into consideration the nonlinear Volterra integro-differential equation

$$\frac{dx}{dt} = -A(t)x + \int_0^t C(t,s)g(s,x(s))ds + h(t,x), \qquad (2)$$

where t is non-negative and real variable, $x \in \Re^n$, $A(.)$ and $C(.)$ have the same properties as in the Volterra integro-differential equation (1), $g:\Re^+ \times \Re^n \to \Re^n$, $h:\Re^+ \times \Re^n \to \Re^n$ are continuous functions with $\Re^+ = [0,\infty)$, and $g(s,0) = 0.$

We will discuss the stability, asymptotic stability, uniform stability of trivial solution, integrability and boundedness of solutions of Volterra integro-differential equation (2) by help of appropriate Lyapunov functionals for the cases of $h(.) \equiv 0$ and $h(.) \neq 0$, respectively.

Briefly, in this wok, new Lyapunov functionals are defined. We apply that functionals to get sufficient conditions guaranteeing the stability, asymptotic stability, integrability, uniform stability and boundedness of solutions of certain non-linear Volterra integro-differential equations of first order. The results obtained have improvements and extensions of the former the results that can found in literature. We give examples to show applicability of the results obtained and for illustrations. In the particular cases, using MATLAB-Simulink, it is clearly shown the behaviors of the orbits of the Volterra integro-differential equations considered.

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On The Tangent Bundle of A Weyl Manifold

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ABSTRACT

This year, we celebrate a century since H. Weyl introduced in 1918 a unified field theory, in order to study a generalized metrical structure. A Weyl structure on a manifold M is described as a conformal class of metrics [g], which is preserved by a torsion-free connection D (called a Weyl connection). The Weyl manifold is said to be Einstein-Weyl if the symmetric part of the Ricci tensor is proportional to the conformal metric. Here, we obtain the behaviour of the Sasaki metric on TM, under the gauge transformations of the metrics in the conformal class [g]. Then we lift several geometric objects from the base manifold to the total space of the tangent bundle. By taking a Weyl structure on the base manifold, we construct a Weyl structure on the total space of the tangent bundle whose conformal class of metrics contains the Sasaki metric on TM. By using the curvature tensor field, we characterize (in terms of Sasaki metric) both Weyl structures on M and on TM to be simultaneously Einstein-Weyl.

The present talk is based on the following papers:

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Coupled Finite and Boundary Element Adaptive Approximations for the Problems of Elasticity

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ABSTRACT

Heterogeneous modeling using finite and boundary element methods based on the domain decomposition method (DDM) we used for an algorithm of investigation linear and nonlinear problems of the elasticity theory. The variant of DDM without of intersection of the subdomains for finite element method (FEM) and direct boundary element method (DBEM), and DDM with intersection of the subdomains for FEM and undirect boundary element method (UBEM) were constructed.

For the problems of the theory of elasticity were constructed the heterogeneous approaches for objects with thin coating and elastic inclusions. Local elastic-plastic fracture mechanic problem was solved with combined schemes. Completely parallel schemes were built for contact problems without friction. We proposed the numerical solution of these problems on the base of an iterative DDM (Dirichlet-Neumann scheme). The theory of Poincare–Steklov operators was used for investigation the convergence of the algorithms. An approach was developed allows us to perform a parallelization of computations, starting with the input information for each subregion, constructing a grid of finite or boundary element, forming local matrices in each of the subregions and solving the system of linear equations on each iteration of the linearization of a nonlinear problem. The algorithms were implemented with C ++ using parallel MPI library.

We proposed *h*–adaptive scheme error estimator for these problems which is based on comparison of FEM and BEM the stress results. The numerical analysis of the problems with isoparametric approximation and mortar functions has indicated that the mesh refinement performed by the algorithm, correctly reveals singularities of stress field near the contact area, and the total number of variables decreases considerably compared to uniform mesh case. The results of testing the proposed approach for modeling example are confirmed the perspective of the proposed

approaches. The validity of the algorithms was verified by solving the model problems.

Key Words: *h*–adaptive finite element method, boundary element methods, domain decomposition methods.

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A Dynamic Private Property Resource Game with Asymmetric Firms

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ABSTRACT

In this paper, we consider a non-cooperative linear state differential game in which two competing firms privately hold the same resource in order to produce a common good, as in Colombo and Labrecciosa (2013a,b), but we suppose that each firm can determine a different growth rate for the resource according to private technology owned by each firm. We are considering, for instance, a forest in which grows a single tree species. The growth rate of the trees depends in part on common natural factors for both firms (e.g. biological characteristics, climate, level of pollution) and in part on the technical efficiency of the firms (which results, for example, in the use of different tending or regeneration methods). We assume that there is one x-efficient firm, which is able to breed the trees to the maximum allowed rate given the "environmental conditions" and the other firm accusing a certain efficiency gap caused by a technology that does not allow it to take full advantage of the resource growth potential. Over time both the natural growth rate of the resource and the technological gap can vary. In particular, the variations of the natural resource growth rate may be positive or negative, both transient and permanent. As regards the technological gap, it is more realistic that it tends to shrink over time in a permanent way. We find an asymmetric linear FNE in which the player's strategy only depends on his own resource stock (regardless of the resource assigned to the other player). Then we show the optimal path obtained by the FNE both for the asset stock and for the firms' output levels and we carry on a short-run and a steady state sensitivity analysis w.r.t. the natural growth rate of the resource and the efficiency gap between the two firms. Finally we compare the steady state framework with the symmetric models with private (Colombo and Labrecciosa, 2013b) and with common resource (Benchekroun, 2003), also from a social welfare point of view.

Key Words: Feedback Nash Equilibrium; Differential Games; Resource exploitation; Private vs common property.

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A Model Suggestion in Order to Write Real Life Mathematics Questions in The PISA and TIMMS Character

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ABSTRACT

Even though the detachment of school mathematics from life has attracted researchers' attention for a long time, there has been an increase in the concentration on the subject since the 2000 as they have been taken into account the PISA results while examining the countries educational policies. While the problems in school mathematics focus on the basic skills of advanced mathematics, mathematical literacy problems, that is life-based problems, focus on the use of basic mathematical knowledge and skills (Steen, Turner and Burkhardt, 2007). Even though there is no definite mark between these two types of problems, while most of the routine problems in textbooks can be solved by recalling learned information, life problems are the problems that measure what can be done with the acquired knowledge, which requires the integration of knowledge and skills. Briefly, PISA applications are like saying "Keep what you have learnt for yourself, tell me what you can do!"

The problems in school books are often not real, but are those problems that are about the assumptions of the truth, and these questions are very easy to write. What has been addressed in this study is the preparation of the second type that is the life problems. The ability to write life problems regarding the PISA requires a good recognition of the characteristics of such questions as well as taking into account the general objectives of mathematics teaching. In addition to the classifications according to the (i) Subject areas (quantity, uncertainty, space and shape, change and relations), (ii) Contexts (personal, social, occupational, scientific) and (iii) Process skills (formulating, executing and interpreting and evaluating) available in OECD's own publications (2016), the articles that address the difficulties involved in writing the questions present serious opportunities in this regard.

Real life questions or problems differ from the conventional question structures by their being contextual, allowing for flexible thinking, argument building and defense requirements, decision making and modeling skills. These characteristics also suggest that in the background of life questions, education system should be constructivist. An education system through a constructivist approach with its nature of teaching students the skills to discuss ideas individually or as a group, deepening their thinking, and ultimately allowing them to produce the skills, naturally allows the skill by which life problems are questioned. The implicit conclusion drawn here is that it is important and necessary, even imperative, to implement the education system in a constructive form in writing real life questions.

The first three of the five phases of the 5E model (Bybee, 1997), which shows the way of implementation of the constructivist approach in teaching, (1) Drawing Attention, (2) Exploring, and (3) Explanation are interconnected and interrelated phases, where the activity presented at the "Exploring" stage is a phase allowing for thinking flexibly, generating an argument and defending it, or even insisting on it in its defense in the learning process, if it is necessary, modeling it. Modeling occurs as a result of the abstraction process as the most basic method in the production of mathematical knowledge. Constructivist education improves the modeling skills of students as it involves the abstract abstraction process by its very nature. Applications involving the acquisition of the details of the concept in the last two steps of the 5E model "Deepening" and "Evaluation" are implemented and serve at this stage for the real life questions to have a certain quality.

These statements reveal that establishing a constructivist approach in the education system constitutes a strong basis for the teacher to write real life questions. These statements also reveal that there is a background to the proposed model, and that this background "the education system should be in a constructivist character".

Real life questions are contextual. Contexts are starting points in writing question, and different dimensions of human life can be taken into account when creating the contexts. These real life situations can be classified as follows:

1) Situations that involve all kinds of decisions for the future

2) Situations that require the creation of mathematical modeling

3) Situations that involve the construction of mathematical knowledge

4) Drawing conclusion from contextual information, graphics, scales, banners, tabulated tables, etc.

5) Comprehension and interpretation of science

6) Clarifying the past and revealing the truth

7) Situations that explain natural phenomena

Taking this detail into account when writing a real life question or problem requires gathering information from scientific data sources, if necessary. The questions written by considering these sources are generally two or three-optioned closed-ended questions. In the first option, a question with numerical applications, which can easily be solved with the aim of introducing the student to the subject, is given. Students encounter the real life questions in the following options.

In addition to the methods regarding writing questions presented above, a practical method is to convert the questions in textbooks into real life questions. On the basis of this method is the "resistance to the demands of the question, resistance to the problem". "An empty pool is filled in 8 hours by the first faucet and in 24 hours filling by the second faucet. The question of “How many hours does it take fill the pool if both faucets are turned on together at the same time? " can eventually lead the real life questions with the ensuing discussion after the opposition with the following statements;

- Why should we fill it?
- Why should we calculate the filling time?

One example may be like this: "Ms. Ayfer heard the news on the radio that there will be water cut for 3 days starting at 17:00 that day. She decided to fill the water tank as a prevention measure for water cut. According to her past experiences, she knows that one of the two water sources of the house fills the water tank in 8 hours and the other source in 24 hours. It is 13:00 o’clock now! Does she have enough time to fill the water tank by using both sources?” There is no need for each question to be converted into a real life question, but this method is a rich source for writing questions.

A 30-40-hour training organized on the basis of both contexts and routine problems may be sufficient to improve the skill of writing real life questions.

Key Words: Mathematical literacy, Real life problems, Constructivist education

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On The Education and Teaching of Mathematics and Misconceptions

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ABSTRACT

The findings of recent years show that quality of education is not at the desired level in science and mathematics , even relatively declined.

In this presentation we will focus on some basic problems of education and teaching of secience and mathematics and suggest some solutions.

We'll also talk about misconceptions that could lead to a lifelong chain of mistakes in education and training. For instance, in mathematics, if the limit, which is the fundamental concept of analysis, is understood as "substituting" the given value, it leads to a misunderstanding of many related concepts.

So, the definition of any concept should be given fully and clearly.

Key Words: Keyword one, keyword two, keyword three.

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A Comparative Analysis of Clustering Algorithms

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ABSTRACT

Number of data increases and data analysis gains more importance day by day. Clustering is one of the most important stages of data analysis. There is no accepted definition of clustering by all researchers today. However, all researchers accept that the elements in the same cluster should be similar, elements in the different clusters should be different from one another and measures of similarities and dissimilarities should be sufficiently clear and practical. The clustering process usually has 4 steps: 1) Extraction and selection of features that represent the data set best 2) Design of the appropriate clustering algorithm for the problem examined 3) Evaluation of the results and validation of the designed algorithm 4) Interpretation of the results and practical explanation of the clusters.

In this presentation, a comparative analysis of clustering algorithms is made. Each of these algorithms has strengths and weaknesses. There is no guarantee that an algorithm that gives very good results for a specific problem will give similar results for another problem. For this reason, it is important to be able to select the appropriate algorithm for the problem considered. Therefore, the algorithms are needed to be well analyzed by comparing them with each other.

Key Words: Keyword one, keyword two, keyword three.

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ABSTRACTS OF ORAL PRESENTATIONS

ALGEBRA AND NUMBER THEORY

m-Adic Residue Codes Over $F_q[v]/(v^s - v)$

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ABSTRACT

Cyclic codes are very important since their algebraic properties, so cyclic codes are one of the most worked of algebraic codes. Quadratic residue codes are a significant class of cyclic codes. Generalizations of quadratic residue codes have studied by some researchers. In particular, m-adic residue codes are a generalization of these codes.

Pless and Brualdi have introduced polyadic codes and have worked have worked idempotent generators of the polyadic codes [3]. Afterwards, Job has defined m-adic residue codes in terms of generator polynomials over fields [4]. Furthermore, Goyal and Raka have worked quadratic residue codes over $F_q[v]/(v^m - v)$ [5]. Next, Kuruz et al. have worked m-adic residue codes over $F_q[v]/(v^2 - v)$ where q is a prime [2]. In this work, we study m-adic residue codes over the ring $F_q[v]/(v^s - v)$ and generalize our results in [2].

Let p be a prime and q be prime power such that p and q are coprime. Let b be a primitive element of Z_p^* and γ be a primitive p^{th} root of unity in some field extension of F_q. Let Q_0 be the set of nonzero m-adic residues modulo p and $Q_i = b^i Q_0$. If q is an m-adic residue modulo p, then the codes generated by polynomials $g_i(x) = \dfrac{x^p - 1}{\prod_{k \in Q_i} (x - \alpha^k)}$ $(i = 0, 1, \ldots, m-1)$ are called even-like families of m-adic residue codes of class I of length p over F_q.

We define the m-adic residue codes over $F_q[v]/(v^s - v)$ in this paper. This paper is a generalization of m-adic residue codes to the ring. We determine the structure of idempotents of m-adic residue codes over $F_q[v]/(v^s - v)$. We represent that the generators of m-adic residue codes over the ring are palindromic under some special conditions.

Key Words: m-adic residue codes, cyclic codes, quadratic residue codes

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2-Crossed Modules from Crossed Squares of Lie Algebras

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ABSTRACT

In this work, we explore the relation between crossed squares and 2-crossed odules of Lie algebras analogue to that given by Arvasi in the commutative algebra case (cf. [3]).

Ellis [6] captured the algebraic structure of a Moore complex of length 2 in his definition of a 2-crossed module of Lie algebras. Akça and Arvasi [1] explain the relationship among crossed modules and 2-crossed modules of Lie algebras [6], and thus by using the image of the higher order Peiffer elements in the Moore complex of a simplicial Lie algebra, in [1], they have constructed a functor from simplicial Lie algebras to 2-crossed modules of Lie algebras as an alternative way to Ellis' construction [6].

By using Artin-Mazur codiagonal functor ([2]), Arvasi in [3] constructed a neat description of the passage from a crossed square of commutative algebras to a 2-crossed module. In this paper, our main aim is to give a way from crossed squares of Lie algebras to 2-crossed modules of Lie algebras. To give this construction, in section 6, we give two ways of going from crossed squares to bisimplicial Lie algebras, and we construct a simplicial Lie algebra from this bisimplicial Lie algebra by applying the Artin-Mazur codiagonal functor, and we obtain a 2-crossed module structure of Lie algebras from the Moore complex of this simplicial Lie algebra.

Key Words: Simplicial Lie Algebra, Crossed Squares, 2-Crossed Module.

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A Generalization Of The Class Of Weakly Normal Rings

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ABSTRACT

Recently, some kinds of normality of rings have been investigated in the literature. For instance, the notion of quasi-normality of rings was defined and investigated by Wei and Li in [1], that is, a ring R is said to be quasi-normal if being ae = 0 implies that eaRe = 0 for every nilpotent element a and any idempotent element e of R. On the other hand, another kind of normality was introduced and studied again by Wei and Li in [2], so-called weakly normal rings, that is, a ring R is said to be weakly normal if for all elements a and r in R and any idempotent element e of R, being ae=0 implies that Rera is a nil left ideal of R. It is seen that the notion of a weakly normal ring is a generalization of the notion of a quasi-normal ring.

In the ring theory, the Jacobson radical of a ring is an important tool in studying the structure of the ring. In the light of before mentioned normality concepts, it is a reasonable question that what kind of properties does a ring gain when it satisfies certain normal property which turns out by its Jacobson radical? In this direction, motivated by the works on the concepts of the quasi-normal rings and weakly normal rings, we will deal with a generalization of the class of weakly normal rings by using the Jacobson radicals of rings.

Key Words: Quasi-normal ring, weakly normal ring, Jacobson radical of a ring.

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A Note On Convolved Fibonacci and Lucas Polynomials

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ABSTRACT

Fibonacci polynomials are a great importance in mathematics. Large classes of polynomials can be defined by Fibonacci-like recurrence relation and yield Fibonacci numbers. Such polynomials, called the Fibonacci polynomials were studied 1883 by the Belgian Mathematician Eugene Charles Catalan and the German Mathematician E. Jacobsthal.

The Fibonacci polynomials, $f_n(x)$ studied by Catalan are defined by the recurrence relation for $n \geq 2$, $f_n(x) = xf_{n-1}(x) + f_{n-2}(x)$ where $f_0(x) = 0$ and $f_1(x) = 1$. The Lucas polynomials, originally studied in 1970 by Bicknell, are defined by for $n \geq 2$ $l_n(x) = xl_{n-1}(x) + l_{n-2}(x)$, where $l_0(x) = 2$ and $l_1(x) = x$.

It is well known that the Fibonacci polynomials and Lucas polynomials are closely related. Obviously, they have a deep relationship with the famous Fibonacci and Lucas sequences. That is $f_n(1) = F_n$ and $l_n(1) = L_n$, where F_n and L_n are the Fibonacci and Lucas numbers.

Many kinds of generalizations of Fibonacci and Lucas polynomials and numbers have been presented in the literature. These polynomials are recursive sequences that generalize several polynomial and number sequences defined by recurrence relation of order two, such as Fibonacci numbers, Pell numbers, Jacobsthal numbers, Fibonacci polynomials, Lucas polynomials, Pell polynomials and so on.

In this paper, we define the extended convolved generalized Fibonacci and Lucas polynomials. Then, we give some recurrence relations for these polynomials. Finally, as applications we obtain some formulas of mixed-multiple sums for generalized Fibonacci and Lucas polynomials.

Key Words: Fibonacci polynomials, Lucas Polynomials, Convolved Fibonacci Polynomials.

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A Note On Nullnorms On The Unit Interval

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ABSTRACT

Nullnorms are generalizations of t-norms and t-conorms with zero element to an arbitrary point from a bounded lattice. Nullnorms are useful tool in many different fields, such as expert systems, neural networks and fuzzy logic. Also, they have been used as aggregators in fuzzy logic in order to maintain as many logical properties as possible. Lately, the notation of the order induced by t-norms, nullnorms and uninorms has been studied widely. First the T-partial order obtained from a t-norm was defined by [5]. Based on these previous studies, the orders denoted by the U-partial order and F-partial order obtained from the uninorm and nullnorm were defined by [4] and [1], respectively. Since the F-partial order is an extension of the T-partial order and S-partial order, then it is important to study the F-partial order to obtain more general conclusions.

In this study, we define an equivalence relation on the class of nullnorms on the unit interval [0,1] and we determine the equivalence classes of the smallest and greatest nullnorms on the unit interval. In [6], [8], the concept of a conjugate of a t-norm was introduced. In the present study, we introduce a conjugate of a nullnorm on the unit interval and we gave a sufficient and necessary condition to ensure that a nullnorm and its conjugate nullnorm are equivalent.

Key Words: Nullnorm, partial order, unit interval.

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About Ordering Based On Nullnorms On Bounded Lattices

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ABSTRACT

The triangular norms (t-norms for short) with 1 as neutral element and triangular conorms (t-conorms for short) with 0 as neutral element were introduced by Schweizer and Sklar in [8]. These operators play an important role in many different theoretical and practical fields, e.g., decision making theory, fuzzy set theory, integration theory, etc. Nullnorms and t-operators were introduced by Calvo, De Baets, Fodor [4] and Mas, Mayor, Torrens [7], respectively. Nullnorms are a generalization of t-norms and t-conorms that were studied more applications. Nullnorms allow the freedom for the zero element a (sometimes called absorbing) to be an arbitrary element from unit interval [0,1], which is 0 for t-norms and 1 for t-conorms. And then Mas, Mayor and Torrens [7] have shown that nullnorms and t-operators are equivalent since they have the same block structures in $[0,1]^2$. That is, if a binary operator F is a nullnorm, then it is also a t-operator and vice versa.

In this paper, we study some properties of an order induced by nullnorms on bounded lattices. We investigate monotonocity property of nullnorms on bounded lattices with respect to the F-partial order. Aşıcı and Karaçal [2] have shown that for the t-norms T_W and $T_\wedge$ on L, T_W is the order-weakest and $T_\wedge$ is the order-strongest t-norms. In this study, we show that for the nullnorms, it need not be that case. That is, smallest and greatest nullnorm on bounded lattice is not order-weakest and order-strongest nullnorm, respectively.

Key Words: Nullnorm, partial order, bounded lattice.

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An Application of Constacyclic Codes to Entanglement Assisted Quantum Codes

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ABSTRACT

Quantum error correcting codes have been developed by researchers to detect and correct the quantum errors occurring in quantum channels during quantum information transfer. There are several constructions for quantum error correcting codes, the important one of which is the Hermitian construction. To construct a quantum error correcting code via Hermitian construction, it is enough to find a Hermitian self-orthogonal linear code over finite field F_{q^2} of order q^2, where q is a prime power.

A basic and principal family of quantum codes is entanglement-assisted quantum codes (EAQCs), which is introduced by Brun *et al.* in [1]. According to the construction given by Brun *et al.* in [1], the requirement of Hermitian self-orthogonality for linear codes over F_{q^2}, which allows us to quantize all linear codes over F_{q^2}. An EAQC of length n and minimum distance d is denoted by $n,k,d;c_{\ q}$ and this EAQC encodes k qubits to n channel qubits via c pairs of maximally entanglement states and corrects up to $\left\lfloor \frac{d-1}{2} \right\rfloor$ errors.

Recently, the construction of entanglement-assisted quantum codes have considered intensively by scholars [2-5]. However, determining the number of shared pairs required for constructing entanglement-assisted quantum codes is a hard task. In this study, by making use of the notion of decomposition for defining sets of constacyclic codes, we construct several new families of entanglement-assisted quantum MDS codes.

Key Words: Entanglement-assisted quantum codes, constacyclic codes, defining sets.

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An Application of Interval-valued Neutrosophic Soft Graphs in a Decision Making Problem

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ABSTRACT

Smarandache (2006) grounded the concept of neutrosophic set theory from philosophical point of view by incorporating the degree of indeterminacy or neutrality as independent component to deal with problems involving imprecise, indeterminate and inconsistent information. The concept of neutrosophic set theory is a generalization of the theory of fuzzy set, intuitionistic fuzzy sets and interval-valued fuzzy sets. The interval valued neutrosophic soft sets constitute a generalization of interval-valued fuzzy soft set theory. The interval-valued neutrosophic soft models give more sensitive, flexibility and conformity to the systems as compared to the interval-valued fuzzy soft models. Graph theory is an extremely useful mathematical tool to solve the complicated problems in different fields. Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. In this study, we applied the concept of interval-valued neutrosophic soft sets to graph structures and describe method of their construction. We also handle an application of interval-valued neutrosophic soft graphs in a decision making problem and then give an algorithm for the selection of optimal object based on given sets of information.

Key Words: Neutrosophic soft set, graph theory, decision-making problem.

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An Approach to Semicommutativity of Rings via Idempotents

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ABSTRACT

A ring R is called semicommutative if for any a, b in R ab=0 implies aRb=0, this ring is also called ZI ring in [1], while, in [2], R is said to be I-semicommutative if ab=0 implies aRb is a subset of I, where I is an ideal of R. Another generalization is made in [3], in which a ring R is called P-semicommutative if ab=0 implies aRb is a subset of P(R) for any a,b in R, where P(R) is the prime radical of R.

In this work, a new kind of rings which behave like semicommutative rings is considered. These are called e-semicommutative rings. That is, a ring R with an idempotent e of R is called e-semicommutative provided ab=0 implies aRbe=0 for any a, b in R.

We investigate properties of this class of rings. We give some examples and determine the structure of e-semicommutative rings. In [4], e-symmetric rings are introduced and investigated. A ring R is called e-symmetric if whenever abc=0, then acbe=0 for every idempotent e and for any a,b,c in R. We also obtain relations between the classes of e-semicommutative rings and certain rings such as semicommutative rings, e-symmetric rings.

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Key Words: Semicommutative ring, P-semicommutative ring, e-symmetric ring.

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An Investigation Of Soft Cryptosystem

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ABSTRACT

In 1999, the flexible set theory introduced by Molodtsov appears to be an effective mathematical tool to deal with uncertainty. This theory was applied to many areas of uncertainty such as information systems, decision making problems, optimization theory, algebraic structures and mathematical analysis. Soft set theory has continued to experience tremendous growth in the mean of algebraic structures since Aktaş and Çağman [5] defined and studied soft groups, soft subgroups, normal soft subgroups, soft homomorphisms, adopting the definition of soft sets. In [4], the same authors introduced two new operations on soft sets, called inverse production and characteristic production depending on the relation forms of soft sets and obtained two isomorphic abelian groups called "the inverse group of soft sets" and "the characteristic group of soft sets". In this study, we redefine the operations inverse and characteristic products of soft sets without using relation forms of soft sets. This leads to simplicity and brevity. Also, we construct two ring structure consisting the representations of uncertain objects as elements. The inverse product and characteristic product defined on soft matrices was used in soft encryption and soft decryption

Key Words: Soft sets, group structure, ring structure, inverse product, characteristic product

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An Overview Of Methods Generalizing Uninorms

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ABSTRACT

The definition of uninorms was first introduced by Yager and Rybalov in 1996 [8]. These operators were comprehensively studied by Fodor, Yager and Rybalov in 1997 [6]. Uninorms on the real unit interval, that are an important generalization of triangular norms and triangular conorms, allow the freedom for the neutral element (sometimes called identity) to be an arbitrary element from the real unit interval. Uninorms with the neutral element 0 are known as triangular conorms and uninorms with the neutral element 1 are known as triangular norms. They have been extensively used in several applications in fuzzy set theory, fuzzy logic, multicriteria decision support and several branches of information sciences. They play an important role not only in theoretical investigations but also in practical applications.

Uninorms on a bounded lattice were introduced by Karaçal and Mesiar in 2015 [7]. Two construction methods are proposed in order to the presence of uninorms on an arbitrary bounded lattice with a neutral element. These methods base on the presence of a triangular norm and a triangular conorm for an arbitrary bounded lattice. By means of these construction methods, the smallest uninorm and the greatest uninorm on bounded lattices are obtained.

In this contribution, we discuss the structure of uninorms defined on bounded lattices. We present a new related construction method to generalize uninorms on an arbitrary bounded lattice having a neutral element. Our method is different from the proposal of Karaçal and Mesiar [7]. In order to show this fact, we provide an illustrative example.

Key Words: Bounded lattice; Uninorm; Neutral element; t-norm; t-conorm

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Binomial Transforms and Its Applications

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ABSTRACT

There are so many studies in the literature that concern about the special number sequences such as Fibonacci, Lucas, Pell, Pell-Lucas and so on. Fibonacci and Lucas numbers have long interested mathematicians for their intrinsic theory and applications. For rich applications of these numbers in science and nature one can see the citations in [1]. For instance the ratio of two consecutive of these numbers converges to the golden section $\alpha = \dfrac{1+\sqrt{5}}{2}$.

The well-known Fibonacci and Lucas sequences are defined by $F_n = F_{n-1} + F_{n-2}$ and $L_n = L_{n-1} + L_{n-2}$, respectively. Fibonacci numbers have been generalized in many ways.

Some matrix transform can be introduced for a given sequences. Binomial transform is one of the these transform, there are also otherwise such as falling and rising binomial transforms [2-5]. The binomial transform B of the integer sequence $A = \{a_n\}$ which is denoted by $B(A) = \{b_n\}$ and defined by $b_n = \sum_{i=0}^{n} \binom{n}{i} a_i$.

In literature, there are many authors have been studied binomial transform. For instance, in [2] Falcon and Plaza applied the binomial transform to the $k-$Fibonacci sequences. In Bhadouria et al. investigated binomial transform of $k-$Lucas sequences using the similar method to [5].

In this paper, we investigated the binomial transform of some special numbers. Then we give recurrence relation, Binet's formula and generating function respectively. Finally, we define a matrix sequence consisting of special numbers and investigate the binomial transform for these new sequences.

Key Words: Binomial Transforms, Special numbers, Matrix sequences.

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Change Base For Reduced Quadratic Lie Algebras

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ABSTRACT

Quadratic modules have been defined by Baues, [2], as an algebraic models for homotopy 3-types. These structure can be thought as 2-dimensional case of crossed modules introduced by Whitehead. The commutative and Lie algebra version of quadratic modules have been studied by Arvasi and Ulualan , [1]. In earlier works of Ulualan, for the quadratic modules of non-abelian groups the change base functor has been constructed. The pullback and induced quadratic modules over groups give these change base functor. Brown and Sivera in [4] proved that the functor Φ_1 :**XMod** →**Gpd** from crossed modules of groupoids to that of groupoids, mapping a crossed module $A \to B$ to its base groupoid B, is a bifibration of categories. For the case of crossed modules of groups, a similar result was appeared in [3] and pursued in [5,6]. In the crossed modules category in Lie algebras and commutative algebras, analogous constructions were given in [7] and [8], respectively. In this work, we give the notion of fibrations and cofibration structure for the forgetful functor from the category of reduced quadratic modules of Lie algebras to the category of Lie algebras whose the class of nilpontency degree is 2. By taking a constant object I in the category of nil(2)-Lie algebras and a morphism $f:J \to I$ in the same category, we will construct a change base functor by constructing the pullback structure for reduced quadratic Lie algebras. Thus we will define the functor from the category of Lie algebras with the same base I to the category of reduced quadratic Lie algebras with the same base J.

Key Words: Quadratic Modules, Crossed Modules, Pullback and pushout.

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Characterization of L-Fuzzy Soft Modules

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ABSTRACT

To solve complicated problems in economics, engineering and environment, we cannot succesfully use classical mathematic methods because of various uncertainties typical for those problems. Uncertainty takes place almost everywhere in our daily life. There are a number of theories have been presented tackle these uncertainty such as fuzzy set theory, rough set theory, vague set theory etc. After these theories, Molodtsov presented the notations of soft set theory in 1999. Soft set is new mathematical tool for dealing with uncertainties that is free from the difficulties affecting existing methods. The works on soft set theory has been progressing rapidly with a wide range- applications not only in the mean of algebraic structures but also in the structures of soft sets, operations of soft sets In this paper, we introduce concepts of L-fuzzy soft module and L-fuzzy soft submodule. We study some algebraic properties of L-fuzzy soft modules and give some related examples of them. Also, we investigate the relation between concepts of L-fuzzy soft module and soft module, and then we show that all L-fuzzy soft modules can be characterized by making use of soft modules. Moreover, we define notions of L-fuzzy soft module homomorphism and L-fuzzy soft module isomorphism and derive their basic properties.

Key Words: L-fuzzy subset, soft module, L-fuzzy soft module.

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Classification of Optimal Additive Toeplitz Codes Over GF(4)

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ABSTRACT

We denote $GF(4)=\{0,1,w,w^2\}$, where $w^2=w+1$. An additive code C over $GF(4)$ of length n is an additive subgroup of $GF(4)^n$. C contains 2^k codewords for some $0\leq k\leq 2n$, and can be defined by a $k\times n$ generator matrix, with entries from $GF(4)$, whose rows span C additively. An additive code with minimum distance d is called an $(n,2^k,d)$ code. If a code has highest possible minimum distance, denoted by $d_{\max}$, it is called optimal.

We say that two additive codes C_1 and C_2 over $GF(4)$ are equivalent provided there is a map sending the codewords of C_1 onto the codewords of C_2 where the map consists of a permutation of coordinates (or columns of the generator matrix), followed by a scaling of coordinates by nonzero elements of $GF(4)$, followed by conjugation of some of the coordinates. The conjugation of $x\in GF(4)$ is defined by $\bar{x}=x^2$. For a code of length n, there is a total of $6^n n!$. Gaborit et al. [3] determined the equivalence or inequivalence of two additive codes over $GF(4)$ by an algorithm.

In this study, we introduce additive toeplitz codes over $GF(4)$. The additive toeplitz codes are a generalization of additive circulant codes over $GF(4)$. We provide some theorems to partially classify optimal additive toeplitz codes (OATC). Then, we give a new algorithm that fully classifies OATC by combining these theorems with Gaborit's algorithm. We classify OATC over $GF(4)$ of length up to 13, except for length 9. We obtain 2 inequivalent optimal additive toeplitz codes (IOATC) that are non-circulant codes of length 5, 92 of length 8, and 39 of length 11. Moreover, we construct 49 IOATC that are non-circulant codes of length 9.

Key Words: Additive codes, Additive circulant codes.

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Cofinitely ⊕-Supplemented Lattices

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ABSTRACT

In this work, cofinitely ⊕-supplemented lattices is defined and some properties of these lattices are investigated. All lattices are complete modular lattices in this work. Let L be a lattice and $1 = \oplus_{i\in I} a_i$ with $a_i \in L$ for every $i\in I$. If $a_i/0$ is cofinitely ⊕-supplemented for each $i\in I$, then L is also ⊕-supplemented.

Some Results

Definition 1 Let L be a lattice. If every cofinite element of L has a supplement in L that is a direct summand of L, then L is called a cofinitely ⊕-supplemented lattice.

Proposition 2 Let L be a lattiice. Then L is cofinitely ⊕-supplemented if and only if for every cofinite element a of L, there exists a direct summand b of L such that $1 = a \vee b$ and $a \wedge b << b/0$.

Proposition 3 Let L be a cofinitely supplemented lattice. Then $1/r(L)$ is cofinitely ⊕-supplemented.

Proposition 4 Let L be a cofinitely ⊕-supplemented lattice, $a\in L$ and $a =(a\wedge m)\oplus(a\wedge n)$ for every $m,n\in L$ with $m\oplus n = 1$. Then $1/a$ is cofinitely ⊕-supplemented.

Key Words: Lattices, Small Elements, Supplemented Lattices, Complemented Lattices.

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Cofinitely ss-Supplemented Modules

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ABSTRACT

The goal of this talk is to give and study a generalization of ss-supplemented modules. First we introduce the concept of modules whose maximal submodules have ss-supplements and then we give some new results in these modules. We show that (1) Every factor module of an (amply) cofinitely ss-supplemented module is (amply) cofinitely ss-supplemented module; (2) If M is a cofinitely ss-supplemented modules with finitely generated M/Rad(M) then M/Rad(M) is ss-supplemented and semisimple; (3) Arbitrary sum of cofinitely ss-supplemented modules is cofinitely ss-supplemented;(4) If M is a cofinitely ss-supplemented module then any M-generated module is cofinitely supplemented; (5) Let M be a cofinitely ss-supplemented R-module. Then every cofinite submodule of M/Rad(M) is a direct summand ; (6) A module M is (amply) cofinitely ss-supplemented if and only if every maximal submodule of M has an (ample) ss-supplement(s) in M; (7) A π-projective cofinitely ss-supplemented module is amply cofinitely ss-supplemented; (8) The left R-module R is amply cofinitely ss-supplemented if and only if R is semiperfect and Rad(R)≤Soc(R) if and only if R is semilocal and Rad(R)≤Soc(R) if and only if every left R-module is (amply) cofinitely ss-supplemented.

Key Words: Semiperfect ring, supplements, ss-supplements.

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Construction of The Binary Simplex Codes and The First Order Reed-Muller Codes

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ABSTRACT

$GF(q)$ denote the finite field with q elements. An $[n,k,d]$ linear code C over $GF(q)$ is a k-dimensional subspace of $GF(q)^n$ with minimum (Hamming) distance d. The vectors in C are codewords of C. Specially, codes over $GF(2)$ are called binary linear codes.

Let $q=2^m$ for some positive integer m, and Tr denote the trace map from $GF(q)$ onto $GF(2)$. For $D=\{d_1,d_2,\ldots,d_n\}\subseteq GF(q)^*$, we define a linear code of length n over $GF(2)$ by

$$C_D=\{(Tr(xd_1),Tr(xd_2),\ldots,Tr(xd_1)):x\in GF(q)\},$$

and call D the defining set of this code C_D. This construction approach of the linear codes was employed by Cunsheng Dings in [1] and [2] for obtaining linear codes with a few weights. Different orderings of the elements of D result in different codes C_D, but the codes are permutation equivalent.

A Hamming code is a linear code for error detection that can detect up to two simultaneous bit errors and is capable of correcting single-bit errors. The duals of the Hamming codes are simplex codes. A code is called constant-weight code if all nonzero codewords of this code have the same weight. The simplex codes are constant-weight codes. Reed-Muller codes are amongst the oldest and most well-known of codes. All codewords, except $(0,0,\ldots,0)$ and $(1,1,\ldots,1)$ codewords, of the first order Reed-Muller code have the same weight.

The objective of this study is to produce the known two binary linear codes by selecting the defining set $D \subseteq GF(q)^*$. The first code is the binary simplex code. The second code is the first order Reed-Muller code.

Key Words: Binary simplex codes, Reed-Muller codes.

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Domination Number of Some Trees

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ABSTRACT

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v) = \{u| \ (u, v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set, if every vertex in G either is element of S or is adjacent to at least one vertex in S. The domiantion number of a graph G is denoted with $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. Fundamental notions of domination theory are outlined in the book [1].

A subset $S \subseteq V$ is a total dominating set, if every vertex in G is adjacent to a vertex of S and the total domination number $\gamma^t(G)$ is the minimum cardinality of a total dominating set.

A vertex v ve-dominates an edge e which is incident to v, as well as every edge adjacent to e. A set $S \subseteq V$ is a ve-dominating set if every edges of a graph G are ve-dominated by at least one vertex of S. The minimum cardinality of a ve-dominating set is named with ve-domination number and denoted with $\gamma_{ve}(G)$

Dendrimers are highly branched trees [2]. A regular dendrimer $T_{k,d}$ is a tree with a central vertex v. Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ ank the radius is k, distance from v to each pendant vertex. In this study we obtain the domination number of regular dendrimers.

Key Words: Domination number, Regular dendrimer.

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Encryption Of Letters Onto Each Other

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ABSTRACT

The privacy of texts has become a problem since the writing was found. Several ways have been tried preventing undesired people reading and making texts much safer (picture, match etc). Its mathematical state is called cryptology. Cryptology, based on mathematics, provides privacy in information. As the discovery of the computer and the widespread use of the internet, the traditional communication place has left electronic communication. Together with "security concept" becoming more important for transactions in electronic environments. Today, cryptography has become an important part of everyday life, as it is involved in a wide range of applications. For example; sim cards, mobile phones, remote controls, online banking, online shopping, satellite receivers, e-signature applications can be given. This work is aimed at creating an alternative function that is resistant to frequency analysis that can be used especially for communication on the internet. The support of the function to be formed in this context is that the number of letters in Turkish alphabet is prime. In that the number of letters is prime, any number can be found inversely to multiplication in number of letters mode. This function can be adapted to any alphabet containing a prime number of letters except for the Turkish alphabet. When looking at the contents of the function, each letter is coded together with the previous letter so that it is avoided to decipher the other letters without any letters being deciphered. So that the chain was formed between the letters. Further, fractional function was used to expand the key space. The ASCII table which is a 7-bits computer language, was used in accordance with the characters in our language so that the function can run at the required speed. As a result of the study, a crypto function which is adapted to computer language, resistant to frequency analysis, key

space wide and secure is obtained. This function can be used to provide confidentiality in binary communication, or it can work as a programming language by working on it.

Key Words: ASCII, encryption, function, modular arithmetic.

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E-Radical Supplemented Modules

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ABSTRACT

In this work, all modules are unital left modules. Let M be an R-module. If every essential submodule of M has a Rad-supplement in M, then M is called an e-radical supplemented (or e-Rad-supplemented) module. In this work, some properties of e-radical supplemented modules are investigated. Let M be an R-module and $M = M_1 + M_2 + ::: + M_n$. If M_i is e-radical supplemented for every $i = 1, 2, ..., n$, then M is also e-radical supplemented.

Some Results

Definition 1 Let M be an R-module and $X \leq M$. If X is a Rad-supplement of an essential submodule of M, then X is called an e-radical supplement (or e-Rad-supplement) submodule in M.

Lemma 2 Let M be an R-module, V be an e-radical supplement in M and $x \in V$. Then $Rx \ll_g M$ if and only if $Rx \ll_g V$.

Corollary 3 Let M be an R-module and V be an e-radical supplement in M. Then $Rad_g V = V \cap Rad_g M$.

Proposition 4 Let M be an e-radical supplemented module. Then $M = RadM$ have no proper essential submodules.

Lemma 5 Let M be an R-module, U be an essential submodule of M and $M_1 \leq M$. If M_1 is e-radical supplemented and $U + M_1$ has a Rad-supplement in M, then U has a Rad-supplement in M.

Key Words: Small Submodules, Radical, Supplemented Modules, Radical (Generalized) Supplemented Modules.

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Extension Of (α, k, b) −Gamma And (α, k, b) −Beta Functions For Conformable Integrals

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ABSTRACT

For a complex number x with $Re x > 0$ define

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt.$$

$\Gamma(x)$ is called the gamma function. Two fundamental properties of the gamma function are that

$$\Gamma(x+1) = x\Gamma(x)$$

and

$$\Gamma(n) = (n-1)!$$

where x is a complex number with positive real part and $n \in \mathbb{N}^+$.
Another important fact is that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Next we define the beta function. Fix x and y complex numbers with positive real parts. We define

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt = \int_0^1 t^{y-1}(1-t)^{x-1}\, dt$$

We have the following relationship between the gamma and the beta functions:

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

when x and y have positive real parts.

For the history and main results on Special function and Conformable Fractional Calculus, we refer the reader to $[1,\ 3-\ 5]$.

In this work, we introduce to extended (α, k) −gamma and extended (α, k) −beta functions. We prove several conformable integral inequalities

generalizing those satisfied by the (α, k) −gamma and (α, k) −beta function. Also, we prove the log-convexity of the (α, k, b) −gamma and (α, k, b) −beta functions.

The Conformable extended gamma function $\Gamma_{b,k}^{\alpha}$ is defined by

$$\Gamma_{b,k}^{\alpha}(x) = \int_0^{\infty} t^{x-1} e^{-\frac{t^{\alpha k}}{\alpha k} - \frac{b^{\alpha k} t^{-\alpha k}}{\alpha k}} d_{\alpha} t$$

where $b \geq 0$, $k > 0$. Clearly, $\Gamma(x) = \lim_{(\alpha,k,b)\to(1,1,0)} \Gamma_{b,k}^{\alpha}(x)$.

The function $\Gamma_{b,k}^{\alpha}(x)$ satisfies the commutative diagram:

$$\begin{tikzcd} \Gamma_{b,k}^{\alpha}(x) \arrow[r, "b \to 0"] \arrow[d, "\alpha \to 1"'] & \Gamma_{k}^{\alpha}(x) \arrow[d, "\alpha \to 1"] \\ \Gamma_{b,k}(x) \arrow[r, "b \to 0"'] & \Gamma_{k}(x) \end{tikzcd}$$

This gives rise to Conformable extended beta function defined by

$$B_{b,k}^{\alpha}(x, y; b) = \frac{1}{\alpha k} \int_0^{\infty} t^{\frac{x}{\alpha k}-1} (1-t)^{\frac{y}{\alpha k}-1} e^{-\frac{b^{\alpha k}}{\alpha k t(1-t)}} d_{\alpha} t$$

(where $Re(x) > 0$, $Re(y) > 0$, $Re(b) \geq 0$, $k > 0$).

When $(\alpha, k, b) \to (1, 1, 0)$, it turns out to be the usual gamma function and beta function [2].

Key Words: k −gamma function, k −beta function, (α, k) −gamma and (α, k) −beta functions, extended gamma function, extended beta function, conformable fractional integrals.

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Finding Characteristic Roots of Matrices of Even Lucas and Fibonacci Numbers

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ABSTRACT

The Fibonacci sequence F_n and the Lucas sequence L_n are number sequences consisting of the result of the addition of each number to the previous one. For Fibonacci sequence two initial values are 0 and 1, for Lucas sequence two initial values are 2 and 1.

Continued fractions can be thought of as a generalization of compound fractions. The Euclidean algorithm is used to find the largest common divisor of two numbers. It is known that this algorithm leads to the development of the continued fraction theory due to its close relation with continued fractions.

In [4] A.H. Değer examined the relation between continued fractions and Fibonacci numbers. So from the matrix connections of continued fractions he found the matrix that gives the even Fibonacci numbers F_{2n}. In our previous work by this motiviation we found the matrix that gives the even Lucas numbers L_{2n}. For even Fibonacci numbers the matrix is

$$S = \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}^n = \begin{bmatrix} (-1)^{n-1}F_{2n-2} & (-1)^n F_{2n} \\ (-1)^{n+1}F_{2n} & (-1)^n F_{2n+2} \end{bmatrix}.$$

For even Lucas numbers the matrix is

$$T = \begin{bmatrix} -3 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}^n = \begin{bmatrix} (-1)^{n-1}L_{2n-2} & (-1)^n L_{2n} \\ (-1)^{n+1}L_{2n} & (-1)^n L_{2n+2} \end{bmatrix}.$$

In this work firstly we examine the determinants of T and S matrices. And then by using the characteristic equations of T and S matrices, we find the characteristic roots of these matrices. To find these matrices we use some Fibonacci and Lucas identities.

Key Words: Fibonacci Sequence, Lucas Sequence, Continued Fractions

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Fuzzy Roughness in LA-Γ-Semigroups

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ABSTRACT

In this paper we give the definition of the notion of fuzzy set valued homomorphism for LA-Γ-semigroups and mention some properties of them. We examine the relation between a set valued and a fuzzy set valued homomorphism of LA-Γ-semigroups. It is demonstrated that by using the level sets, a set valued homomorphism is derivable from a fuzzy set valued homomorphism and vice versa is also true. We also explain the connection between a fuzzy set valued homomorphism and a fuzzy relational morphism. It is obtained that the composition of two fuzzy set valued homomorphisms is a fuzzy set valued homomorphism of LA-Γ-semigroups.

Furthermore this paper offers an investigation on the rough approximations of a generalized fuzzy approximation space constructed on an LA-Γ-semigroup and formed by a fuzzy set valued homomorphism of LA-Γ-semigroups. Especially, we focus on some algebraic features of fuzzy subsets in point of maintaining of some properties under the fuzzy lower and upper rough approximations. It is deduced that a fuzzy LA-Γ-subsemigroup of a semigroup under the upper rough approximation is a fuzzy LA-Γ-subsemigroup. In some theorems connected with the lower rough approximations which are formed by R-implications and also S-implications, as hypotheses, some conditions are determined for a fuzzy set valued homomorphism of LA-Γ-semigroups. It is demonstrated that a fuzzy LA-Γ-subsemigroup of a semigroup under the lower rough approximation is a fuzzy LA-Γ-subsemigroup according to these conditions. Similar problems are examined for other fuzzy concepts of LA-Γ-subsemigroup.

Key Words: (Fuzzy) set valued homomorphism, Fuzzy lower and upper rough approximations, LA-Γ-semigroup.

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Generalized Fuzzy Soft Groups

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ABSTRACT

Uncertainty takes place almost everywhere in our daily life. There are a number of theories have been presented tackle these uncertainty such as fuzzy set theory, rough set theory, vague set theory etc. In 1999, Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting the existing methods. Some researches have studied algebraic properties of fuzzy soft sets. Firstly, Maji et al. (2001) defined fuzzy soft set and established some results on them. This study aims to extend the notion of group to inside the algebraic structures of L-fuzzy soft sets. We firstly give some new notions such as product, extended product, restricted product of two L-fuzzy soft sets. By using these new notions we then introduce concept of L-fuzzy soft groups and study some of their properties. We also compare L-fuzzy soft groups to the related concept of soft groups and then we show that L-fuzzy soft groups are more general concept than soft groups. Furthermore, we obtain a necessary and sufficient condition to be L-fuzzy soft group of a given L-fuzzy soft set. We finally define L-fuzzy soft function and L-fuzzy soft group homomorphism, and then give theorem of homomorphic image and homomorphic pre-image under a L-fuzzy soft function.

Key Words: L-fuzzy subset, soft group, L-fuzzy soft group.

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G-Supplemented Lattices

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ABSTRACT

In this work, we firstly define g-supplemented lattice and characterize some fundamental properties of these lattices. Also we investigate the relationship between supplemented lattices and g-supplemented lattices. We characterize g-supplemented lattices via supplemented lattices. All lattices are complete modular lattices. Let L be a lattice. If every element of L has a g-supplement in L, then L is called a g-supplemented lattice. Every supplemented lattices also g-supplemented.

RESULTS

Definition 1 *Let L be a lattice and $a, b \in L$. If $1 = a \vee b$and $1 = a \vee t$ with $t \trianglelefteq b/0$ implies that t=b, then b is called a g-supplement of a in L. If every element of L has a g-supplement in L, then L is called a g-supplemented lattice.*

Lemma 2 *Let L be a lattice and $a, b \in L$. Then b is a g-supplement of a in L if and only if $1 = a \vee b$ and $a \wedge b \ll_g b/0$.*

Lemma 3 *Let L be an lattice, $a, b \in L$ and b/0 be g-supplemented. If $a \vee b$ has a g-supplement in L, then a also has a g-supplement in L.*

Proposition 4 *Let L be a lattice, $a_1, a_2 \in L$ and $1 = a_1 \vee a_2$. If $a_1/0$ and $a_2/0$are g-supplemented, then L is also g-supplemented.*

Corollary 5 *Let L be a lattice, $a_1, a_2, \dots, a_n$ and $1 = a_1 \vee a_2 \vee \dots \vee a_n$. If $a_i/0$ is g-supplemented for every $i = 1, 2, \dots, n$ then L is also g-supplemented.*

Key Words: Essential Elements, Radical, Small Elements, Supplemented Lattices.

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Interval-valued Neutrosophic Soft Graphs

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ABSTRACT

The concept of neutrosophic set which is a powerful mathematical tool for dealing with incomplete, indeterminate andinconsistent information was firstly proposed by Smarandache in 2006. Neutrosophic sets are generalization of the theory of fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. The neutrosophic sets are characterized by a truth-membership function *T*, an indeterminacy-membership function *I* and a falsity membership function *F* independently, which are within the real standard or nonstandard unit interval $]^{-}0,1^{+}[$. Graph theory was firstly introduced by Euler in 1736. A graph is used to create a relationship between a given set of elements. Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. The interval-valued neutrosophic soft soft models give sensitive, flexibility and conformity to the systems. In this study, we introduce notations of interval-valued neutrosophic soft graph and complete interval-valued neutrosophic soft graph and investigate some related properties of them. We then discuss various methods of their construction and investigate some of their related properties. We also present several different types operations including cartesian product, union and intersection on interval-valued neutrosophic soft graphs and give many related examples.

Key Words: Interval-valued neutrosophic set, Interval-valued neutrosophic soft set, graph theory.

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Locally Nilpotent p-Groups Whose Proper Subgroups Are Hypercentral-by-Chernikov

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ABSTRACT

If $\mathcal{X}$ is a group theoretical property or class of groups then a group G is a $\mathcal{X}$-group if G has the property $\mathcal{X}$ or is a member of the class $\mathcal{X}$. Let G be a group and $\mathcal{X}$ be a property of groups. If every proper subgroup of G satisfies $\mathcal{X}$ but G itsellf does not satisfy it, then G is called a minimal non-$\mathcal{X}$group (We denote the classes of minimal non-$\mathcal{X}$ group by $MN\mathcal{X}$-group). In this work we study locally nilpotent minimal non-$\mathcal{X}$ groups, where $\mathcal{X}$ stands for hypercentral-by-Chernikov. It was shown in [1] that if N be a normal nilpotent subgroup and U be a nilpotent subnormal subgroup of any group then NU is nilpotent. In this study, a generalizations of this situation was given. Let N be a normal an N0 closed $\mathcal{X}$ subgroup (see [2]) of G for a class N0-closed $\mathcal{X}$of groups and U be an N0-closed $\mathcal{X}$ subnormal subgroup of G. Then UN is an N0-closed $\mathcal{X}$ subgroup of G. In addition, the results for the nilpotent-by-Chernikovgroups of Asar [3] were also extended to hypercentral-by-Chernikov groups in this study. Thus, the following results were obtained. Let G be a Baer p-group and every proper subgroup is N0 closed $\mathcal{X}$-by-Chernikov for a class N0 closed $\mathcal{X}$. Then every proper subgroup of G which is generated by a subset of finite exponent is N0 closed $\mathcal{X}$. Also we show that if G is a Baer p-group and G has a normal hypercentral N subgroup such that G/N Chernikov. Then G/N is nilpotent.

Key Words: $\mathcal{X}$-group, $\mathrm{MN}\mathcal{X}$-groups, hypercentral-by-Chernikov grup, Bare group.

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Mathematic Teaching With Short Film: First Order Equation With One Unknown

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ABSTRACT

The equations that play an important role in the transition from arithmetic to algebra affect students' advanced algebra learning significantly. For this reason, first-order unknown equations have an important place in the secondary school mathematics curriculum. However, the topic of equations is not understood by most students [1]. It has been seen that students have many reasons for difficulty in understanding the equations.

These are as following; misunderstandings of equations [2], can not simplify algebraic expressions, difficulties in the transition from arithmetic to algebra [3] and trouble with writing algebraic verbal expressions as equations [4].

Stated that students have insufficient information about associate equation to daily life and they perceive the equations as mathematical systems consisting of only some letters and signs [5].

In this study, the equations are related to everyday life and screened then these scenario has been turned into short films. By watching the created short films, it is aimed to feel students that the subject of equations is related to real life, to make meaningful learning for the students and to make the lesson fun. Ten students in 7th grade participated in the study. In order to observe the change for the students, workshops were distributed to students before and after the work and observations were also made during the course. According to the findings, short films have been found to be effective on students' adaptation to the lesson and in relation with the subject and the daily life. Teaching mathematics with a short film was found to be positive by students and it was seen that short films serve the purpose of studying. At the end of the study, it was proposed that mathematics education with short film could be applied to other mathematics subjects and other courses.

Key Words: Mathematic education, short film, equations.

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Modules with ss-supplements over Commutative Domains

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ABSTRACT

The aim of this talk is to discuss cofinitely ss-supplemented and amply cofinitely ss-supplemented modules over commutative domains. First we introduce the concept of submodules $Loc_S(M)$ and $Cof_S(M)$ of a module M and then we give some new characterizations for (amply) ss-supplemented modules over a Dedekind domain by means of these modules. In particular we prove that (1) $\sigma: M \to M'$ be a homomorphism from a module M to a module M'. Then $\sigma(Loc_S(M)) \leq Loc_S(M')$; (2) $Loc_S(M) \leq Tor(M)$ for any R-module M; (3) Let M be a cofinitely ss-supplemented R-module. Then M/Tor(M) does not contain a maximal submodule; (4) Let an amply cofinitely ss-supplemented R-module $M = M_1 \oplus M_2$ be a direct sum of submodules M_1, M_2 such that M_1 has a maximal submodule. Then M_2 is a torsion module; (5) The following statements are equivalent for a commutative domain R. R is h-local if and only if $Loc_S(M)= Tor(M)$ for every (cyclic) R-module M if and only if every torsion R-module is cofinitely ss-supplemented if and only if every cyclic torsion R-module is (cofinitely) ss-supplemented if and only if an R-module M is cofinitely ss-supplemented if and only if M/Tor(M) does not contain a maximal submodule; (6) Let R be a one dimensional domain. Then an R-module M is cofinitely ss-supplemented if and only if M/Tor(M) is a divisible module.

Key Words: ss-supplement, h-local domain, one dimensional domain.

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Modules With the Properties (δ-E*) and (δ-EE*)

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ABSTRACT

Zöschinger generalized injective modules to modules with the property (E), that is, M has the property (E) if M has a supplement in every extension. Moreover, a module M has the property (EE) if M has ample supplements in every extension, i.e. for $M \leq N$, if $M + K = N$, K contains a supplement of M in N. A module N is said to be a cofinite extension of M provided $M \leq N$ and N/M is finitely generated. By adopting this fact, a module N is called a coatomic extension of M if $M \leq N$ and the factor module N/M is coatomic (see [8]).

In this paper we study on modules with the property $(\delta\text{-}E^*)$ and $(\delta\text{-}EE^*)$ as a generalization of E^*-modules and EE^*-modules. Even these modules can be seen as a generalization of $(\delta - E)$-modules and $(\delta - EE)$-modules. Let R be a ring and M be an R-module. M is said to be a $(\delta\text{-}E^*)$-module (respectively, a $(\delta\text{-}EE^*)$-module if M has a δ-supplement (respectively, ample δ-supplements) in every coatomic extension N, i.e. N/M is coatomic. We study some basic properties of these modules. We prove that the class of $(\delta\text{-}E^*)$-modules is closed under direct summands. We show that a module M is a $(\delta\text{-}EE^*)$-module iff every submodule of M is a $(\delta\text{-}E^*)$-module. In addition Generalized strongly δ-supplementes modules are introduced and it is proven that if M is a $(\delta\text{-}E^*)$-module over a δ-semilocal ring, then M is generalized strongly δ-supplemented. Under proper conditions a factor module of a module with $(\delta\text{-}E^*)$ is again a $(\delta\text{-}E^*)$-module. Moreover, a ring R is left δ-semiperfect ring iff every finitely generated left R-module is a $(\delta\text{-}E^*)$-module iff every finitely generated left R-module a $(\delta\text{-}EE^*)$-module. At the end, over a left V-ring every $(\delta\text{-}E^*)$-module M with $\delta(M) = 0$ is injective.

Key Words: coatomic extension, δ-supplement, $(\delta\text{-}E^*)$-module, $(\delta\text{-}EE^*)$-module, δ-perfect ring.

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On A Weighted Class Of ω −Caputo Fractional Derivatives, Their Representations And Properties

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ABSTRACT

Recently, many researchers are dealing with some well-known equations in the extended terms of fractional integro-differential operators, see e.g.$[3,4,7,8]$. These kind of extensions imply several expected and unexpected properties of the solutions of the considered equation, we refer $[1,2]$. This is the base for studying in this paper the $Re\alpha > 0$ order, $\boldsymbol{\omega}$ −weighted fractional derivatives of the functions f from several spaces $\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}]$, which are some subsets of the set $\boldsymbol{AC}[\boldsymbol{a},\boldsymbol{b}]$ of absolutely continuous functions on $[a,b]$, see in $[1,8]$. This paper gives some $\boldsymbol{\omega}$ −weighted extensions of the results in $[5,6]$.

In the most general case of $\boldsymbol{\omega}$ −weights, some normed functional spaces $\boldsymbol{X^p_\omega}(\boldsymbol{a},\boldsymbol{b})(\boldsymbol{1} \leq \boldsymbol{p} \leq \infty)$, $\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}]$ and a generalization of the fractional integro-differentiation operator are introduced and analysed. The boundedness of the $\boldsymbol{\omega}$ −weighted fractional operator over $\boldsymbol{X^p_\omega}(\boldsymbol{a},\boldsymbol{b})$ is proved. Some theorems and lemmas on the properties of the invertions of the mentioned operator and several representations of functions from $\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}]$ are established. A general ω −weighted Caputo fractional derivative of order $\boldsymbol{\alpha}$ is studied over $\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}]$**.** Some representations and other properties of this fractional derivative are proved. Some conclusions are presented.

$\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}]$ is defined by

$$\boldsymbol{AC^n_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}] := \left\{\boldsymbol{f}: [\boldsymbol{a},\boldsymbol{b}] \to \mathbb{C} \middle| \boldsymbol{\gamma^{n-1}f} \in \boldsymbol{AC}[\boldsymbol{a},\boldsymbol{b}], \boldsymbol{\gamma} = \frac{\boldsymbol{1}}{\boldsymbol{\omega'(x)}}\frac{\boldsymbol{d}}{\boldsymbol{dx}}\right\}$$

where $\boldsymbol{\omega} \in \boldsymbol{\Omega}$, $\boldsymbol{n} \in \mathbb{N}$ and $\boldsymbol{AC^1_{\gamma,\omega}}[\boldsymbol{a},\boldsymbol{b}] = \boldsymbol{AC}[\boldsymbol{a},\boldsymbol{b}]$.

Key Words: $\boldsymbol{\omega}$ −weighted fractional derivatives and integrals, Functional Spaces, Caputo Derivative, Absolutely Continuous Functions.

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On Fuzzy Retract of a Fuzzy Loop Space

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ABSTRACT

Homotopy theory is one of the main areas of algebraic topology. In [4] the concept of *H*-spaces (Hopf spaces) is introduced from the viewpoint of homotopy theory, then a grouplike space which is a group up to homotopy, called an *H*-group (Hopf group) is defined. An *H*-space is a pointed space (X, x_0) with a constant map $c: X \to X, x \to x_0$, for all $x \in X$ and a continuous multiplication $m : X \times X \to X$ such that x_0 is a homotopy identity, that is, the diagram

$$\begin{tikzcd} X \arrow[r, "{(c,1_X)}"] \arrow[dr, "1_X"'] & X \times X \arrow[d, "m"] & X \arrow[l, "{(1_X,c)}"'] \arrow[dl, "1_X"] \\ & X & \end{tikzcd}$$

commutes up to homotopy: $mo(c, 1_X) \simeq 1_X \simeq mo(1_X, c)$. Also if an H-space (X, x_0) has a homotopy commutative multiplication m (i.e. $mo(m \times 1_X) \simeq mo(1_X \times m)$) and has a homotopy invers n (i.e. there exist a function $n: X \to X$ such that $mo(n, 1_X) \simeq c \simeq mo(1_X, n)$), then (X, x_0) is called a H-group. If there exist a function $T: X \times X \to X \times X$, $T(x, y) = (y, x)$ such that $moT \simeq m$, then H-space (X, x_0) is called an abelian H-space.

In [5] Zadeh introduced the concepts of fuzzy sets. After Chang developed the theory of fuzzy topological spaces [2], basic concepts from homotopy theory were discussed in fuzzy settings. In this direction, Zheng [3] introduced the concept of fuzzy paths. Also in [1], fuzzy homotopy concepts in fuzzy topological spaces were conceived.

In this study firstly we define fuzzy H-space and fuzzy H-group. Then we show that a fuzzy loop space ΩX is an fuzzy H-group with the continuous multiplication

$m: \Omega X \times \Omega X \to \Omega X, m\big(\alpha(E), \beta(D)\big) = \gamma(E + D)$.

Then we show that a fuzzy retract of a fuzzy loop space is a fuzzy H-space. Besides, a fuzzy deformation retract is defined and it is shown that a fuzzy deformation retract of a fuzzy loop space is an fuzzy H-group. Also we show that if ΩX is an abelian fuzzy H-group, then the deformation retract of ΩX is an abelian fuzzy H-group.

Key Words: Fuzzy Loop Space, Fuzzy H-space, Fuzzy Retract, Fuzzy H-homomorphism

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On Graph Polynomials Of Some Trees

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ABSTRACT

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v) = \{u | (u, v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \cup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set, if every vertex in G either is element of S or is adjacent to at least one vertex in S. The domiantion number of a graph G is denoted with $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. Fundamental notions of domination theory are outlined in the book [1].

The domination polynomial of a simple graph G is calculated with [2] $D(G, x) = \sum_{i=1}^{n} d(G, i)x^i$ such that $d(G, i)$ is the number of the dominating sets of G of size i.

In this study we calculate the domination polynomials of caterpillar graphs and for a special case we obtain the domination polynomials of comb graphs. A caterpillar graph is a tree consisted a path and vertices directly connected to this path. Comb graphs are a special form of caterpillar graphs which are obtained by adding a vertex to every vertex of a path with an edge.

Key Words: Domination Polynomial, Caterpillar graph, Comb Graph.

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On Injective Modules

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ABSTRACT

Throughout this paper, all rings are associative with identity and modules are unital left modules. Let $0 \to M \to N \to K \to 0$ be a short exact sequence of modules. Then, N is called *an extension* of M by K. Without restriction of generality we will assume that $M \leq N$. Let M be a module. M is said to be *injective* if the functor Hom(., M) is exact. It is known in [5, Theorem 2.15] that M is injective if and only if it is a direct summand in every extension N. As a dual notation of projective modules, injective modules are very important in the module theory and the class of injective modules is extensively studied by many authors.

Let M be a module. We call M *strongly injective* if whenever $M+K = N$ with $M \leq N$, there exists a submodule L of K such that $M \oplus L = N$. It is clear that every strongly injective module is injective. Over left V-rings (that is, every simple module is injective) linearly compact modules are strongly injective. Every finitely generated artinian injective module is strongly injective.

In this talk, we study on strongly injective modules and give some properties of these modules. We prove that a module M is strongly injective if and only if it is semisimple injective. This gives us, the class of strongly injective modules is closed under submodules and factor modules.

Key Words: injective module, strongly injective module.

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On Left (σ,τ)-Jordan Ideals And Generalized Derivations

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ABSTRACT

Let R be a prime ring with characteristic not 2 and σ, τ, α, β, λ, μ, γ automorphisms of R. For each r, s∈R we set $[r,s]_{(\sigma,\tau)}=r\sigma(s)-\tau(s)r$ and $(r,s)_{(\sigma,\tau)}=r\sigma(s)+\tau(s)r$. Let U be an additive subgroup of R. If $(R,U)_{(\sigma,\tau)}\subset U$ then U is called a left (σ,τ)-Jordan ideal of R. A derivation d is an additive mapping on R which satisfies d(rs)=d(r)s+rd(s),∀r, s∈R. An additive mapping h:R→R is said to be a right generalized (σ,τ)-derivation of R associated with d if h(xy)=h(x)σ(y)+τ(x)d(y), for all x, y∈R and h is said to be a left generalized (σ,τ)-derivation of R associated with d if h(xy)=d(x)σ(y)+τ(x)h(y), for all x, y∈R. h is said to be a generalized (σ,τ)-derivation of R associated with d if it is both a left and right generalized (σ,τ)-derivation of R associated with d.Let h:R→R be a nonzero left (resp. right)- generalized (α,β)-derivation associated with (α,β)- derivation d (resp. d_1). Let W, V be nonzero left (σ,τ)-Jordan ideals of R.

The main object in this paper is to study the situations. (1) h(W)=0, (2) $[b,W]_{(\lambda,\mu)}=0$ or $[W,b]_{(\lambda,\mu)}=0$, (3) $(b,W)_{(\lambda,\mu)}=0$ or $(W,b)_{(\lambda,\mu)}=0$, (4) $b[W,a]_{(\lambda,\mu)}=0$ or $[W,a]_{(\lambda,\mu)}b=0$ (5) $b(W,a)_{(\lambda,\mu)}=0$ or $(W,a)_{(\lambda,\mu)}b=0$, (6) $bW\subset C_{(\lambda,\mu)}(V)$ or $Wb\subset C_{(\lambda,\mu)}(V)$. (7) $(h(R),b)\,)_{(\lambda,\mu)}a=0$ or $a(h(R),b)_{(\lambda,\mu)}b=0$.

Key Words: Prime ring, generalized derivation, (σ,τ)-Jordan Ideal.

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On Low Dimensional Leibniz Algebras

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ABSTRACT

Leibniz algebras are generalization of Lie algebras. As an immediate consequence, every Lie algebras are Leibniz algebras. Recall that a Leibniz algebra L is finite dimensional if the dimension of algebra L as a vector space over a field is finite. The condition to be finite dimensional is very strong. That is why the majority of results on Leibniz algebras were obtained for finite dimensional Leibniz algebras. In literature, there are many studies on one dimensional and two dimensional Leibniz algebras. The structure of three dimensional Leibniz algebras are more complicated than the structure of one dimensional and two dimensional Leibniz algebras. Investigation of Leibniz algebras, having dimensions 3 and 4 has been conducted in many papers, but only for the case of algebra over a field of characteristic 0.

In this note, firstly we show that 1-dimensional Leibniz algebras are abelian. Then we deal with the structure of 2-dimensional Leibniz algebras and we give two non-isomorphic non-Lie 2-dimensional Leibniz algebras. Our main goal is to investigate three dimensional non-Lie Leibniz algebras. Moreover, we prove that if L is a three dimensional non-Lie Leibniz algebra, then there exists a one Leibniz algebra which is isomorphic to L.

Key Words: Leibniz algebra, Lie algebra, dimension of Leibniz algebra.

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On Multiplicative Generalized Semiderivation in Semiprime Rings

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ABSTRACT

Let R be a semiprime ring with center $Z(R)$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$ and is called semiprime if for any $a \in R, aRa = (0)$ implies that $a = 0$. Let S be a nonempty subset of R. A map $F: R \to R$ is called centralizing on S if $[F(x), x] \in Z(R)$ for all $x \in S$ and is called commuting on S if $[F(x), x] = 0$ for all $x \in S$. A mapping F on R is said to be a multiplicative generalized semiderivation of R if there exists a multiplicative semiderivation d associated with a map g on R such that (i) $F(xy) = F(x)y + g(x)d(y) = d(x)g(y) + xF(y)$ and (ii) $F\big(g(x)\big) = g\big(F(x)\big)$; for all $x, y \in R$.

In [4], Daif and Bell proved that if R is a semiprime ring with a nonzero ideal K and d is a derivation of R such that $d([x,y]) = \pm[x,y]$ for all $x, y \in K$, then K is a central ideal. In particular, if $K = R$, then R is commutative. In this line of investigation, it is more interesting to study the identities replacing derivation d with a multiplicative generalized semiderivation F. The main object of the present paper to study the following identities: (i) $F([x,y]) = \pm[x,y]$ (ii) $F([x,y]) = \pm(xoy)$ (iii) $F(xoy) = \pm(xoy)$
(iv) $F(xoy) = \pm[x,y]$ (v) $F([x,y]) = x^m[x,y]x^n$ (vi) $F([x,y]) = x^m(xoy)x^n$
(vii) $F(xoy) = x^m[x,y]x^n$ (viii) $F(xoy) = x^m(xoy)x^n$ for all $x, y \in R$, $m, n \in \mathbb{N}$, where $F: R \to R$ is a multiplicative generalized semiderivation associated with the multiplicative derivation $d: R \to R$ and any map g on R.

Key Words: Semiprime ring, semiderivation, generalized semiderivation, multiplicative derivation.

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On Skew-Centrosymmetric Matrices

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ABSTRACT

Matrices have wide applications in many fields of science, for example numerical solution of certain differential equations, digital signal processing, information theory, statistics, linear systems theory and so on.

Recently, there has been huge interest on matrix operations, such as determinant, permanent, Pfaffian, etc., and there are many papers on linear algebraic properties of matrices.

If it is compared with others, Pfaffian is a new concept in mathematics, and computing Pfaffian of matrices has been studied less than others. For example, Tam et. al. [1] studied the determinants of the sum of orbits of two real skew symmetric matrices, under similarity action of orthogonal group and the special orthogonal group and the Pfaffian case and the complex case. In [2], Hamel considered Pfaffians from a combinatorial point of view and produced a number of vector-based identities. In [3], the authors obtained some relating Pfaffians and Hafnians with determinants and permanents.

At this study, we show that the Pfaffians and determinants of some skew centrosymmetric matrices can be computed by a paired two-term recurrence relation, or a general number sequence of second order. Note that, a matrix is said to be centrosymmetric matrix if $A=JAJ^{-1}$ where J is anti-diagonal matrix whose entries are "1" and others are "0".

Key Words: Skew centrosymmetric matrix, Pfaffian

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On Some Relations Between Continued Fractions and Fibonacci Numbers

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ABSTRACT

Continued fractions have an important place in the theory of rational approximation theory and facilitate the formation of many transcendental numbers. Also these fractions can be thought of as a generalization of compound fractions. The Euclidean algorithm is used to find the largest common divisor of two numbers. It is known that this algorithm leads to the development of the continued fraction theory due to its close relation with continued fractions.

The Fibonacci sequence F_n is a number sequence consisting of the result of the addition of each number to the previous one. The two initial values are 0 and 1. When a number is divided by its previous number, a sequence approaching the golden ratio is obtained. This golden ratio is a number associated with aesthetics, seen in nature, in art and in every area of life.

In this work, we examine the relationship between the value of the terms F_{2n} of the Fibonacci number sequence with a special periodic continued fraction. We derive the results of these terms of the Fibonacci sequence from the matrix connections of continued fractions. We also found some relations using the recurrence relations of continued fractions to obtain other terms of the Fibonacci sequence as well.

Key Words: Continued Fractions, Fibonacci Numbers

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On Some Special Classes Of Nullnorms On Bounded Lattices

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ABSTRACT

The concepts of t-operators and nullnorms were introduced by Mas, Mayor, Torrens in 1999 [7] and Calvo, De Baets, Fodor in 2001 [1], respectively. It was stated that nullnorms and t-operators are equivalent in 2002 by [8] since they have the same block structures on the real unit interval. Namely, if a binary operation F is a nullnorm then it is also a t-operator and on the other hand, once F is a t-operator then it is also a nullnorm. Nullnorms are important generalizations of triangular norms with 0 as the zero element and triangular conorms with 1 as the zero element. A nullnorm is a function on the real unit interval that has to satisfy the properties of commutative, associative, non-decreasing in each variable and admits a zero element to be an arbitrary point from the real unit interval. The zero element of nullnorms is 0 we are in triangular norms case, while the zero element is 1 we are in triangular conorms case. Nullnorms have also numerous application fields such as fuzzy logic, expert systems, neural networks, utility theory, fuzzy system modeling and multicriteria decision support.

It is studied nullnorms (particularly idempotent nullnorms) defined on bounded lattices in [2]. It is showed that there need not always exist an idempotent nullnorm on an arbitrary bounded lattice having a zero element. Moreover, a construction method for idempotent nullnorms on a bounded lattice L is introduced under an additional assumption on L.

In this contribution, we study some special class of nullnorms defined on a bounded lattice with a zero element different from the top and the greatest element. We present several basic properties of such nullnorms on bounded lattices and add an illustrative example to demonstrate their structures.

Key Words: Bounded lattice; Nullnorm; Zero element; t-norm; t-conorm

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On Sophie Germain Primes

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ABSTRACT

This article consists of two parts. An elementary method for eliminating 2m prime pairs is given by Lampret [S. Lampret, Sieving 2m-prime pairs, Notes on Number Theory and Discrete Mathematics Vol. 20, 2014, No.3, 54-46.], where m is an arbitrary positive integer. 2m-prime pairs are related the twin prime pairs since a 2m-prime pair is a twin prime pair if m=1. Lampret gave a characterization for 6n-prime pairs of the form (6k - 1, 6k + 6n - 1). In first part of this paper, the Sophie Germain prime and connected safe prime pairs are referred to as SG-S-prime pairs in short. By using Lampret's results, we focus on a characterization to obtain SG-S-prime pairs owing to an eliminating method. Thus it is formed instructions for a sieve as an elementary method to find the SG-S-prime pairs. Moreover we give an example in which we use our instructions to obtain the SG-S-prime pairs up to 250.

Wilson's fundamental theorem in number theory gives a characterization of prime numbers via a congruence. A theorem based on Wilson's Theorem is formulated by Clement [P. A. Clement, Congruences to sets of primes, Am. Math. Mon. 56, 1949, 23-25]. Clement has a characterization of twin primes (n,n+2). In second part of this paper, by a congruence, we give a characterization of Sophie Germain primes in the light of the inspiration of Clement's theorem.

Key Words: Prime numbers; Sophie Germain primes; Twin primes

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On The Distance-Sum-Connectivity Matrix

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ABSTRACT

Let G be a simple, connected graph with n nodes and m edges. If two vertices v_i and v_j are adjacent then we use the notation $v_i \sim v_j$. The Distance-Sum-Connectivity Matrix of a graph G is defined by

$$^{\delta}X = \begin{Bmatrix} (\delta(i)\delta(j))^{-1/2} & i\sim j \\ 0 & otherwise \end{Bmatrix}$$

where $\delta(i) = \sum_{j=1}^{v} {}^{v}D_{ij}$ and D is the distance matrix.

Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be eigenvalues of the graph G. In this presentation, we find the eigenvalues of the distance-sum-connectivity matrix of a graph. Since the distance-sum-connectivity matrix is a real symmetric matrix, its eigenvalues must be real and may be ordered as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. λ_1 is called the spectral radius of the graph G. Using the some known lemmas, we obtain upper bounds for these eigenvalues. We give a bound for the spectral radius λ_1 and the second greatest eigenvalue λ_2 .

A lot of results for the energy of a graph have been known. Secondly, we define the incidence energy of a graph G. Also, we obtain a bound for the sum of two incidence energy of the distance-sum-connectivity matrix of a graph and we find two different bounds for the incidence energy of the distance-sum-connectivity matrix of a graph. For these bounds, we use the Cauchy-Schwarz inequality, Ozeki's inequality Besides, these bounds contain the edges, the vertices, the degrees and the determinant of the distance-sum-connectivity matrix of a graph.

Lastly, we define the matching energy and we give some upper bounds on the matching energy using the Polya-Szego inequality and Maclaurin's Symmetric Mean inequality. Also, we set a conclusion for the regular graph and the bipartite graph.

Key Words: Distance-Sum-Connectivity Matrix, Energy, Bounds.

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On The Dodgson Condensation Method

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ABSTRACT

In mathematics and especially linear algebra, determinant is an important tool and has a wide use area. They first were developed in the 18th century. In 1885 the great French mathematician Cauchy gave the first systematic and modern approach of determinant. Then determinant become comprehensive in some areas of mathematics, science and social sciences. Although small size determinants are quite simple to calculate, the degree of difficulty increases exponentially with the size of the matrix. The determinant of a matrix of arbitrary size can be defined by the Leibniz formula or the Laplace formula. But, cofactor expansion is not the only method to express the determinant of a matrix in terms of smaller determinants. The condensation method can also be taken into this category. The condensation method, discovered by Charles Dodgson (better known by his pseudonym Lewis Carroll as the creator of Alice in Wonderland) in 1866, provides a more efficient way of calculating determinants of large matrices. For a 5x5 matrix, it reduces the amount of computations by nearly half compared to a standard cofactor expansion method. Dodgson based his method on Jacobi's Theorem, which after some rearrangement provided a useful algorithm for finding determinants [1,2].

The main aim of this work is to introduce in detail the condensation method.

Key Words: Determinant, condensation method, Charles Dodgson.

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On the LOG and EXP Operators on Isoboric Polynomials

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ABSTRACT

In Rearick [2,3], the notions of "Logarithm" and "Exponential" operators of arithmetic functions were introduced. These operators were inverses of one another. The Logarithm operator takes convolution products to sums in *A* (set of Arithmetic functions denoted by *A*), and the Exponential operator takes sums to convolution products. He denoted these operators by *L* and *E*. The Exponential operator was then used to define "Sine," "Cosine" and "Tangent" [1]. Huilan Li and Trueman Machenry defined two operators (LOG and EXP operators) inspired by Rearick's work by using the logarithm and exponential functions of arithmetic the functions in [1]. Arithmetic functions(or number-theoretic function), which are a special type of function, whose domain is the positive integers and whose range is reel or complex numbers. The LOG operators on generalized Fibonacci polynomials giving generalized Lucas polynomials. The EXP is the inverse of LOG. In particular, LOG takes a convolution product of generalized Fibonacci polynomials to a sum of generalized Lucas polynomials, and EXP takes the sum to the convolution product [1]. Let P_n is a weighted isobaric polynomial, for a fixed *k* and *n*>0,

$$LOG(P_n) = -t_{n-1}P_1 - 2t_{n-2}P_2 - ... - (n-1)t_1P_{n-1} + nP_n .$$

Parallel to the definition of elementary hyperbolic trigonometric functions in the [2], Li and Machenry defined "hyperbolic" SINE and "hyperbolic" COSINE;

$$S(G) = \frac{1}{2}(EXP(G) - \overline{EXP(G)}),\ C(G) = \frac{1}{2}(EXP(G) + \overline{EXP(G)}) .$$

Where *G* is generalized Lucas polynomial.

In this presentation, we will obtain some properties of the hyperbolic trigonometric operators given above.

Key Words: generalized Fibonacci polynomials, generalized Lucas polynomials, hyperbolic trigonometric operators.

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On The Properties Of Hyperbolic Trigonometric Operators

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ABSTRACT

Arithmetic functions which are a special type of function, whose domain is the positive integers and whose range is a reel or complex numbers. The set of Arithmetic functions denoted by *A*. Some example of an arithmetic function are Euler totient function, Jordan totient function, Möbius function and Liouville function. In this study, we only use real-valued arithmetic function. If g and h are two arithmetic functions one defines a new arithmetic function $f = g * h$, the Dirichlet convolution of g and h, by $f(n) = (g*h)(n) = \sum_{d|n} g(d)h(\frac{n}{d})$. Where the sum extends over all positive divisors *d* of *n.* $(A,*)$ algebraic structure is an Abel monoid, and each element has no inverse. The set of inverse elements of *A* are $P = \{g \in A : g(1) > 0\}$, and the inverse of these functions is called Dirichlet inverse. In [1] Rearick defined following *L* operator to show the isomorphism between $(P,*)$ and $(A,+)$. $Lf(n) = \sum_{d|n} f(d) f^{*-1}(\frac{n}{d}) \log d$. Rearick [1,2] defined the exponential operator *E*, which is the inverse operator of *L* operator, so that a new trigonometric operator family has emerged. Parallel to the definition of elementary hyperbolic trigonometric functions in the analysis, the sine (S), cosine (C) and tangent (T) operators defined by, $\forall g \in A$,

$$Sg = \frac{1}{2}(Eg - E(-g)),\ \mathrm{C}g = \frac{1}{2}(Eg + E(-g)) \text{ and } Tg = Sg * Cg^{-1}.$$

In this work, we will obtain some new properties of the hyperbolic trigonometric operators given above.

Key Words: Arithmetic functions, Dirichlet convolution, hyperbolic trigonometric operators.

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On the The Maximum Determinant of Lower Hessenberg (0,1) Matrix

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ABSTRACT

Hessenberg matrix is a kind of square matrix, a lower Hessenberg matrix has zero entries above the first superdiagonal, and an upper Hessenberg matrix has zero entries below the first subdiagonal. If the entries of the lower Hessenberg matrix are composed only of 0 and 1, this matrix is called the Lower Hessenberg (0,1) matrices. In 1993 Li Ching [2] developed a method to calculate the maximum determinant of n×n Lower Hessenberg (0,1) matrices. There are 2^n (possibly nonzero) terms in the determinant of an n×n Lower Hessenberg (0,1) matrices, so this is a trivial upper bound for the determinant [2]. Ching defined n×n Lower Hessenberg (0,1) key matrices D_n and made the necessary calculation by using this matrix. In 2002 Cahill et al. [1] gave a method to calculate the determinant of Hessenberg matrices. In this study, we will first give Cahill's main theorem. Our main focus will be to explain Ching's maximum determinant method obtained in [2]. Firstly, we will prove that determinant of key matrices are Fibonacci numbers by using Cahill's Hessenberg matrix determinant method. Then we will obtain the maximum determinant of an n×n Lower Hessenberg (0,1) matrices step by step. This method is essentially based on the effect of elementary row operations on the determinant.

Key Words: Maximum determinant, Hessenberg (0,1) matrix, Fibonacci numbers.

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Parametric Soft Semigroups

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ABSTRACT

While probability theory, fuzzy set theory,rough set theory, vague set theory and the interval mathematics are useful approaches to describing uncertainty, each of these theories has its inherent difficulties. Since its inception by Molodtsov in 1999, soft set theory has been regarded as a new mathematical tool for dealing with uncertainties and it has seen a wide-ranging applications in the mean of algebraic structures such as groups, semirings, rings, BCK/BCI-algebras, BL-algebras, near-rings etc.. Nowadays, it has promoted a breadth of the discipline of Informations Sciences with intelligent systems, approximate reasoning, expert and decision support systems, self-adaptation and self-organizational systems, information and knowledge, modeling and computing with words. The parameter set of the soft set may be any set, whereas the universe set is semigroup. In this study, the parameter set of the soft set is semigroup, whereas the universe set is any set. This provides us to operate on sets easily with respect to inclusion relation and intersection of. In this study, we give concept of soft intersection product and soft characteristic function and obtain their basic properties. Also, the notation of parametric soft semigroup is defined and study with respect to soft set operations and soft intersection product. Moreover, the concept of parametric soft homomorphism and some characterization theorems are given, and several related properties are investigated.

Key Words: Semigroup, soft set, soft semigroup.

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Soft Lattice Structures and Their Properties

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ABSTRACT

Because of various uncertainties arise in complicated problems in economics, engineering, environmental science, medical science and social science, methods of classical mathematics may not be successfully used to solve them. Mathematical theories such as probability theory, fuzzy set theory and rough set theory were established by researchers to model uncertainties appearing in the above fields but all these theories have their own difficulties. To overcome these difficulties, D. Molodtsov introduced the concept of soft set as a new mathematical tool for dealing with uncertainties. Since Molodtsov proposed an approach for modeling vagueness and uncertainty, called soft set theory in 1999, works on soft set theory has been progressing rapidly with a wide range- applications not only in the mean of algebraic structures but also in the structures of soft sets, operations of soft sets. Lattice is a partially ordered set in which supremum and infimum exist for every pair of elements in the set. Lattice structure of soft sets is an interesting topic to the researchers. In this study, we initiate the structure of soft lattices by using soft set theory. The notions of soft lattices, soft distributive lattices, soft modular lattices, soft lattice ideals are introduced and several related properties are investigated. Also the concept of soft lattice homomorphism and some characterization theorems are given.

Key Words: Soft set, soft lattice, soft lattice homomorphism.

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Soft Linear Transformations between Vector Spaces

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ABSTRACT

The complexities of modeling uncertain data in economics, engineering, environmental science, sociology, medical science and many other fields can not be successfully dealt with by classical methods. While probability theory, fuzzy set theory,rough set theory, vague set theory and the interval mathematics are useful approaches to describing uncertainty, each of these theories has its inherent difficulties. Since Molodtsov proposed an approach for modeling vagueness and uncertainty, called soft set theory in 1999, works on soft set theory has been progressing rapidly with a wide range- applications not only in the mean of algebraic structures but also in the structures of soft sets, operations of soft sets. In this study, we give the concept of soft vector space which extends the notion of vector spaces by including some algebraic structures in soft set theory, and we construct some basic properties of it by using vector spaces and Molodtsov's definition of soft sets. Also, we introduce the notions of soft v-linear transformations, soft b-linear transformations and investigate some related properties with illustrating examples. Moreover, soft v-linear transformation are investigated with respect to the soft linear transformatic image and we show that some structures of soft vector spaces are preserved under soft b-linear transformations.

Key Words: Soft set, vector space, soft vector space.

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Solution of Nonlinear Fredholm Integral Equation

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ABSTRACT

We begin our study nonlinear Fredholm integral equations of the second kind are characterized by fixed limits of integration of the form

$$u(x) = f(x) + \lambda \int_a^b K(x,t)F\big(u(t)\big)dt \tag{1}$$

where the unknown function u(x) occurs inside and outside the integral sign, λ is a parameter, and a and b are constants. For this type of equations, the kernel K(x, t) and the function f(x) are given real-valued functions, and F(u(x)) is a nonlinear function of u(x) such as u2(x),sin(u(x)), and eu(x). We will mostly use degenerate or separable kernels. A degenerate or a separable kernel is a function that can be expressed as the sum of product of two functions each depends only on one variable. Such a kernel can be expressed in the form

$$K(x,t)=\sum_{i=1}^{n} g_i(x)f_i(t) \tag{2}$$

Several analytic and numerical methods have been used to handle the nonlinear Fredholm integral equations.

We will present an existence theorem for the solution of nonlinear Fredholm integral equations. The proof of this theorem can be found in [1–2] among other references. We first rewrite the nonlinear Fredholm integral equation of the second kind by

$$u(x) = f(x) + \lambda \int_a^b G(x,t,u(t))\,dt. \tag{3}$$

The specific conditions under which a solution exists for the nonlinear Fredholm integral equation are:

(i) The function f(x) is bounded, |f(x)| < R, in a≤ x≤ b.

(ii) The function G(x, t, u(t)) is integrable and bounded where

$$|G(x, t, u(t))| < K, \quad \text{in } a\le x, t\le b. \tag{4}$$

(iii) The function G(x, t, u(t)) satisfies the Lipschitz condition

$$|G(x, t, z) - G(x, t, z')| < M|z - z'|. \quad (5)$$

Using the successive approximations method, it is proved in [1] that the series obtained by this method converges uniformly for all values of λ for

$$\lambda < \frac{1}{k(b-a)}, \quad (6)$$

where k is the larger of the two numbers $K\left(1 + \frac{R}{|\lambda| K(b-a)}\right)$ and M.

In this study,the solution of the nonlinear Fredholm integral equation are investigated.

The nonlinear Fredholm integral equation we have dealt with is solved numerically and analytically by direct computation, series solution and Adomian decomposition methods.

Keywords : Direct computation method , Series solution method, Adomian decomposition method.

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Some Binomial Double Sums

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ABSTRACT

The numbers, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,..., known as Fibonacci numbers have been named by the nineteenth-century French mathematician Edouard Lucas after Fibonacci of Pisa, one of the best mathematicians of the Middle Ages, who referred to them in his book Liber Abaci (1902) in connection with his rabbit problem. Edouard Lucas studied Fibonacci numbers extensively, and the simple generalization 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123,….

DeMoivre was the first to write the Fibonacci numbers as a recursion: $F_n = F_{n-1} + F_{n-2}$. Together with the initial values $F_0 = 0$ and $F_1 = 1$, this defines the Fibonacci sequence as a recurrence relationship. The Lucas numbers are the sequence of integers defined by the linear recurrence equation $L_n = L_{n-1} + L_{n-2}$ with $L_0 =2$ and $L_1 = 1$.

In 1843, Binet gave a formula which is called "Binet formula" for the usual Fibonacci numbers F_n by using the roots of the characteristic equation $x^2 - x - 1 = 0$: $F_n = (\alpha^n - \beta^n)/(\alpha - \beta)$ where α is called Golden Proportion.

In this study, we compute the binomial sums involving Fibonacci numbers. All the sums obtained are expressed neatly as products of Fibonacci and Lucas numbers. While proving the obtained results we use the Binet formulas for Fibonacci and Lucas numbers, Binomial theorem, and some new Fibonacci-Lucas identities.

Key Words: Fibonacci numbers, Lucas numbers, binomial sums.

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Some Equations in Semiprime Rings With Multiplicative Generalized Semiderivation

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ABSTRACT

The first study on derivations of prime rings was conducted in 1957 by Posner [7]. In his paper, Posner describes the definition of derivation in any ring as follows: An additive mapping d on R is a derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. The idea of multiplicative derivation was put forward in 1991 by Daif [3] as follows: A mapping d on R is said to be multiplicative derivation of R if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. These maps are not additive, so it is a generalization of the derivations in a sense. In 1991, Bresar [2], the concept of derivation has been generalized as follows: Let d be a derivation of R, an additive mapping F on R is called generalized derivation of R associated with d if $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$.

On the other hand, in 1983, J. Bergen [1] has been defined the concept of semiderivation of ring as follows: An additive mapping d on R is called a semiderivation of ring if there exist a map g on R such that (i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and (ii) $d(g(x)) = g(d(x))$; for all $x, y \in R$. If g is an identity map on R, the concept of semiderivation covers the concept of derivation.

The principal aim in this study is to motivate the definition of semiderivation given by Bergen. As multiplicative generalized semiderivation is an extended notion of semiderivation, derivation and generalized derivation. The main objective of this paper is to take care of this definition and investigate the some certain identities on a semiprime ring R admitting multiplicative generalized semiderivation.

Key Words: Semiprime ring, semiderivation, generalized semiderivation, multiplicative derivation.

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SP-Fuzzy Soft Ideals in Semigroups

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ABSTRACT

In this paper, the definition of a new concept which is a member of the class $\underline{(U,P)}$ and which is referred to as UP-fuzzy soft subset of a soft set on the class (U,P) is introduced, where $\underline{(U,P)}$ denotes the fuzzy soft class and (U,P) denotes the soft class with the universal set U and the set of parameters P. We give the definitions of the complement and α-level soft set of a UP-fuzzy soft subset of a soft set. It is demonstrated that UP-fuzzy soft subsets provide De Morgan rules for restricted union and restricted intersection.

Furthermore, considering a semigroup S as an universal set, this paper presents some new algebraic notions which are called SP-fuzzy soft subsemigroup and SP-fuzzy soft left (right, bi-, quasi, interior, prime, semiprime) ideal of a soft semigroup. We examine some basic properties such as restricted union, extended union, restricted intersection, extended intersection and product of the families of SP-fuzzy soft subsemigroups and SP-fuzzy soft left (right, bi-, quasi, interior, prime, semiprime) ideals. It is obtained that the restricted intersection of the family of SP-fuzzy soft subsemigroups is a SP-fuzzy soft subsemigroup of the restricted intersection of the family of soft subsemigroups. Moreover it is indicated a characterization of α-level soft sets by SP-fuzzy soft sub algebraic structures. For instance, we show that an α-level soft set of a SP-fuzzy soft subset is a soft subsemigroup for all $\alpha \in [0,1]$ if and only if the SP-fuzzy soft subset is a SP-fuzzy soft subsemigroup.

Key Words: (Fuzzy) soft set, SP-fuzzy soft subset, SP-fuzzy soft subsemigroup.

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ss-Radical And Strongly ss-Radical Supplemented Modules

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ABSTRACT

In this talk, all rings are associative with identity and all modules are unitary left modules .Let R be such a ring and M be a R-module.By Rad(M) and Soc(M), we will denote the radical of M and the Socle of M, respectively. A submodule N of M is called "*small*" in M, denoted $N \ll M$, if $M \neq N + K$ for every proper submodule K of M. A module M is called "*supplemented*" if every submodule N of M has a supplement, ie., a submodule K minimal with respect to $N + K = M$. K is a supplement of N in M if and only if $N + K = M$ and $N \cap K << K$. An R-module M is said to be "*radical supplemented*" if $Rad(M)$ has a supplement in M and it is said to be "*strongly radical supplemented*" if every submodule N of M with Rad(M) $\subseteq$N has asupplement in M. In [1], Zhou and Zhang generalized the concept of socle of a module M to that of $Soc_s(M)$ by considering the class of all simple submodules M that are small in M in place of the class of all simple submodules of M, that is, $Soc_S(M) = \sum\{N \ll M | N \text{ is simple}\}$ It is clear that $Soc_S(M) \subseteq Rad(M)$ and $Soc_S(M) \subseteq Soc(M)$. In [6], V is a ss-supplement of U in M if and only if $M = U + V$ and $U \cap V \subseteq Soc_S(V)$ A module M is called "*ss-supplemented*" if every submodule U of M has a ss-supplement V in M.

In this work, we define the concept of ss-radical supplemented modules and strongly ss-radical supplemented modules. In particular, we obtain the basic properties of these modules.

Key Words: Radical, ss-supplement

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Strongly Local Modules and Rings

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ABSTRACT

In this talk, all rings are associative with identity and all modules are unitary left modules. Let R be a ring and M be an R-module. $U \subseteq M$ will mean that U is a submodule of M. $Rad(M)$ and $Soc(M)$ will indicate radical and socle of M. A submodule N of M is called small in M, denoted $N \ll M$, if $M \neq N + K$ for every proper submodule K of M. A non-zero module M is called hollow if every proper submodule of M is small in M and is called local if the sum of all proper submodules of M is also a proper submodule of M. Note that local modules are hollow and hollow modules are clearly amply supplemented. A ring R is called local ring if ${}_RR$ (or R_R) is a local module. In [1], Zhou and Zhang generalized the concept of socle of a module M to that of $Soc_S(M)$ by considering the class of all simple submodules M that are small in M in place of the class of all simple submodules of M, that is, $Soc_S(M) = \sum\{N \ll M | N\ is\ simple\}$ It is clear that $Soc_S(M) \subseteq Rad(M)$ and $Soc_S(M) \subseteq Soc(M)$. We call a module M strongly local if it is local and $Rad(M) \subseteq Soc(M)$. We call a ring R left strongly local ring if ${}_RR$ is a strongly local module. Then we have that the following implications on modules:

simple $\Longrightarrow$ strongly local $\Longrightarrow$ local

In this present paper, we give some properties of strongly local modules and ring.

Key Words: local module, hollow module, strongly local module.

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Structure of Cyclic Codes over $\mathbb{Z}_{p^s} + u\mathbb{Z}_{p^s}$ and Some Special Family of Them

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ABSTRACT

Cyclic codes, which are a significant class of linear codes are vital for the communication theory, because of the natural coherence between their mathematical structure and its reflection to electronic circuit structures and linear feedback shift registers (LFSR).

Although, some non-linear codes may have better parameters, capability of error correcting for instance, they are more difficult to find and study since they are not as systematic as linear codes. Some well-known and best parameter non-linear codes are shown to be the images of linear codes over $\mathbb{Z}_4$. Hence, various finite rings have been considered and their structures are being explored [1, 2]. In this study, we establish the structure of the cyclic codes of length n over the ring $\mathbb{Z}_{p^s} + u\mathbb{Z}_{p^s}$ as an on-going search of [3] and [4] where p is any prime number such that $\gcd(p,n)=1$ and s is any positive integer with $u^2=0$. This is equivalent to determining the structure of ideals of the quotient ring $(\mathbb{Z}_{p^s} + u\mathbb{Z}_{p^s})[x]/<x^n-1>$ because of the fact that a linear code C of length n over a ring R is cyclic if and only if the corresponding polynomials of its codewords form an ideal structure in the quotient ring $R[x]/<x^n-1>$. In this way, we determine the number of these ideals by giving a formula. We also give the size of these dual codes and present some examples.

Key Words: cyclic codes, codes over rings, linear codes, local rings, non-chain rings.

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Two New Classes of EAQMDS Codes From Constacyclic Codes

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ABSTRACT

Quantum error-correcting codes were introduced for security of quantum information. A q-ary quantum code Q, denoted by parameters $[[n,k,d]]_q$, is a q^k dimensional subspace of the Hilbert space $\mathbb{C}^{q^n}$. A quantum code $\mathcal{C}$ with parameters $[[n,k,d]]_q$ satisfy the quantum Singleton bound: $k \leq n-2d+2$. If $k=n-2d+2$, then $\mathcal{C}$ is called a quantum maximum-distance-separable (MDS) code [1]. In recent years, many researchers have been working to find quantum MDS codes via constacyclic codes. Construction of good quantum codes via classical codes is an important task for quantum information and quantum computing.

Entanglement-assisted quantum error correcting (EAQEC for short) codes use pre-existing entanglement between the sender and receiver to improve information rate. Recently, a few papers have been devoted for obtaining EAQEC codes derived from classical error correcting codes. These papers can be summarized as follows. In [2], based on classical quaternary constacyclic codes, some parameters for quantum codes were obtained. In [3], a decomposition of the defining set of negacyclic codes has been proposed and by virtue of the proposed decomposition four classes of EAQEC codes have been constructed. Fan et al., have constructed five classes of entanglement-assisted quantum MDS (EAQMDS for short) codes based on classical MDS codes by exploiting one or more pre-shared maximally entangled states [4]. Qian and Zhang have constructed some new classes of maximum distance separable (MDS) linear complementary dual (LCD) codes with respect to Hermitian inner product and as an application, they have constructed new families of MDS maximal EAQEC codes in [5]. In [6], Lu et al. constructed six classes of q-ary EAQMDS codes based on classical negacyclic MDS codes. In [7], Guenda et al. have shown that the number of shared pairs required to construct an EAQEC code is related to the hull of the classical codes. Using this fact, they gave methods to

construct EAQEC codes requiring desirable amounts of entanglement. Further, they constructed maximal entanglement EAQEC codes from LCD codes.

In this work, by virtue of a decomposition of the defining set of constacyclic codes we have constructed two new classes of entanglement-assisted quantum maximum distance separable codes of length $n=\frac{q^2+1}{5}$, where $q=2^e$. Moreover, we have constructed two new classes of maximal-entanglement entanglement-assisted quantum codes.

Key Words: Entanglement-assisted quantum error-correcting codes, Constacyclic codes, MDS codes.

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ANALYSIS

A New Semi-Normed Sequence Space Of Non-Absolute Type

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ABSTRACT

The purpose of the this study is to introduce the sequence space

$$\ell_p\left(E,B(r,s)\right)=\left\{x=(x_n)\in\omega:\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}rx_j+\sum_{j\in E_{n+1}}sx_j\right|^p<\infty\right\},$$

where $E=(E_n)$ is a partition of finite subsets of the positive integers, $r,s\in\mathbb{R}/\{0\}$ and $p\geq 1$. Furthermore, we denote that $\ell_p\left(E,B(r,s)\right)$ is a sequence space of non absolute type. The topological and algebrical properties of this space are examined and some inclusion relations concerning our space are given. By the applying the first isomorphism theorem we indicated that the quotient space $\ell_p\left(E,B(r,s)\right)$/M is linearly isomorphic to the space ℓ_p that is we show that the operator T from the space $\ell_p\left(E,B(r,s)\right)$ into ℓ_p is linear, surjective and although it preserves the semi-norm is not injective . The matrix domain given in this paper specify by a certain non triangle matrix, so we should not expect that related space is normed sequence spaces. Also, we prove that the set $\ell_p\left(E,B(r,s)\right)$ becomes a vector space with coordinatwise addition and scalar multiplication, which is a complete semi-normed space. Moreover, we adduce that except for the case p=2, the space $\ell_p\left(E,B(r,s)\right)$ is not a semi-inner product space. After this the Schauder base of this space are computed. Finally, we show that the operator A defined from ℓ_p into $\ell_p\left(E,B(r,s)\right)$ is bounded and also we tend to compute the norm of the operator A and the problem of finding the norm of certain matrix operators such as Copson and Hilbert from ℓ_p into $\ell_p\left(E,B(r,s)\right)$ is considered.

Key Words: Block sequence spaces, matrix domain, operator norm.

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A Note on Superposition Operators on Linear n-Normed Spaces

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ABSTRACT

Let X be a real linear space which has dimension d with $n \leq d$. Let $\mathbb{N}$ and w_X denote the set of all natural numbers and the set of all sequences defined on real vector space X. The $n-norm$ is a function satisfying following four properties:

1. $\|(z_1, z_2, ..., z_n)\|_n = 0$ if and only if $z_1, z_2, ..., z_n$ are linearly dependent,
2. $\|(z_1, z_2, ..., z_n)\|_n$ is invariant under permutation,
3. $\|(z_1, z_2, ..., \alpha z_n)\|_n = |\alpha| \|(z_1, z_2, ..., z_n)\|_n$ for any real number α,
4. $\|(z_1, z_2, ..., z_{n-1}, x+y)\|_n \leq \|(z_1, z_2, ..., z_{n-1}, x)\|_n + \|(z_1, z_2, ..., z_{n-1}, y)\|_n$

for all $z_1, z_2, ..., z_n \in X$. It is known that $(X, \|.\|_n)$ is a $n-Banach$ space. The concept of 2-normed spaces was firstly introduced by Gahler $[6]$ and Misiak $[1]$ generalized these spaces to linear $n-$normed spaces. Many authors have introduced new $n-$normed sequence spaces and studied their various properties. Let A and B be two sequence spaces. A superposition operator P_g acts from A to B is defined by $P_g(x) = (g(k, x_k))$, where $g : \mathbb{N} \times X \to X$ is a function such that $g(k, 0) = 0$. The superposition operators were studied by Dedagich and Zabreiko $[4]$, Petrantuarat and Kemprasit $[7]$, Kolk and Raidjoe $[3]$, Sağır and Güngör $[2]$, Oğur $[5]$ and others.

Let X be a linear $n-$normed space. Let define $n-$normed sequence spaces as following;

$$\ell^{n,p} := \left\{ x = (x_k) \in w_X : \sum_{k=1}^{\infty} \|x_k, z_1, z_2, ..., z_{n-1}\|^p < \infty \text{ for all } z_1, z_2, ..., z_{n-1} \in X \right\},$$

$$\ell^{n,\infty} := \left\{ x = (x_k) \in w_X : \sup_k \|x_k, z_1, z_2, ..., z_{n-1}\| < \infty \text{ for all } z_1, z_2, ..., z_{n-1} \in X \right\}$$ and

$$c_{n,0} := \left\{ x = (x_k) \in w_X : \lim_{k\to\infty} \|x_k, z_1, z_2, ..., z_{n-1}\| = 0 \text{ for all } z_1, z_2, ..., z_{n-1} \in X \right\}.$$

It is easy to check that these spaces are Banach space with the norms

$$\|x\|_{n,p} = \left(\sum_{k=1}^{\infty} \|x_k, z_1, z_2, ..., z_{n-1}\|^p \right)^{\frac{1}{p}} \quad , \quad \|x\|_{n,\infty} = \sup_k \|x_k, z_1, z_2, ..., z_{n-1}\|$$ for every $z_1, z_2, ..., z_n \in X.$, respectively. In this paper, we study on characterization of superposition operators P_g acts from the spaces $c_{n,0}$, $\ell^{n,\infty}$ and $\ell^{n,p}$, $1 \le p < \infty$, to $\ell^{n,1}$ such that $p_g(x) = \left(g(k, x_k)\right)$, where $g : \mathbb{N} \times X \to X$.

Key Words: $n-$normed space, superposition operator, sequence space

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Accretive Canonical Type Quasi-Differential Operators for First Order

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ABSTRACT

It is known that a linear closed densely defined operator in any Hilbert space is called accretive if its real part is non-negative and maximal accretive if it is accretive and it does not have any proper accretive extension [1].

Note that the study of abstract extension problems for operators on Hilbert spaces goes at least back to J.von Neumann [2] such that in [2] a full characterization of all selfadjoint extensions of a given closed symmetric operator with equal deficiency indices was investigated.

Class of accretive operators is an important class of non-selfadjoint operators in the operator theory. Note that spectrum set of the accretive operators lies in right half-plane.

The maximal accretive extensions of the minimal operator generated by regular differential-operator expression in Hilbert space of vector-functions defined in one finite interval case and their spectral analysis have been studied by V. V. Levchuk [3].

In this work, using the method Calkin-Gorbachuk all maximal accretive extensions of the minimal operator generated by linear canonical type quasi-differential operator expression in the weighted Hilbert space of the vector functions defined at right semi-axis are described. Lastly, geometry of spectrum set of these type extensions will be investigated.

Key Words: Accretive operator, quasi-differential operator, spectrum.

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Canonical Type First Order Boundedly Solvable Differential Operators

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ABSTRACT

The general information on the degenerate differential equations in Banach spaces can be found in book of A. Favini and A. Yagi [1]. The fundamental interest to such equations are motivated by applications in different fields of life sciences.

The solvability of the considered problems may be seen as boundedly solvability of linear differential operators in corresponding functional Banach spaces. Note that the theory of boundedly solvable extensions of a linear densely defined closed operator in Hilbert spaces was presented in the important works of M. I. Vishik in [2], [3]. Generalization of these results to the nonlinear and complete additive Hausdorff topological spaces in abstract terms of abstract boundary conditions have been done by B. K. Kokebaev, M. O. Otelbaev and A. N. Synybekov in [4]-[6]. Another approach to the description of regular extensions for some classes of linear differential operators in Hilbert spaces of vector-functions at finite interval has been offered by A. A. Dezin [7] and N. I. Pivtorak [8].

Remember that a linear closed densely defined operator on any Hilbert space is called boundedly solvable, if it is one-to-one and onto and its inverse is bounded.

The main goal of this work is to describe of all boundedly solvable extensions of the minimal operator generated by first-order linear canonical type differential-operator expression in the weighted Hilbert space of vector-functions at finite interval in terms of boundary conditions by using the methods of operator theory. Later on, the structure of spectrum of these type extension will be investigated.

Key Words: Boundedly solvable operator, differential-operator expression, spectrum.

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Compactness of Upper Triangular One-Band Block Operator Matrices

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ABSTRACT

It is known that the traditional infinite direct sum of Hilbert spaces $H_n,\ n \geq 1$ is defined as

$$H = \mathop{\oplus}_{n=1}^{\infty} H_n = \left\{ u = (u_n) : u_n \in H_n,\ \ n \geq 1,\ \ \sum_{n=1}^{\infty} \|u_n\|_{H_n}^2 < +\infty \right\}.$$

Note that H is a Hilbert space with norm induced by the inner product

$$(u,v)_H = \sum_{n=1}^{\infty} (u_n, v_n)_{H_n},\ u,v \in H,$$

(see [1]).

It is known that many of the physical problems of today arising in the modelling of processes of multiparticle quantum mechanics, quantum field theory and in the physics of rigid bodies support to study a theory of linear operators in the direct sum of Hilbert spaces (see [2] and references in it).

Investigation of these problems in direction of spectral analysis of finite or infinite dimensional real and complex entries special matrices (upper and lower triangular double-band or third-band or Toeplitz types) in sequences spaces $w,\ c,\ c_0,\ bs,\ bw_p,\ l_p$ have been widely studied in the current literature.

On the other hand some spectral analysis of 2×2 and 3×3 types block operator matrices have been studied in [3], [4]. Note that the structure of spectrum of diagonal operator matrices has been surveyed in [5]. The compactness properties and belonging to Schatten-von Neumann classes of diagonal operator matrices in the direct sum of Hilbert spaces has been examined in [6].

In this work the compactness properties of upper triangular one-band block operator matrices in the infinite direct sum of Hilbert spaces has been studied. And also belonging to Schatten-von Neumann Classes these type operators will be investigated.

Key Words: Direct sum of Hilbert spaces, upper triangular one-band block operator matrix, Schatten-von Neumann classes.

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Discreteness of Spectrum of Normal Differential Operators for First Order

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ABSTRACT

It is well-known that in the operator theory, for any differential operator the important questions are:

1. which differential expression; it is generated by in the corresponding functional space?
2. which boundary conditions, it is generated by?
3. which special class does it belong to?
4. in which cases its spectrum is discrete?

(see [1]).

Remember that a densely defined closed operator N in any Hilbert space is called formally normal if $D(N) \subset D(N^*)$ and $\|Nf\| = \|N^* f\|$ for all $f \in D(N)$, where N^* is the adjoint to the operator N. If a formally normal operator has no formally normal extension, then it is called maximal formally normal operator. If a formally normal operator N satisfies the condition $D(N) = D(N^*)$, then it is called a normal operator [2].

Generalization of J. Von Neumann's theory to the theory of normal extensions of formally normal operators in Hilbert space has been obtained by E. A. Coddington in [2]. The first results in the area of normal extension of unbounded formally normal operators in a Hilbert space are also due to Y. Kilpi [3], [4] and R. H. Davis [5]. Some applications of this theory to two-point regular type first order differential operators in Hilbert space of vector functions can be found in [6] (also see references therein).

In this work we will consider the differential-operator expression given by

$$l(u) = (\alpha u)'(t) + Au(t)$$

in the weighted Hilbert space $L_\alpha(H,(a,\infty))$, where H is a Hilbert space, $a \in \mathbb{R}$, $\alpha : (a,\infty) \to (0,\infty)$, $\alpha \in C(a,\infty)$, $\dfrac{1}{\alpha} \in L^1(a,\infty)$, $A : D(A) \subset H \to H$ is a selfadjoint operator, $A \geq E$ and $E : H \to H$ is an identity operator. Connected with this differential expression it can be constructed the minimal and maximal operators in $L_\alpha(H,(a,\infty))$ (see [7]). In this case, it can be shown that the minimal operator is formally normal, but it is not maximal. The all normal extensions of the minimal operator and their spectrum have been studied in [8].

In this work under the condition $A^{-1} \in C_\infty(H)$, we will investigate the discreteness of spectrum of normal extensions in detail. Later on, the asymptotical behavior of eigenvalues of any normal extension will be examined.

Key Words: Differential operator, formally normal and normal operator, spectrum.

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Explicit Upper Bounds For The Spectral Distance Of Two Trace Class Operators

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ABSTRACT

Given an arbitrary compact operator A on a separable Hilbert space an interesting question of practical importance is to determine its spectrum σ(A). One way of tackling it numerically is to reduce the infinite-dimensional problem to a finite-dimensional one by manufacturing a sequence of finite rank operators $(A_k)^{\infty}_{k=1}$ converging to A in operator norm and using the fact that the eigenvalues of A_k, converge to the eigenvalues of A. In practice, one has to stop after a finite number of steps. If there is interest in error estimates, the problem arises how to explicitly bound the distance of the spectrum of a finite rank approximant A_k to the spectrum of A in a suitable sense. A popular choice is the Hausdorff metric. In order to make this question more precise, the main concern of the present article is the following. Given two trace class operators A and B on a separable Hilbert space we provide an explicitly computable upper bound for the Hausdorff distance of their spectra involving only the distance of A and B in operator norm and the singular values of A and B. By specifying particular asymptotics of the singular values our bound reproduces or improves existing bounds for the spectral distance. The proof is based on lower and upper bounds for determinants of trace class operators of independent interest.

Key Words: Spectral distance, spectral variation, trace class operators, determinants.

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Generalized Cesaro Summability of Order β

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ABSTRACT

The theory of sequence of fuzzy numbers was introduced by Matloka [5], where he proved some basic theorems related to sequences of fuzzy numbers. Later, the notion of statistical convergence for sequences of fuzzy numbers was defined and studied by Nuray and Savaş [6]. Çolak [4] generalized the statistical convergence by ordering the interval [0,1] and defined the statistical convergence of order α and strong p-Cesàro summability of order α, where $0<\alpha\leq 1$ and p is a positive real number. Altinok et al. [2] introduced the concepts of statistical convergence of order β and strong p-Cesàro summability of order β for sequences of fuzzy numbers. Aizpuru et al. [1] defined the f-density of the subset A of $\mathbb{N}$ by using an unbounded modulus function. After then, Bhardwaj [3] introduced strong Cesaro summability of order α with respect to a modulus function f for real sequences. The purpose of this paper is to generalize the study of Bhardwaj [3] and Çolak [4] applying to sequences of fuzzy numbers so as to fill up the existing gaps in the summability theory of fuzzy numbers. On account of this, in the present paper, we define the concept strongly (Δ^m,f)-Cesaro summability of order β for $\beta \in (0,1]$ using an unbounded modulus function f and generalized difference operator Δ^m in sequences of fuzzy numbers and give some inclusion relations.

Key Words: Fuzzy sequence, statistical convergence, Cesàro summability, modulus function, difference operator.

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Hermite-Hadamard Inequality for Strongly GA-Convex Functions

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ABSTRACT

Since the definition that forms the basis of the convexity is expressed by inequality, there is very important place of inequality in convex functions. Many inequalities have been established for convex functions but the most famous inequality is the Hermite-Hadamard inequality, due to its rich geometrical significance and applications [1].

The $f: I \subseteq \mathbb{R} \to \mathbb{R}$ is a convex function defined on the interval *I* of real numbers and $a, b \in I$ with $a < b$. The following inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

holds. This double inequality is known in the literature as Hermite-Hadamard inequality for convex functions [4].

Convexity theory has been generalized and extended in various directions using innovative techniques. One of the most important classes of convex functions is strongly convex functions. The strongly convex functions also play an important role in optimization theory and mathematical economics, see [1].

Definition 1. Let $I \subseteq \mathbb{R}$ be an interval and *c* be a positive number. A function $f: I = [a,b] \subset \mathbb{R} \to \mathbb{R}$ is called strongly convex with modulus $c > 0$, if

$$f\big((1-t)x + ty\big) \leq (1-t)f(x) + tf(y) - ct(1-t)\|y-x\|^2 \in I, \ \ \forall x, y \in I, t \in [0,1].$$

Definition 2. Let I be a interval, $f: I \subset \mathbb{R}^+ \to \mathbb{R}$ is said to be strongly GA-convex with modulus $c > 0$, if

$$f(x^{1-t}y^t) \leq (1-t)f(x) + tf(y) - ct(1-t)\|\ln y - \ln x\|^2$$

for all $x, y \in I \ \ ve \ \ t \in [0,1]$.

In this paper we obtain the Hermite-Hadamard inequality for strongly GA-convex function. Using this strongly GA-convex function we get the new theorem and corollary.

Key Words: Hermite-Hadamard inequality, convex functions, strongly convex functions.

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Hermite-Hadamard Inequality for Strongly p-Convex Functions

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ABSTRACT

Since the definition that forms the basis of the convexity is expressed by inequality, there is very important place of inequality in convex functions. Many inequalities have been established for convex functions but the most famous inequality is the Hermite-Hadamard inequality, due to its rich geometrical significance and applications [1].

The $f: I \subseteq \mathbb{R} \to \mathbb{R}$ is a convex function defined on the interval *I* of real numbers and $a, b \in I$ with $a < b$. The following inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

holds. This double inequality is known in the literature as Hermite-Hadamard inequality for convex functions [4].

Convexity theory has been generalized and extended in various directions using innovative techniques. One of the most important classes of convex functions is strongly convex functions. The strongly convex functions also play an important role in optimization theory and mathematical economics, see [1].

Definition 1. Let $I \subseteq \mathbb{R}$ be an interval and *c* be a positive number. A function $f: I = [a,b] \subset \mathbb{R} \to \mathbb{R}$ is called strongly convex with modulus $c > 0$, if

$$f\big((1-t)x + ty\big) \leq (1-t)f(x) + tf(y) - ct(1-t)\|y-x\|^2 \in I, \ \ \forall x,y \in I, t \in [0,1].$$

Definition 2. Let $I \subset (0,\infty)$ be a interval, $f: I \to \mathbb{R}$ is said to be strongly p-convex with modulus $c > 0$, if

$$f\big([(1-t)x^p + ty^p]^{1/p}\big) \leq (1-t)f(x) + tf(y) - ct(1-t)\|y^p - x^p\|^2$$

for all $x, y \in I \;\; ve \;\; t \in [0,1]$.

In this paper we obtain the Hermite-Hadamard inequality for strongly p-convex function. Using this strongly p-convex function we get the new theorem and corollary.

Key Words: Hermite-Hadamard inequality, p-convex functions, strongly p-convex functions.

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Hermite-Hadamard Type Inequalities For Quasi-Convex Functions Via Fractional Integrals

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ABSTRACT

Let $f:I\ \mathrm{R}\rightarrow \mathrm{R}$ be a convex mapping defined on the interval I of real numbers and *a, b in I* with $a<b$.The following double inequality:

$$f\left(\frac{a+b}{2}\right)\le \frac{1}{b-a}\int_a^b f(x)dx\le \frac{f(a)+f(b)}{2}$$

is known in the literature as the Hadamard inequality for convex mapping. Note that some of the classical inequalities for means can be derived from the double inequality above for appropriate particular selections of the mapping *f.* Both inequalities hold in the reversed direction if *f* is concave. At the entrance of the work, we give defines and theorems because we will use them throughout the study It is seen that some of our results correspond to some of known inequalities in these theorems. Also the history of integral inequalities, convex functions theory and quasi-convex functions is briefly mentioned. On the main part of the work, we give a fractional integral identity for differentiable functions that was derived by Yetgin et. al. By using this identity, we obtain some new general integral inequalities for fractional integrals. These general integral inequalities are obtained for the functions whose derivatives in absolute value at certain power are quasi-convex. We have examined these inequalities for different values of n and have finally obtained new integral inequalities.

Key Words: Quasi-Convex Function, Convex Functions, General Integral Inequalities, Hermite-Hadamard Inequalities, Fractional Integral.

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Improvement of Hermite-Hadamard Type and Midpoint Type Inequalities for Convex Functions via Conformable Fractional Integral

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ABSTRACT

Mathematical inequalities are one of the most significant instruments in many branches of mathematics such as functional analysis, theory of differential and integral equations, interpolation theory, harmonic analysis, probability theory, etc. They have very useful applications area in mechanics, physics and other sciences. The first comprehensive study in inequalities was made in the first half of the 20. century and in the second half of the century inequalities attracted the attention of many researchers. Especially, in recent years, so many study have been published. that inequalities may be considered as a different area of mathematics.

Convexity theory featured an important and fundamental role in the developments of various branches of engineering, financial mathematics, economics and optimization. In literature, there are hundreds studies for Hermite Hadamard type inequality by using the left and right fractional integrals (such as Riemann Liouville fractional integrals, Hadamard fractional integrals, Conformable fractional integrals etc.). In all of them, the left and right fractional integrals are used together. As much as we know, the first study for Hermite-Hadamard type inequality by using only the right Riemann-Liouville fractional integral is given in [6] by Kunt et al. In this paper, it is proved that conformable fractional Hermite-Hadamard inequality and conformable fractional Hermite Hadamard Fejér inequality are just results of Hermite Hadamard Fejér inequality. After this, a new conformable fractional Hermite Hadamard inequality which is not a result of Hermite Hadamard Fejér inequality and better than given in [7] by Set et. al. is obtained. Also, a new equality is proved and some new conformable fractional midpoint type inequalities are given. Our results have some relations with the results given in [5, 6].

Key Words: Convex functions, Hermite-Hadamard inequality, right conformable fractional integral, Trapezoid type inequalities, Midpoint type inequalities.

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Lacunary Statistical Convergence for Sequences of Dual Numbers

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ABSTRACT

Dual numbers was first introduced by Clifford in [1]. This concept has lots of applications; to screw systems, modelling plane joint, iterative methods for displacement analysis of spatial mechanisms, inertial force analysis of spatial mechanisms etc. The dual numbers has the form $d = a + b\varepsilon$, where ε is the dual unit and $\varepsilon^2 = 0$, $\varepsilon \neq 0$. $D = R[\varepsilon] = \{a + b\varepsilon : a,b \in R\}$ is called dual number system. It is two dimensional commutative associative algebra over the R. It was studied in summability theory by some authors. (see [2])

Zygmund introduced the idea of statistical convergence in [3]. Further, Fast and Schoenberg introduced statistical convergence to assign limit to sequences which are not convergent in the usual sense independently in the same year (see [4],[5]). Rapid developments were started after the papers of Fridy [6] and Šalát [7]. They used the asymptotic density of a set $A \subset N$, the set of positive integers as follows:

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n} \left| \{k \le n : k \in A\} \right|$$

whenever the limit exists. $|\{.\}|$ denotes the cardinality of the enclosed set. A sequence (x_k) of numbers is said to be statistically convergence to a number L provided that for every $\varepsilon > 0$, $\lim_{n\to\infty} \frac{1}{n} \left| \{k \le n : |x_k - L| \ge \varepsilon\} \right| = 0$. This notion is used an effective tool to resolve many problems in ergodic theory, fuzzy set theory, trigonometric series and Banach spaces in the past years.

By a lacunary sequence $\theta=(k_r)$ $(r=0,1,2,...)$ where $k_0=0$, we mean an increasing sequence of non-negative integers with $k_r-k_{r-1}\to\infty$ as $r\to\infty$. $I_r=(k_{r-1},k_r]$ denotes the intervals determined by θ, $h_r=k_r-k_{r-1}$ and $q_r=\frac{k_r}{k_{r-1}}$. Let $K\subset N$. θ-density of a set K was defined by $\delta_\theta(K)=\lim_{r\to\infty}\frac{1}{h_r}|K\cap I_r|$. A sequence (x_k) is called lacunary statistically convergent to L provided that for every $\varepsilon>0$, $\lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k\in I_r:|x_k-L|\geq\varepsilon\right\}\right|=0$ (see [8]). In this study, we introduce the notion of lacunary statistical convergence for sequences of dual numbers. We investigate some basic properties of this new concept.

Key Words: Statistical convergence, dual numbers, lacunary sequence.

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Lucas Difference Sequence Spaces $c_0(\hat{L},\Delta)$ and $c(\hat{L},\Delta)$

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ABSTRACT

Define the sequence $\{f_n\}_{n=0}^{\infty}$of Fibonacci numbers and define the sequence $\{L_n\}_{n=0}^{\infty}$ of Lucas numbers given by the linear recurrence relations

$$f_0 = 0, \qquad f_1 = 1 \text{ and } f_n = f_{n-1} + f_{n-2}, n \geq 2,$$
$$L_0 = 2, \;\; L_1 = 1 \text{ and } L_n = L_{n-1} + L_{n-2}, n \geq 2.$$

Fibonacci and Lucas numbers have many interesting properties and applications in arts, sciences and architecture. For example, the ratio sequences of Fibonacci and Lucas numbers converges to the golden ratio which is important in sciences and arts. Also, some basic properties of Fibonacci and Lucas numbers [4] are given as follows:

$$\sum_{k=1}^{n} f_k^2 = f_n f_{n+1} \text{ and } \sum_{k=1}^{n} f_k = f_{n+2} - 1, \;\; n \geq 1$$
$$\sum_{k=1}^{n} L_{2k} = L_{2n+1} - 1 \text{ and } \sum_{k=1}^{n} L_k = L_{n+2} - 3.$$

Recently, Karakaş and Karabudak [1] established the regular matrix E by using Lucas numbers. Then, by introducing the sequence space $X(E)$ with the help of matrix E, they showed that this spaces is a BK-space, where $E = (L_{nk})$ is defined by

$$L_{nk} = \begin{cases} \dfrac{L_{k-1}^2}{L_n L_{n-1} + 2}, & 1 \leq k \leq n, \\ 0, & k > 0 \end{cases}$$

for all $k, n \in \mathbb{N}$ and

$$X(E) = \{x = (x_k) \in w : y = (y_k) \in X\}.$$

In [1], defined the sequence $y = (y_k)$ by the $\hat{L}$ transform of a sequence $x = (x_k)$,

$$y = (y_k) = E_k(x) = \frac{1}{L_k L_{k-1} + 2} \sum_{i=1}^{k} L_{i-1}^2 x_i.$$

Now, let us define the sets $c_0(\hat{L},\Delta)$ and $c(\hat{L},\Delta)$ whose $\hat{L} = (\hat{L}_{nk})$ transforms is in the well-known sequence spaces c_0 and c, respectively,

$$c_0(\hat{L},\Delta) = \left\{x = (x_k) \in w: \lim_{n\to\infty}\left(\frac{L_nL_{n-1}+2}{L_{n-1}^2}x_n - \frac{L_{n-1}L_{n-2}+2}{L_{n-1}^2}x_{n-1}\right) = 0\right\}$$

$$c(\hat{L},\Delta) = \left\{x = (x_k) \in w: \lim_{n\to\infty}\left(\frac{L_nL_{n-1}+2}{L_{n-1}^2}x_n - \frac{L_{n-1}L_{n-2}+2}{L_{n-1}^2}x_{n-1}\right) \text{exist}\right\}$$

where $\hat{L} = (\hat{L}_{nk})$ is the double band matrix defined by the sequence (L_n) of Lucas numbers as follows

$$L_{nk} = \begin{cases} -\dfrac{L_{n-1}L_{n-2}+2}{L_{n-1}^2} & , \quad k = n-1 \\ \dfrac{L_nL_{n-1}+2}{L_{n-1}^2} & , \quad k = n \\ 0 & , \quad \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$.

The main purpose of this study is to introduce the sequence spaces $c_0(\hat{L},\Delta)$and $c(\hat{L},\Delta)$. Furthermore, the inclusions $c_0 \subset c_0(\hat{L},\Delta)$and $c \subset c(\hat{L},\Delta)$ strictly hold, the basis of the sequence spaces $c_0(\hat{L},\Delta)$ and $c(\hat{L},\Delta)$ is constructed and alpha-, beta- and gamma-duals of these spaces are determined. Finally, the classes of matrix transformations from the new sequence spaces to the spaces ℓ_∞, c and c_0 are characterized.

Key Words: Lucas sequence spaces, matrix domain, matrix transformations.

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Midpoint Type Inequalities for the Multiplicatively P-Functions

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ABSTRACT

In this paper, we introduce a new class of extended multiplicatively P-function. Some new Hermite Hadamard type inequalities are obtained. Some special cases are discussed. Results represent significant refinement and improvement of the previous results. Ideas of this paper may stimulate further research about multiplicatively P-function. We should especially mention that the definition of multiplicatively P-function is given for the first time in the literature by us.

In introduction, firstly, the concepts of convexity, Hermite Hadamard inequality, P-function and multiplicatively P-functions are given. After the definition of multiplicatively P-function is given, some algebraic properties for multiplicatively P-function are investigated. The classical Hermite-Hadamard inequality provides estimates of the mean value of a continuous convex function, so Hermite Hadamard integral inequality is very important for convexity theory. The history of this inequality begins with the papers of Ch. Hermite and J. Hadamard in the years 1883-1893. Some refinements of the Hermite Hadamard inequality on convex functions have been extensively studied by a number of authors (e.g., [1, 2, 3, 4, 5, 6]) and the Authors obtained a new refinement of the Hermite Hadamard integral inequality for convex functions.

In our paper, by using an integral identity together with both the Hölder and the Power-mean integral inequality we establish several new inequalities for multiplicatively P-functions.

Key Words: Convex function, Multiplicatively P-function, Hölder Integral inequality and Power-Mean Integral inequality, Hermite-Hadamard type inequality.

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Modelling of Turkey Health Survey Data with Artificial Neural Networks

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ABSTRACT

Nowadays, the increasing amount and variety of data bring along situations where classical statistical methods may be insufficient. Therefore, various methods need to be used. Data mining is one of these methods. The main purpose of the data mining is to extract meaningful information from the increasingly large volume data. At the same time, it includes discovering the unpredictable relationships between these complex data and revealing the results in an understandable way [1, 2]. Increased technological developments have increased the interest in data mining applications because the convenience of collecting and storing data is reflected in the cost of data processing [3]. Data must first be passed through the pre-processing step to be able to apply data mining techniques. This step is a very important process in data mining and it is a large part of the total process. Artificial neural networks, which is one of the methods in data mining, are described as an algorithm that is affected by the basic behaviour of biological nerve cell and expose a mathematical model. According to the definition of biological nerve cell, the artificial nerve cell collects signals from other nerve cells and when the total signal amplitude is greater than a certain value, it transmits the signal of the artificial nerve cell to another nerve cell [4].

In this study, "Turkey Health Survey" data which carried out by TURKSTAT every two years is used. It is aimed to estimate the general health status of persons with the data of 2016 using the model obtained with the data of 2014 to determine the variables affecting the general health status of the persons in the study. For this purpose, “Turkey Health Survey" data of 2014 and 2016 which is obtained from TURKSTAT are used. While the Turkey Health Survey included the data of 26075 observation units and 471 variables in 2014, it contained 23606 observation units

and 445 variables in 2016. The information about the "0-7" and "7-14" age range was extracted from the data of the years 2014 and 2016. Since the 2014 and 2016 surveys have changed over time, the data set has been determined by using variables that are common every two years and subtracting some variables. As a result, 19129 observation units and 80 variables for 2014, 15866 observation units and 80 variables for 2016 were used. Then, SAS Enterprise Guide 5.1 and SPSS MODELER IBM 16.0 programs were used appropriately for the analysis. After determining the target variable, the artificial neural network algorithm was applied to the 2014 year data and the most effective variables in the "general health condition" were determined as “duration of sports”, “muscle strengthening period”, “disease health status” and “weight”, respectively. When the artificial neural network algorithm was applied to 2016 year data, the most effective variables in the "general health condition" were determined as “restriction of vital activity”, “duration of sports activity", “disease health status” and “weight”, respectively. As a result, it is seen that the values realized by the model correctly predicted the general health status of the persons by 63.23%.

Key Words: Data Mining, Artificial Neural Network, Statistics, Turkey Health Survey, TURKSTAT.

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Multiplicatively P-functions

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ABSTRACT

In this paper, we introduce a new class of extended multiplicatively P-function. Some new Hermite-Hadamard type integral inequalities are derived. Some special cases are discussed. Results represent significant refinement and improvement of the previous results. Ideas of this paper may stimulate further research. We should especially mention that the definition of multiplicatively P-function is given for the first time in the literature by us.

Convexity theory has appeared as a powerful technique to study a wide class of unrelated problems in pure and applied sciences. One of the most important integral inequalities for convex functions is the Hermite-Hadamard inequality. The classical Hermite-Hadamard inequality provides estimates of the mean value of a continuous convex function. Some refinements of the Hermite-Hadamard inequality on convex functions have been extensively investigated by a number of authors (e.g., [3, 4, 5, 6]) and the Authors obtained a new refinement of the Hermite-Hadamard integral inequality for convex functions.

The main purpose of this paper is to establish new estimations and refinements of the Hermite-Hadamard inequality for functions whose derivatives in absolute value are multiplicatively P-functions. After the definition of multiplicatively P-function is given, some algebraic properties for multiplicatively P-function are investigated. Then, by using an integral identity together with both the Hölder and the Power-mean integral inequality we establish several new inequalities for multiplicatively P-functions.

Key Words: Convex function, Multiplicatively P-function, Hölder and Power-Mean Integral inequality, Hermite-Hadamard type inequality.

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Multiplication Operators On Grand Lorentz Spaces

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ABSTRACT

Let (X,Σ,μ) be a σ-finite measure space and f be a complex-valued measurable function defined on X. The distribution function of f is defined by

$$D_f(\lambda)=\mu\left(\left\{x\in X:\left|f(x)\right|>\lambda\right\}\right) \text{ for all } \lambda\geq 0.$$

By f^*, we mean the non-increasing rearrangement of given function f as

$$f^*(t)=\inf\left\{\lambda>0:D_f(\lambda)\leq t\right\}=\sup\left\{\lambda>0:D_f(\lambda)>t\right\},\ t>0.$$

For a measurable function f on $X=(0,1)$, $\|f\|_{p,q)}$ is defined as

$$\|f\|_{p,q)}=\begin{cases}\sup\limits_{0<\varepsilon<q-1}\left(\dfrac{q}{p}\varepsilon\displaystyle\int_0^1 t^{\frac{q}{p}-1}\left(f^*(t)\right)^{q-\varepsilon}\mathrm{dt}\right)^{\frac{1}{q-\varepsilon}} & ;1<q<\infty\\ \sup\limits_{0<t<1} t^{\frac{1}{p}}f^*(t) & ;q=\infty.\end{cases}$$

The grand Lorentz spaces $L_{p,q)}$ consists of those complex-valued measurable functions defined on $X=(0,1)$ such that $\|f\|_{p,q)}<\infty$. Cleary, if $p=q$, then $L_{p,q)}$ is equal to grand Lebesgue space $L_{p)}$.

Let $u:X\to\mathbb{C}$ be a measurable function such that $u\cdot f\in M(X,\Sigma)$ whenever $f\in M(X,\Sigma)$ where $M(X,\Sigma)$ is the set of all measurable functions defined on X. This gives rise to a linear transformation $M_u:M(X,\Sigma)\to M(X,\Sigma)$ defined by $M_u(f)=\mathrm{u}\cdot f$, where the product of functions is pointwise. In case if $M(X,\Sigma)$ is a topological vector space and M_u is a continuous (bounded) operator, then it is called a multiplication operator induced by u.

Multiplication operators have been studied on various function spaces by various authors such as Abrahamse [1], Komal [5], Nakai [6], Singh [7] and Takagi [8]. Along the line of their arguments we will study the multiplication operators on the grand Lorentz spaces $L_{p,q)}$. For this purpose, we will characterize the invertibility of M_u on $L_{p,q)}$ and find necessary and sufficient conditions for Compact multiplication operators.

In this paper, multiplication operators on grand Lorentz spaces are defined and the fundamental properties such as boundedness, closed range, invertibility, compactness and closedness of these are characterized.

Key Words: Grand Lorentz space, Multiplication operator, Compact operator

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New Inequalities for Operator (alpha,*m*)-preinvex Functions in Hilbert Spaces

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ABSTRACT

A. G. Ghazanfari et all. [1] established a Hermite- Hadamard type inequality for operator preinvex functions and an estimate of the right hand side of a Hermite-Hadamard type inequality in which some operator preinvex functions of selfadjoint operators in Hilbert spaces are involved. S.-H. Wang and X.-M. Liu [2] introduced the concept of operator s-preinvex function. They established some new Hermite-Hadamard type inequalities for operator s-preinvex functions, and provided the estimates of both sides of Hermite-Hadamard type inequality in which some operator s-preinvex functions of positive selfadjoint operators in Hilbert space was involved. And then, S.-H. Wang and X.-W.Sun [3] similarly introduced the concept of operator alpha-preinvex function and some inequalities. E. Unluyol et all. [4] defined the operator *m*-preinvex. They established some algebraic properties of operator *m*-preinvex functions. And then, they obtained new inequalities for operator *m*-preinvex functions in terms of continuous functions of self adjoint operators in Hilbert spaces. E. Unluyol and H. Karbuz [5] defined the operator (alpha,*m*)-preinvexity, and obtained some new integral inequalities. So, In this paper, firstly we introduced some definitions, theorems and lemmas for operator (alpha,*m*)-preinvex function in Hilbert spaces. Secondly we obtained new theorems and finally established new inequalities in terms of Hermite-Hadamard inequality.

Key Words: Hilbert space, preinvex, operator (alpha,*m*)-preinvex.

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New Inequalities on Lipschitz Functions

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ABSTRACT

This study is about getting for Lipschitz functions of some Hermite Hadamard type inequalities that are obtained for p-convex functions. Inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The Hermite Hadamard integral inequality is the first fundamental result for convex functions with a natural geometrical interpretation and many applications, has attracted and continues to attract much interest in elementary mathematics and the other sciences.

Many mathematicians have devoted their efforts to generalize, refine, counterpart and extend it for different classes of functions such as: convex functions, quasi-convex functions, harmonically convex functions, p-convex functions, s-convex functions, Godunova-Levin class of functions, log-convex and r−convex functions, p−functions, etc. or apply it for special means (Harmonic means, geometric means, aritmetic means, p−logarithmic means, identric mean, Stolarsky means, etc.).

In this study, at first, some basic definitions and theorems of the condition of Lipschitz, convex functions, p-convex functions and harmonic convex functions are given. Then, some inequalities Hermite Hadamard type obtained for p-convex functions are given for Lipschitz mappings. Also some applications for special means are given. Finally, we have new inequalities for Lipschitz functions by means of Hermite Hadamard type inequalities which are used for p-convex functions.

Key Words: Convex function, p-convex function, Lipschitz function.

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New Integral Inequalities for Operator *h*-preinvex Functions in Hilbert Spaces

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ABSTRACT

A. G. Ghazanfari et all. [1] established a Hermite- Hadamard type inequality for operator preinvex functions and an estimate of the right hand side of a Hermite-Hadamard type inequality in which some operator preinvex functions of selfadjoint operators in Hilbert spaces are involved. S.-H. Wang and X.-M. Liu [2] introduced the concept of operator s-preinvex function. They established some new Hermite-Hadamard type inequalities for operator s-preinvex functions, and provided the estimates of both sides of Hermite-Hadamard type inequality in which some operator s-preinvex functions of positive selfadjoint operators in Hilbert space was involved. And then, S.-H. Wang and X.-W.Sun [3] similarly introduced the concept of operator alpha-preinvex function and some inequalities. E. Unluyol et all.[4] defined the operator *h*-preinvex functions in a Hilbert space. Then they obtained some algebraic properties of this class. Moreover, they established some new integral inequalities for operator *h*-preinvex functions in terms of continuous functions of self adjoint operators in Hilbert spaces. Also E. Unluyol et all. [5] obtained some new inequalities for operator *h*-preinvex functions in a Hilbert space. So, in this paper firstly, we gave some definitions about operator h-preinvex functions. Secondly, we established new theorems and finally we got some new integral inequalities for operator *h*-preinvex functions in terms of self adjoint operators of continuous functions in Hilbert spaces.

Key Words: Hilbert space, preinvexity, operator *h*-preinvexity.

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New Type Integral Inequalities for n-Times Differentiable Prequasiinvex Functions

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ABSTRACT

Convexity plays a central role in mathematical economics, engineering, management science, and optimization theory. In recent years, research on generalized convexity has increased rapidly. In almost all branches of mathematics as well as in other areas of science, convexity and inequalities play an important role. Inequalities present an attractive and active field of research not only mathematics but also the other sciences. In recent years, various inequalities for convex functions and their variant forms are being developed using innovative techniques by researchers. Convexity theory has appeared as a powerful technique to study a wide class of unrelated problems in pure and applied sciences. In recent years, many mathematicians have been studying about preinvexity and types of preinvexity, see for example [1, 2, 3, 4, 5, 6, 7, 8] and references there in.

In this study, firstly, the concepts of convexity, Hermite-Hadamard inequality for convex functions, quasi-convex function, invex set, Hermite-Hadamard type integral inequalities for the preinvex functions, prequasiinvex function and condition C are given. Then, by using the an identity, Hölder integral inequality and Power mean integral inequality, we present new type integral inequalities for functions whose powers of nth derivatives in absolute value are prequasiinvex functions. This study is a generalization of some articles. The results obtained in special cases coincide with the well-known results in the literature.

Key Words: Invex set, Preinvex function, Prequasiinvex function, Hölder Integral inequality, Power-mean integral inequality.

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Non-Newtonian Superposition Operators on $\ell_p(N)$, $c_0(N)$ and $c(N)$ into $\ell_1(N)$

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ABSTRACT

In this paper, we define a non-Newtonian superposition operator ${}_N P_f$ where $f:\mathbb{N}\times\mathbb{R}(N)_\alpha \to \mathbb{R}(N)_\beta$ by ${}_N P_f((x_k)) = \left(f(k,x_k)\right)_{k=1}^{\infty}$ for every non-Newtonian real sequence (x_k) and we characterize non-Newtonian superposition operators on $\ell_\infty(N)$, $\ell_p(N)$, $c_0(N)$ and $c(N)$ into $\ell_1(N)$.

Under the assumption that $f(k,.)$ is continuous on $\mathbb{R}$ for every $k\in\mathbb{N}$, Chew and Lee [1] have characterized $P_f:\ell_p \to \ell_1$ and $P_f:c_0\to\ell_1$ for $1\le p<\infty$. Dedagich and Zabreiko [2] have given the necessary and sufficient conditions for the superposition operators on the sequence spaces ℓ_p, ℓ_∞ and c_0. The purpose of this paper is to generalize these works respect to the non-Newtonian calculus which have created by Grossman and Katz [3].

Firstly, the information about the studies that are done until today and the application areas, are briefly given. Non-Newtonian calculus is introduced which is an alternative to the classical calculus, definitions, theorems and properties were given which will be used at this study. The non-Newtonian superposition operator is defined using the definition in classical calculus. Then we characterize ${}_N P_f:\ell_\infty(N)\to\ell_1(N)$, ${}_N P_f:c_0(N)\to\ell_1(N)$, ${}_N P_f:c(N)\to\ell_1(N)$ and ${}_N P_f:\ell_p(N)\to\ell_1(N)$, respectively. The applications of the given theorems are explained with examples. Finally, we show that such ${}_N P_f$ which acting from $\ell_\infty(N)$ to $\ell_1(N)$, is *-continuous if and only if $f(k,.)$ is *-continuous for every $k\in\mathbb{N}$.

Key Words: Continuity, non-Newtonian superposition operator, non-Newtonian sequence spaces.

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On I_σ-Convergence Of Folner Sequences On Amenable Semigroups

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ABSTRACT

The concepts of σ-uniform density of subsets *A* of the set *N* of positive integers and corresponding I_σ-convergence were introduced by Nuray et al. Furthermore, inclusion relations between I_σ-convergence and invariant convergence were given.

In this study, the concepts of σ-uniform density of subsets *A* of the set natural number of positive integers and corresponding I_σ-convergence of functions defined on discrete countable amenable semigroups were introduced. Furthermore, for any Folner sequence inclusion relations between I_σ-convergence and invariant convergence also I_σ-convergence and p strongly invariant summability were given. We introduce the concept of I_σ-statistical convergence and I_σ-lacunary statistical convergence of functions defined on discrete countable amenable semigroups. In addition to these definitions, we give some inclusion theorems. Also, we make a new approach to the notions of strongly λ -summability, σ-convergence and λ-statistical convergence of Folner sequences by using ideals and introduce new notions, namely, I_σ-strongly-λ-summability, I_σ-λ-statistical convergence of Folner sequences. We mainly examine the relation between these two methods as also the relation between I_σ-statistical convergence and I_σ-λ-statistical convergence of Folner sequences introduced by the author recently.

We introduce the concepts of I_σ-statistical asymptotically equivalent, $I_{\sigma,\lambda}$-statistical asymptotically equivalent, I_σ-asymptotically lacunary statistical equivalent and strongly I_σ-asymptotically lacunary equivalent functions defined on discrete countable amenable semigroups, and establish certain inclusion theorems.

Key Words: Folner sequence, amenable group, invariant, asymptotically equivalent

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On (Δ^m,f)-Statistical Convergence Of Order β

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ABSTRACT

In order to generalize the concept of convergence of real sequences, the notion of statistical convergence was introduced by Fast [3] and Schoenberg [6], independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory and Number theory. The concepts of fuzzy sets and basic operations on a fuzzy set were introduced by Zadeh [7] as an extension of the concept of classical set Matloka [4] defined the notion of fuzzy sequence and introduced bounded and convergent sequences of fuzzy real numbers and showed that every convergent sequence of fuzzy numbers is bounded. After then, Nuray and Savaş [5] defined the notion of statistical convergence for sequences of fuzzy numbers. Çolak [2] generalized the statistical convergence by ordering the interval [0,1] and defined the statistical convergence of order α and strong p-Cesàro summability of order α, where $0<\alpha\leq 1$ and p is a positive real number. Altinok et al. [1] introduced the concepts of statistical convergence of order β and strong p-Cesàro summability of order β for sequences of fuzzy numbers. In the present paper, we define the concepts of (Δ^m,f)-statistical convergence of order β for β ϵ (0,1] using an unbounded modulus function f and generalized difference operator Δ^m in sequences of fuzzy numbers and examine some inclusion theorems.

Key Words: Fuzzy sequence, statistical convergence, modulus function, difference operator.

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On *I*-Asymptotically Lacunary Statistical Equivalence of Functions on Amenable Semigroups

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ABSTRACT

Amenable semigroups were studied by Day [1]. The concept of summability in amenable semigroups was introduced in [6], [7]. In [8], Douglas extended the notion of arithmetic mean to amenable semigroups and obtained a characterization for almost convergence in amenable semigroups.

Nuray F. Rhoades B.E. [4] gave the notions of convergence and statistical convergence, statistical limit point and statistical cluster point to functions on discrete countable amenable semigroups were introduced.

Nuray F. Rhoades B.E. [5] defined the notions of asymptotically, statistically, almost statistically and strong almost asymptotically equivalent functions defined on discrete countable amenable semigroups. In addition to these definitions, they gave some inclusion theorems. Also, they proved that the strong almost asymptotically equivalence of the functions f(g) and h(g) defined on discrete countable amenable semigroups does not depend on the particular choice of the Folner sequence.

The purpose of the study [2] was to extend the notions of *I*-convergence, *I*-limit superior and *I*-limit inferior, *I*-cluster point and *I*-limit point to functions defined on discrete countable amenable semigroups. Also, he made a new approach to the notions of strongly λ-summability and λ-statistical convergence by using ideals and introduced new notions, namely, *I*-strongly λ-summability and *I*-λ-statistical convergence to functions defined on discrete countable amenable semigroups.

This study presents the notion of *I*-asymptotically lacunary statistical equivalence which is a natural combination of *I*-asymptotically equivalence, lacunary statistical equivalence for functions defined on discrete countable amenable semigroups. We introduce new concepts, and establish certain inclusion theorems.

Key Words: Asymptotic equivalence, lacunary sequence, *I*-convergence.

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On Non-Newtonian Measure For Closed Sets

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ABSTRACT

Let α be a generator, namely, α is a one-to-one function whose domain is real numbers and whose range is a subset A of $\Re$. We know that each generator produces exactly one arithmetic and conversely, each arithmetic is produced by one generator. For instance, the identity function I generates the classical arithmetic and the exponential function $\exp$ generates multiplicative (geometric) arithmetic. Let take a generator α such that have the following basic algebraic operations;

α -addition $\quad x \dot{+} y = \alpha(\alpha^{-1}(x) + \alpha^{-1}(y))$

α -subtraction $\quad x \dot{-} y = \alpha(\alpha^{-1}(x) - \alpha^{-1}(y))$

α -multiplication $\quad x \dot{x} y = \alpha(\alpha^{-1}(x) x \alpha^{-1}(y))$

α -division $\quad x \dot{/} y = \alpha(\alpha^{-1}(x) / \alpha^{-1}(y))$

α -order $\quad x \dot{<} y (x \dot{\leq} y) \Leftrightarrow \alpha^{-1}(x) < \alpha^{-1} y (\alpha^{-1}(x) \leq \alpha^{-1}(y))$

for every $x, y \in A$. For example, if we choose the function $\alpha = \exp$ as a generator

$$\alpha : \Re \to \Re_{\exp} \subset \Re$$

then, the basic algebraic operations are defined as follows:

α -addition $\quad x \dot{+} y = e^{(Inx + Iny)} = x.y$

α -subtraction $\quad x \dot{-} y = e^{(Inx - Iny)} = x / y$

α -multiplication $\quad x \dot{x} y = e^{Inx.Iny)} = y^{Inx} = x^{Iny}$

α -division $\quad x \dot{/} y = e^{Inx/Iny)} = x^{1/Iny}$

The set of non-Newtonian real numbers is defined as $\Re(N) = \{\alpha(x) : x \in \Re\}$.

Non-Newtonian calculus was created by Katz and Grossman $[1]$ as an alternative to classic calculus between 1967-1970. The first arithmetic calculus is then defined as

geometric, harmonic and quadratic calculus. Grossman $[2]$ also studied some properties of derivatives and integrals in non-Newtonian calculus. Bashirov et al. $[3]$ have recently studied some basic properties of derivatives and integrals in multiplicative calculus and gave the results with applications. Later, Duyar, Sağır and Oğur $[4]$ gave some basic topological properties of non-Newtonian calculus. Recently, Duyar ad Sağır $[5]$ introduced the concept of the non-Newtonian measure for open sets. For more details see $[6][7][8]$.

In this work, we study on non-Newtonian measure for closed sets and give some basic results.

Key Words: non-Newtonian calculus, non-Newtonian Lebesgue measure, non-Newtonian real line.

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On Qi Type Integral Inequalities and Their Applications

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ABSTRACT

It is very well known that inequalities have played a fundamental role in the development of almost all the fields of pure and applied sciences and are continuing to do so. Inequalities present very active and fascinating field of research. In the last few decades, much significant development in the classical and new inequalities, particularly in analysis has been witnessed.

Especially an integral inequality which is called Qi Inequality by mathematics community, has been studied by many authors. Significant development in this area has been achieved for the last two decades.

Feng Qi proposed an open integral problem at the end of [1] which was actually posed by himself in the preprint version [2]:

Problem 1: Under what conditions does the inequality

$$\int_a^b [f(x)]^t dx \geq \left(\int_a^b f(x)dx\right)^{t-1}$$

hold for $t > 1$?

Since then, this Problem 1 has been stimulating much interest of many mathematicians and affirmative answer to it has been established. Also, some other Qi type integral inequalities were proposed by researchers; some of them are given as follows:

Problem 2: Under what conditions does the inequality

$$\int_a^b [f(x)]^{n+2} dx \geq \left(\int_a^b f(x)dx\right)^{n+1}$$

hold for $n \in N$?

Problem 2 is special case of Problem 1.

The following two problems are extentions of Problem 1 and 2.

Problem 3: Under what conditions does the inequality

$$\int_a^b [f(x)]^\alpha dx \geq \left(\int_a^b f(x)dx\right)^\beta$$

hold for α and β ?

Problem 4: Under what conditions does the inequality

$$\int_a^b [f(x)]^\alpha dx \geq \left(\int_a^b f^\gamma(x)dx\right)^\beta$$

hold for α, β and γ ?

To the best of our knowledge, till now there have been many mathematicans and articles devoted to generalizing and applying the Qi's integral inequality and to answering Problem 1.

The main purpose of this talk is to provide a brief survey of Qi type integral inequalities applications such as discrete calculus, including extensions, generalizations and variations of Qi's integral Inequality.

Key Words: Qi type integral inequality, integral inequality.

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On The Fine Spectra Of The Jacobi Matrix Over Certain Sequence Spaces

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ABSTRACT

As it is well known, the matrices play an important role in operator theory. The spectrum of an operator generalizes the notion of eigenvalues for matrices. The spectral theory is an important branch of mathematics due to its application in other branches of science. It has been proved to be a standard tool of mathematical sciences because of its usefulness and application-oriented scope in different fields. In ecology, the spectral values may determine whether a food web will settle into a steady equilibrium. In aeronautics, the spectral values may determine whether the flow over a wing is laminar or turbulent. In electrical engineering, it may determine the frequency response of an amplifier or the reliability of a power system.

In the calculation of the spectrum of an operator over a Banach space, we mostly deal with three disjoint parts of the spectrum, which are the point spectrum, the continuous spectrum, and the residual spectrum. Also, spectrum has decomposition which is approximate point spectrum, defect spectrum and compression spectrum such that these are not necessarily disjoint.

In the last year, several authors have investigated spectral divisions of generalized difference matrices. In 2011, Amirov, Durna, and Yildirim [1] calculated approximate point spectrum, defect spectrum and compression spectrum of operators using the relationship between spectral divisions of operators easily. After this study, these spectral parameters are taken into account by the authors while the fine division of spectrum was found. So far the studies, approximate point spectrum, defect spectrum and compression spectrum of operators were calculated using fine division of spectrum. Generally, it is obtained that operator has the dense image or bounded inverse using the injectivity and surjectivity of its adjoint while the fine

division of operator was investigated. However, it may not be possible to find the adjoint operator. Even if the adjoint operator is found, it may not be possible to examine the character of obtained series while injectivity and surjectivity of operator were investigated. For example, on ℓ_∞, it is not possible to mention the adjoint of the operator in the usual sense since ℓ_∞ does not have the Schauder basis in the usual sense. Therefore we will first calculate approximate point spectrum, defect spectrum and compression spectrum of Jacobi matrix with the help of the relationship between the spectral division of operator and spectral division of its adjoint. Moreover, we will find the fine division of spectrum of Jacobi matrix which is given by Goldberg [2] with the help of it.

Key Words: approximate point spectrum, defect spectrum, compression spectrum, Jacobi matrix.

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On the Hermite-Hadamard Type Inequalities for Strongly Convex Functions via Katugampola fractional integrals

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ABSTRACT

Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be a convex function, then,

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \quad (1)$$

is known in the literature as Hermite--Hadamard inequality for convex mappings.

Note that some of the classical inequalities for means can be derived from (1) for appropriate particular selections of the mapping f.

It is well known that the Hermite--Hadamard's inequality plays an important role in nonlinear analysis.

Strongly convex functions have been introduced by Polyak, he used them for proving the convergence of a gradient type algorithm for minimizing a function. They play an important role in optimization theory, mathematical economics and nonlinear programming etc.

A function $f: I \to \mathbb{R}$, is said to be strongly convex in the classical sense with modulus $c \geq 0$,

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) - ct(1-t)(x-y)^2$$

where all $x, y \in I \;\; t \in [0,1]$.

The aim of this study is to establish Hermite-Hadamard type inequality by using Katugampola fractional integrals via strongly convex functions.

Also we give some new inequalities and equalities of right-hand side of Hermite-Hadamard type are given for functions.

Definition: Let $[a, b] \in \mathbb{R}$ be a finite interval. Then the left and right side Katugampola fractional integrals

$$\left({}^{\rho}I_{a^+}^{\alpha} f\right)(x) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)}\int_a^x (x^{\rho} - t^{\rho})^{\alpha-1} f(t) t^{\rho-1} dt, \;\; x > a$$

and

$$\left({}^{\rho}I_{b-}^{\alpha}f\right)(x)=\frac{\rho^{1-\alpha}}{\Gamma(\alpha)}\int_{x}^{b}(t^{\rho}-x^{\rho})^{\alpha-1}f(t)t^{\rho-1}dt,\ \ x>b$$

with $a<x<b$, and $\rho>0$, if integrals exist.

Key Words: Hermite-Hadamard inequalities, Strongly Convex functions, Katugampola Fractional Integral,Riemann-Liouville Fractional Integrals.

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On the Variable Exponent $L_{p,\gamma}$ Spaces and Maximal Operators

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ABSTRACT

It is well known that many classical operators in harmonic analysis such as maximal operators, fractional integrals and Riesz transforms are bounded on the variable Lebesgue space $L_{p(\cdot)}$ whenever the Hardy–Littlewood maximal operator is bounded on the variable exponent Lebesgue spaces $L_{p(\cdot)}$ (see [1, 2, 3, 4]). They do so by applying the theory of weighted norm inequalities. As applications we prove that the boundedness of Calderon Zygmund singular integral operators in variable exponent Lebesgue spaces $L_{p(\cdot)}$. These operators are singular integral operators with type convolution that is these related to Laplace operators. It is well known that translation operator is not bounded in variable exponent Lebesgue spaces $L_{p(\cdot)}$. Therefore, we don't have a notion about the boundedness of Riesz Bessel transforms generated by generalized translation operator for Laplace Bessel equations, [5]. Thus, we will introduce the boundedness of translation operator related to Laplacean Bessel operator [5]. Let $p(\cdot) \in \mathcal{P}(\mathbb{R}_+^n)$ and define the translation operator by $T^y f(x)$. It was shown that $T^y f(x)$ maps $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$ to $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$ for any $y \in \mathbb{R}_+^n$ if and only if $p(\cdot)$ is constant. In this study, we present regularity conditions, which ensure this operator is bounded on the variable Lebesgue spaces $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$. Then, we prove the boundedness of this operator.

Keywords: Laplace-Bessel operator, generalized translation operator, maximal operator, classical Riesz transforms, variable Lebesgue spaces.

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On the Variable Exponent $L_{p,\gamma}$ Spaces and Riesz Transforms

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ABSTRACT

We know that many classical operators in harmonic analysis such as maximal operators, potential operators and Riesz Bessel transforms are bounded on the Lebesgue space $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$ whenever the Hardy–Littlewood maximal operator generated by generalized shift operator related to Laplace Bessel equation is bounded on the Lebesgue spaces $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$, [5]. We do so by applying the theory of weighted norm inequalities. As applications we prove that the boundedness of Riesz Bessel transforms generated by generalized shift operators related to Laplace Bessel operator in Lebesgue spaces. Are these classical operators bounded in variable exponents Lebesgue spaces? The question will form the basis of our talk. However, since the ordinary translation operator is not bounded in variable exponent Lebesgue spaces, we don't have a notion about the boundedness of Riesz Bessel transforms generated by generalized translation operator for Laplace Bessel equations. Since, we say that Riesz Bessel transforms generated by generalized shift operators are bounded from $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$ to $L_{p(\cdot),\gamma}(\mathbb{R}_+^n)$ for any $y \in \mathbb{R}_+^n$ if and only if $p(\cdot)$ is constant, (see, [1, 2, 3, 4]). To prove the boundedness of these transforms, we shall show that we need to take into account the additional conditions of variable exponents.

Keywords: Laplace-Bessel operator, Generalized translation operator, Maximal operator, Classical Riesz Bessel transforms, Riesz Bessel transforms, Variable exponent Lebesgue spaces.

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Quantitative Perturbation Theory For Operators Belonging To Compactness Classes

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ABSTRACT

Perturbation theory is the study of the behaviour of characteristic data of a mathematical object when replacing it by a similar nearby object. More narrowly, spectral perturbation theory is concerned with the change of spectral data of linear operators (such as their spectrum, their eigenvalues and corresponding eigenvectors) when the operators are subjected to a small perturbation. There are two sides to spectral perturbation theory, a qualitative one and a quantitative one. Qualitative perturbation theory focusses on questions such as the continuity, differentiability and analyticity of eigenvalues and eigenvectors, while quantitative perturbation theory attempts to provide computationally accessible bounds for the smallness of the change in the spectral data in terms of the smallness of the perturbation.The present article addresses the following problem of fundamental importance in both qualitative and quantitative perturbation theory. If A and B are two operators belonging to same compactness class acting on a separable Hilbert space which are close, then how close are their spectra σ(A) and σ(B)? In order to make this question more precise, the main concern of the present article is to provide explicit upper bounds for the Hausdorff distance of the spectra of two arbitrary operators A and B belonging to new classes of compact operators on a separable Hilbert space in terms of the distance of the two operators A and B in operator norm. These new classes of operators, termed compactness classes, generalise Bandtlow's exponential classes from [2]. The basic idea here is to group together all operators the singular values of which decay at a certain speed.

Key Words: Quantitative perturbation theory, Compactness class, Hausdorff distance.

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Some Algebraic and Topological Properties of New Lucas Difference Sequence Spaces

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ABSTRACT

Define the sequence $\{f_n\}_{n=0}^{\infty}$of Fibonacci numbers and define the sequence $\{L_n\}_{n=0}^{\infty}$ of Lucas numbers given by the linear recurrence relations

$$f_0 = 0, \qquad f_1 = 1 \text{ and } f_n = f_{n-1} + f_{n-2}, n \geq 2,$$
$$L_0 = 2,\; L_1 = 1 \text{ and } L_n = L_{n-1} + L_{n-2}, n \geq 2.$$

Fibonacci and Lucas numbers have many interesting properties and applications in arts, sciences and architecture. For example, the ratio sequences of Fibonacci and Lucas numbers converges to the golden ratio which is important in sciences and arts. Also, some basic properties of Fibonacci and Lucas numbers [4] are given as follows:

$$\sum_{k=1}^{n} f_k^2 = f_n f_{n+1} \text{ and } \sum_{k=1}^{n} f_k = f_{n+2} - 1,\; n \geq 1$$
$$\sum_{k=1}^{n} L_{2k} = L_{2n+1} - 1 \text{ and } \sum_{k=1}^{n} L_k = L_{n+2} - 3.$$

Recently, Karakaş and Karabudak [1] established the regular matrix E by using Lucas numbers. Then, by introducing the sequence space $X(E)$ with the help of matrix E, they showed that this spaces is a BK-space, where $E = (L_{nk})$ is defined by

$$L_{nk} = \begin{cases} \dfrac{L_{k-1}^2}{L_n L_{n-1} + 2}, & 1 \leq k \leq n, \\ 0, & k > 0 \end{cases}$$

for all $k, n \in \mathbb{N}$ and

$X(E) = \{x = (x_k) \in w : y = (y_k) \in X\}$.

In [1], defined the sequence $y = (y_k)$ by the $\hat{L}$ transform of a sequence $x = (x_k)$,

$$y = (y_k) = E_k(x) = \frac{1}{L_k L_{k-1} + 2} \sum_{i=1}^{k} L_{i-1}^2 x_i.$$

Now, let us define the sets $\ell_p(1 \leq p \leq \infty)$ as the sets of all sequence spaces whose $\hat{L} = (L_{nk})$ transforms is in the well-known sequence spaces ℓ_p and ℓ_∞, respectively, namely,

$$\ell_p(\hat{L},\Delta) = \left\{x = (x_k) \in w : \sum_{n=1}^{\infty} \left| \frac{L_n L_{n-1} + 2}{L_{n-1}^2} x_n - \frac{L_{n-1} L_{n-2} + 2}{L_{n-1}^2} x_{n-1} \right|^p < \infty \right\}$$

$$\ell_\infty(\hat{L},\Delta) = \left\{x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \frac{L_n L_{n-1} + 2}{L_{n-1}^2} x_n - \frac{L_{n-1} L_{n-2} + 2}{L_{n-1}^2} x_{n-1} \right| < \infty \right\}$$

where $\hat{L} = (\hat{L}_{nk})$ is the double band matrix defined by the sequence (L_n) of Lucas numbers as follows

$$L_{nk} = \begin{cases} -\dfrac{L_{n-1}L_{n-2}+2}{L_{n-1}^2} & , \quad k = n-1 \\ \dfrac{L_n L_{n-1}+2}{L_{n-1}^2} & , \quad k = n \\ 0 & , \quad \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$.

The main purpose of this study is to introduce the sequence spaces $\ell_p(\hat{L},\Delta)$and $\ell_\infty(\hat{L},\Delta)$. Furthermore, the inclusions $\ell_p \subset \ell_p(\hat{L},\Delta)$ and $\ell_\infty \subset \ell_\infty(\hat{L},\Delta)$ strictly hold, the basis of the sequence spaces $\ell_p(\hat{L},\Delta)$ and $\ell_\infty(\hat{L},\Delta)$ is constructed and alpha-, beta- and gamma-duals of these spaces are determined. Finally, the classes of matrix transformations from the new sequence spaces to the spaces ℓ_∞, c and c_0 are characterized.

Key Words: Lucas sequence spaces, matrix domain, matrix transformations.

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Some Convergence Theorems for Multipliers

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ABSTRACT

Let A be a commutative semisimple Banach algebra and let Σ_A be the Gelfand space of A equipped with the ω^*-topology. By $\hat{a}$, we denote the Gelfand transform of $a \in A$. A linear mapping $T: A \to A$ is called a multiplier of A if

$$T(ab) = (Ta)b \quad (= a(Tb))$$

holds for all $a, b \in A$. For each multiplier T on A, there is a uniquely determined bounded continuous function $\hat{T}$ on Σ_A such that

$$\widehat{(Ta)}(\zeta) = \hat{T}(\zeta)\hat{a}(\zeta) \qquad \forall a \in A, \forall\, \zeta \in \Sigma_A$$

The function $\hat{T}$ is often called the Helgason-Wang representation of T [4, Proposition 4.3.9]. If the Gel'fand transformation on A is injective, then A is said to be semisimple [3]. Recall that a commutative Banach algebra A is said to be regular if given a closed subset S of Σ_A and $\in \Sigma_A \backslash S$, there exists an $a \in A$ such that $\hat{a}(\zeta) \neq 0$ and $\hat{a}(S) = \{0\}$ [3]. Note that a multiplier T on A is said to be power bounded if $\sup_{n\geq 0}\|T^n\| < \infty$. If T is a power bounded multiplier on A, then $I_T := \{a \in A : \lim_{n\to\infty}\|T^n a\| = 0\}$ is a closed ideal of A associated with T. Subset of Σ_A associated to any multiplier T of A

$$\mathcal{E}_T = \{\zeta \in \Sigma_A : |\hat{T}(\zeta)| = 1\}$$

plays an important role in the study of Banach algebras. if T is a power bounded multiplier on a regular semi-simple Banach algebra, then $hull(I_T) = \mathcal{E}_T$. (see, [1, Theorem 2.6] and [5, Proposition 2.1]). We have the following theorem.

Theorem Let A be a commutative semisimple regular Banach algebra and let T be a power bounded multiplier on A.If $\mathcal{E}_T$ is a compact set, then

$$\lim_{n\to\infty}\|T^{n+1}a - T^n a\| = 0, \qquad \forall a \in A$$

if and only if $\hat{T}(\mathcal{E}_T) = \{1\}$.

Key Words: Banach algebra, Multiplier, Power bounded.

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Some General Integral Inequalities for Lipschitzian Functions via Conformable Fractional Integral

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ABSTRACT

This study is about getting of some new general integral inequalities for Lipschitz functions by using conformable fractional integrals. Inequalities play an important role in almost all branches of mathematics as well as in other areas of science. Mathematical inequalities are one of the most significant instruments in many branches of mathematics such as functional analysis, theory of differential and integral equations, interpolation theory, harmonic analysis, probability theory, etc. They have very useful applications area in mechanics, physics and other sciences. The first comprehensive study in inequalities was made in the first half of the 20. century and in the second half of the century inequalities attracted the attention of many researchers. Especially, in recent years, so many study have been published. that inequalities may be considered as a different area of mathematics.

Convexity theory featured an important and fundamental role in the developments of various branches of engineering, financial mathematics, economics and optimization. The Hermite Hadamard integral inequality is the first fundamental result for convex functions with a natural geometrical interpretation and many applications, has attracted and continues to attract much interest in elementary mathematics and the other sciences. In literature, there are hundreds studies for Hermite Hadamard type inequality by using the left and right fractional integrals (such as Riemann Liouville fractional integrals, Hadamard fractional integrals, Conformable fractional integrals etc.).

In this paper, we obtain some generalized integral inequalities for lipschitzian functions via conformable fractional integrals. The results obtained give Hermite-Hadamard, Bullen and Simpson type inequalities in special cases.

Key Words: Convex functions, Lipschitzian functions, Hermite-Hadamard inequality, conformable fractional integral.

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Some Inequalities Related to η-Strongly Convex Functions

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ABSTRACT

The relationship between theory of convex functions and theory of inequalities has occured as a result of many researches investigation of these theories. A very intersting result in this regard is due to Hermite and Hadamard independently that is Hermite-Hadamard's inequality.

The convexity property of a given function plays an important role in obtaining integral inequalities. Proving inequalities for convex functions has a long and rich history in mathematics.

A function $f: I \subset \mathbb{R} \to \mathbb{R}$ be a convex function defined on a interval I of real numbers $a, b \in I$ and $a < b$, given by

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

The classical Hermite-Hadamard's inequalities have attracted many researchers since 1893. Researchers investigated Hermite- Hadamard inequalities involving fractional integrals according to the associated fractional integral equalities and different types of convex functions.

Strongly convex functions have been introduced by Polyak, he used them for proving the convergence of a gradient type algorith for minimizing a function. They play an important role in optimization theory and mathematical economics.

Now, we introduce η $-$strongly convex the following definition.

Definition: A function $f: I \to \mathbb{R}$ is called convex with respect to η $-$strongly convex $c \geq 0,$

$f(tx + (1-t)y) \leq f(y) + t\eta\big(f(x), f(y)\big) - ct(1-t)\eta^2(x, y)$

or

$f(tx + (1-t)y) \leq f(y) + t\eta\big(f(x), f(y)\big) - ct(1-t)(x-y)^2$

For all $x, y \in I$ and $t \in [0,1]$.

1. When $c = 0$, we obtain η-convex function.

2. When $c = 0$ and $\eta(x, y) = x - y$, we obtain classic convex function.

The aim of this paper, is to establish some new inequalities of Hermite-Hadamard type by using η -strongly convex fuction. Moreover, we also consider their relevances for other related known results.

Key Words: Hermite-Hadamard inequalities, Strongly convex functions, Convex functions.

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Some New Inequalities for Lipschitz Functions via a Functional

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ABSTRACT

This study is about getting of some new integral inequalities for Lipschitz functions by using a functional defined via a Lipschitz function. Inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The Hermite Hadamard integral inequality is the first fundamental result for convex functions with a natural geometrical interpretation and many applications, has attracted and continues to attract much interest in elementary mathematics and the other sciences.

Many mathematicians have devoted their efforts to generalize, refine, counterpart and extend it for different classes of functions such as: convex functions, quasi-convex functions, harmonically convex functions, p-convex functions, s-convex functions, Godunova-Levin class of functions, log-convex and r−convex functions, p−functions, etc. or apply it for special means (Harmonic means, geometric means, aritmetic means, p−logarithmic means, identric mean, Stolarsky means, etc.).

In this study, at first, some basic definitions and theorems of the condition of Lipschitz, convex functions, p-convex functions and harmonic convex functions are given. Then, some new type integral inequalities are obtained by using a functional defined via a Lipschitz function. Also some new Hermite-Hadamard type inequalities are obtained as a corollary of main theorems. Finally, we have new Hermite Hadamard type iinequalities for Lipschitz functions by means of inequalities which are used for p-convex functions.

Key Words: Convex function, p-Convex function, Lipschitz function.

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Some New Inequalities for Operator *m*-preinvex Functions in Hilbert Spaces

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ABSTRACT

A. G. Ghazanfari et all. [1] established a Hermite- Hadamard type inequality for operator preinvex functions and an estimate of the right hand side of a Hermite-Hadamard type inequality in which some operator preinvex functions of selfadjoint operators in Hilbert spaces are involved. S.-H. Wang and X.-M. Liu [2] introduced the concept of operator s-preinvex function. They established some new Hermite-Hadamard type inequalities for operator s-preinvex functions, and provided the estimates of both sides of Hermite-Hadamard type inequality in which some operator s-preinvex functions of positive selfadjoint operators in Hilbert space was involved. And then, S.-H. Wang and X.-W.Sun [3] similarly introduced the concept of operator alpha-preinvex function and some inequalities. E. Unluyol et all. [4] defined the operator *m*-preinvex functions in a Hilbert space. Then they obtained some algebraic properties of this class. Moreover, they established some new integral inequalities for operator *m*-preinvex in terms of continuous functions of self adjoint operators in Hilbert spaces.

So, in this paper firstly, we gave some definitions about operator *m*-preinvex functions. Secondly, we established new theorems and finally we got some new integral inequalities for operator *m*-preinvex in terms of self adjoint operators of continuous functions in Hilbert spaces.

Key Words: Hilbert space, preinvex, operator *m*-preinvex.

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Some New Integral Inequalities for Multiplicatively Geometrically P-Functions

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ABSTRACT

In this study, we introduce a new class of extended multiplicatively geometric P-functions. Some new Hermite Hadamard type integral inequalities are derived. Results represent significant refinement and improvement of the previous results. Ideas of this paper may stimulate further research. We should especially mention that the definition of multiplicatively geometrically P-function is given for the first time in the literature by us.

Inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The Hermite Hadamard integral inequality is the first fundamental result for convex functions with a natural geometrical interpretation and many applications, has attracted and continues to attract much interest in elementary mathematics and the other sciences.

Firstly, the concepts of convexity, Hermite Hadamard inequality, P-function, multiplicatively P-functions, geometrically arithmetically convex function, P-geometric-arithmetic function and geometrically multiplicatively P-functions are given. After the definition of multiplicatively P-function is given, some algebraic properties of it are investigated.

The main purpose of this paper is to establish new estimations and refinements of the Hermite Hadamard inequality for functions whose derivatives in absolute value are multiplicatively geometrically P-function. For this, we will use our Lemma with both the Hölder and the Power mean integral inequality.

Key Words: Convex function, Multiplicatively P-function, Multiplicatively Geometrically P-function, Hölder Integral inequality and Power-Mean Integral inequality, Hermite-Hadamard type inequality.

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Some Properties Of Convex Functions Via Non-Newtoian Calculus

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ABSTRACT

Two operations, differentiation and integration, are basic in calculus and analysis. In fact, they are the infinitesimal versions of the subtraction and addition operations on numbers, respectively. From 1967 till 1970 Michael Grossman and Robert Katz gave definitions of a new kind of derivative and integral, converting the roles of substraction and addition into division and multiplication, respectively and thus established a new calculus, called Non-Newtonian Calculus. Since these calculi has emerged, it has become a seriously alternative to the classical analysis developed by Newton and Leibnitz. Every property in classical calculus has an analogue in non-Newtonian calculus. As an alternative to classical calculus, Grossman and Katz [1] introduced the non-Newtonian calculus consisting of the branches of geometric, anageometric and bigeometric calculus *etc. S*ome study with regard to multiplicative calculus is given by D.Campell [2], Bashirov et al. [3] presented concepts of the multiplicative calculus in detail and some applications to the properties of derivative and integral operators of this calculus. Furthermore, some researchers [4–8] have shown that multiplicative analysis can be used effectively in the solution of problems in some science and engineering fields. So in this paper, we investigated to a new view of some properties of convex functions in terms of Non-Newtonian Calculus. Moreover, paper our aim is to bring up this calculus to the attention of researchers and demonstrate its usefulness.

Key Words: Non-Newtonian Calculus, alpha- arithmetic, alpha-convexity.

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Strongly (∆,f)-Cesaro Summability of Order β

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ABSTRACT

The concept of statistical convergence which is a powerful mathematical tool for studying the convergence problems of numerical problems through the concept of density was introduced by Fast [2] and Schoenberg [5], independently. The concepts of fuzzy sets and basic operations on a fuzzy set were introduced by Zadeh [6] as an extension of the concept of classical set. The theory of sequence of fuzzy numbers was introduced by Matloka [3], where he proved some basic theorems related to sequences of fuzzy numbers. Later, the notion of statistical convergence for sequences of fuzzy numbers was defined and studied by Nuray and Savaş [4]. Çolak [2] defined the concepts of statistical convergence of order β and strong p-Cesàro summability of order β for classical sequences and later, Altınok et al [1] generalized these concepts for sequences of fuzzy numbers.

In this article, we define the spaces $w^{\beta}(\Delta, F, f)$, $w^{\beta,0}(\Delta, F, f)$, and $w^{\beta,\infty}(\Delta, F, f)$, where f is an unbounded modulus function, Δ is a difference operator and $\beta \in (0,1]$ is a real number, for sequences of fuzzy numbers and examine some inclusion relations between them. We didn't allowed β to exceed 1 in the $w^{\beta}(F, p)$ of Altinok et al. [1], but we consider β as any positive real number and it can exceed 1 in the spaces $w^{\beta}(\Delta, F, f)$, $w^{\beta,0}(\Delta, F, f)$, and $w^{\beta,\infty}(\Delta, F, f)$.

Key Words: Fuzzy sequence, statistical convergence, Cesàro summability, modulus function, difference operator.

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Subdivision Of The Spectra Of The Generalized Difference Operator $B(r,s)$ on The Sequence Space cs

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ABSTRACT

We know that there is strictly the relationship between matrices and operators. The eigenvalues of matrices are contained in the spectrum of an operator. The spectral theory is a generalization of a set of eigenvalues of a linear operator in a finite-dimensional vector space to an infinite dimensional vector space. The spectral theory of finite-dimensional linear algebra may be provided as an attempt to expand the known decomposition results in similar situations in the infinite dimension.

Many investigators studied the spectrum and fine spectrum of linear operators on some sequence spaces. In 2005, Altay and Başar [1] determined spectra and the fine spectra of generalized difference operator $B(r,s)$ on c_0 and c. In 2008, Bilgiç and Furkan [2] determined spectra and the fine spectra of generalized difference operator $B(r,s)$ on ℓ_p and bv_p, $(1 \le p < \infty)$. In the last year, the spectral divisions of generalized difference matrices have studied. For example, in [5], Das calculated the spectrum and fine spectrum of the matrix $U(r_1,r_2;s_1,s_2)$ over the sequence space c_0. In [3] Başar, Durna and Yildirim investigated partition of the spectra for generalized difference operator $B(r,s)$ over certain sequence spaces. In [6], Tripathy and Das determined the spectra and fine spectra of $U(r,s)$ on the sequence space

$$cs = \left\{ x = (x_n) \in w : \lim_{n\to\infty} \sum_{i=0}^{n} x_i \text{ exists} \right\},$$

which is a Banach space with respect to the norm $\|x\|_{cs} = \sup_n \left| \sum_{i=0}^{n} x_i \right|$. In [4], Das and Tripathy examined the spectra and fine spectra of the matrix $B(r,s,t)$ on the sequence space cs.

In this study, we determine the approximate point spectrum, the defect spectrum and the compression spectrum of the generalized difference matrix

$$B(r,s)=\begin{pmatrix} r & 0 & 0 & \cdots \\ s & r & 0 & \cdots \\ 0 & s & r & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (s\neq 0).$$

over the sequence space cs.

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Key Words: approximate point spectrum, defect spectrum, compression spectrum, generalized difference matrix.

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The Discreteness Of The Spectrum Of The Operator Schrödinger Equation And Some Properties Of The S-Numbers Of The Inverse Schrödinger Operator

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ABSTRACT

In this study, we investigate the discreteness of the spectrum for the operator L, acting in the Hilbert space $L_2(H,\ [0,\infty))$, which is given by the formula

$$Ly = -\frac{d^2y}{dx^2} + \frac{A(A+I)}{x^2}y + Q(x)y,$$

where A and $Q(x)$ are self-adjoint operators defined in the separable Hilbert space H; and I is the identity operator.

In [1], an investigation has been done on the discreteness of the negative spectrum for the operator given in the form of

$$Ly = -\frac{d^2y}{dx^2} + \frac{A(A+I)}{x^2}y - Q(x)y$$

We denote $q(x) = \inf_{\|f\|=1}(Q(x)f,f)$. The condition $\lim_{x\to\infty}\int_x^{x+w} q(t)dt = \infty$ is a sufficient condition, but not a necessary one for the discreteness of the spectrum of the operator L, where $w > 0$. On the other hand, for all $k \in Z$ and for $w > 0$, the condition

$$\lim_{x\to\infty}\left(\int_x^{x+w} Q(t)dt\, e_k, e_k\right) = \infty$$

is necessary, but not sufficient for the discreteness of the spectrum of L, where $\{e_k\}_{k=1}^{\infty}$ are the eigenvectors of the operator A. Let the numbers $\gamma_1 \le \gamma_2 \le \cdots \le \gamma_n \le \cdots$ denote the eigenvalues of the operator A; $\gamma_k \sim ak^{\beta}$, $\beta > 0$, $a > 0$, and the numbers $\{\lambda_k\}_{k=1}^{\infty}$ be the eigenvalues of L. Then, the following evaluation holds:

$$\sum_{n=1}^{\infty}\frac{1}{\lambda_n^p(L)}=\|L^{-1}\|_{\sigma_p}^p\leq c(p)\int_0^{\infty}\frac{x^{\frac{1}{p}}}{q(x)^{p-\frac{1}{2}-\frac{1}{2\beta}}}$$

Key Words: Operator equations, Eigenvalues, Discreteness of spectrum, Hilbert space.

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The Disjoint Spectrum of the Weighted Mean, Cesaro and Rhaly Operators Various Sequence Spaces.

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ABSTRACT

Let X be a Banach space and $B(X)$ denote the linear space of all bounded linear operators on X. Given an operator $L \in B(X)$, the set

$$\rho(L) := \{\lambda \in \mathbb{C} : \lambda I - L \ \text{ bijection}\} \quad (1.1)$$

is called the resolvent set of L, its complement

$$\sigma(L) := \mathrm{K} \setminus \rho(L) \quad (1.2)$$

the spectrum of L.

Recall that a number $\lambda \in \mathbb{C}.$ is called eigenvalue of L if the equation $Lx = \lambda x$ has a nontrivial solution $x \in X$. Any such x is then called eigenvector.

We will call the set of eigenvalues

$$\sigma_p(L) := \{\lambda \in \mathbb{C} : Lx = \lambda x \ \text{ for some } \ x \neq 0\}. \quad (1.4)$$

We say that $\lambda \in \mathbb{C}$ belongs to the continuous spectrum $\sigma_c(L)$ of L if the resolvent operator (1.3) is defined on a dense subspace of X and is unbounded. Furthermore, we say that $\lambda \in \mathrm{K}$ belongs to the residual spectrum $\sigma_r(L)$ of L if the resolvent operator (1.3) exists, but its domain of definition (i.e. the range $R(\lambda I - L)$ of $(\lambda I - L)$ is not densein X; in this case $R(\lambda; L)$ may be bounded or unbounded. Together with the point spectrum (1.4), these two subspectra form a disjoint subdivision

$$\sigma(L) = \sigma_p(L) \cup \sigma_r(L) \cup \sigma_c(L) \quad (1.5)$$

of the spectrum of L.

Given a bounded linear operator L in a Banach space X, we call a sequence $(x_k)_k$ in X a Weyl sequence for L if $\|x_k\| = 1$ and $\|Lx_k\| \to 0$ as $k \to \infty$.

In what follows, we call the set

$$\sigma_{ap}(L) := \{\lambda \in \mathbb{C} : \text{there is a Weyl sequence for } \lambda I - L\} \tag{1.6}$$

the approximate point spectrum of L. Moreover, the subspectrum

$$\sigma_{\delta}(L) := \{\lambda \in \mathbb{C} : \lambda I - L \ \text{ is not surjective}\} \tag{1.7}$$

is called defect spectrum of L.

There is another subspectrum,

$$\sigma_{co}(L) := \left\{\lambda \in \mathbb{C} : \overline{R(\lambda I - L)} \ \neq X\right\} \tag{1.8}$$

which is often called compression spectrum in the literature.(see [1])

In this study, the relationship between the subdivisions of spectrum which are not necessary to be disjoint and Goldberg's classification is given. Moreover, these subdivisions for following summability methods are studied.

A weight mean matrix A is a lower triangular matrix with entries $a_{nk} = p_k / P_n,$ where $p_0 > 0,\ p_n \geq 0 \text{ for } n > 0, \text{ and } P_n = \sum_{k=0}^{n} p_k.$ The necessary and sufficient condition for the regularity of A is that $\lim P_n = \infty.$ $p_k = 1$ alınırsa $C_1 = (c_{nk}),$ matrisini elde ederiz.. In 1975, Reade [2] and in 1975, Wenger [3] determined spectra and the fine spectra of Cesaro operator C_1 on c, the space of convergent sequences, respectively.

We assume that; given a scalar sequence of $a = (a_n),$ a Rhaly matrix $R_a = (a_{nk})$ is the lower triangular matrix where $a_{nk} = a_n,\ k \leq n \text{ and } a_{nk} = 0$ otherwise.

In [4], the spectrum of the Rhaly operators on c_0 and c, under the assumption that $\lim_n (n+1) a_n = L \neq 0$ has been determined. Also in [6-8] the spectrum of Rhaly operator over some kinds of spaces has been determined.

In this study, we will investigate the disjoint spectrum of the weighted mean, Cesaro and Rhaly operators various sequence spaces.

Key Words: spectrum, fine spectrum, approximate point spectrum, defect spectrum, compression spectrum, weighted mean operators, Rhaly operators, Cesáro operators.

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The Lower And Upper Bounds Of Discrete Generalized Cesaro Operators And Its Fine Spectrum On l^p $(1<p<\infty)$

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ABSTRACT

Family of the lower triangular matrix $A_t = (c_{nk})$ defined by $c_{nk} = \frac{t^{n-k}}{(n+1)}$, 0 < t < 1 is called generalized Cesaro operators. The matrix reduces to the Cesàro matrix by setting t=1. In 1982, Rhaly [3] showed that the A_t Discrete Generalized Cesàro Operators on the l^2 Hilbert space were a bounded compact linear operator and computed its spectrum. Also in [2], lower bounds for these classes were obtained under certain restrictions on l^p (1 < p < 1) by Rhoades in [2].

There are many different ways to subdivide the spectrum of a bounded linear operator; some of them are motivated by applications to physics (in particular, quantum mechanics). In this study, the relationship between the subdivisions of spectrum which are not necessary to be disjoint and Goldberg's classification is given In this study, we will investigate Goldberg's classification of Spectrum of discrete generalized Cesaro on l^p (1<p<∞) sequence space.

Bennet [1] gave a method of how to find the lower bounds of infinite matrices with positive entries: [Let (x_n) be a monotone decreasing nonnegative sequence, $A \in B(l^p)$ with nonnegativee entries, 1<p<∞ . Then

$$\|Ax\|_p \geq L\|x\|_p$$

where

$$L^p = \inf_r (r+1)^{-1} \sum_{j=0}^{\infty} \left(\sum_{k=0}^{r} a_{jk} \right)^p =: \inf_r \ f(r) \text{ .]}$$

Using this theorem, the lower bounds of the various operators on the l^p sequence space have been determined. we will determine these lower bounds of the generalized Cesaro operator.

Key Words: Lower bound; Cesaro operator; weighted mean operators, discrete generalized Cesaro operators, spectrum, fine spectrum

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Trapozoidal Type Inequalities Related To Geometrically Aritmetic Convex Functions With Application

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ABSTRACT

Twentieth century mathematics has played the power of mathematical inequalities which has given rise to a large number of new results and problems and has led to new areas of mathematics. Convexity theory featured an important and fundamental role in the developments of various branches of engineering, financial mathematics, economics and optimization. In recent years, the concept of convex functions and its variant forms have been extended and generalized using innovative techniques to study complicated problems. Convex functions are related to integral inequalities. Related to the arithmetic means, we have geometric means. The geometric means have applications in electrical circuit theory and other branches of sciences. For example, the total resistance of a set of parallel resistors is obtained by adding up the reciprocal of the individual resistance value and then considering the reciprocal of their total. Some authors introduced the concepts of the geometrically aritmetic convex functions and establishes some integral inequalities of Hermite Hadamard type related to the geometrically aritmetic convex functions. In this paper, we have given $M(t)$ for geometrically aritmetic convex function. A mapping $M(t)$ is considered to get some preliminary results and a new trapezoidal form of Fejer inequality related to the geometrically aritmetic convex functions.

Key Words: Geometrically convex, Fejer inequality, Trapezoidal form.

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Trapozoidal Type Inequalities Related To Ha Convex Functions With Application

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ABSTRACT

Twentieth century mathematics has recognized the power of mathematical inequalities which has given rise to a large number of new results and problems and has led to new areas of mathematics. Convexity theory played an important and fundamental role in the developments of various branches of engineering, financial mathematics, economics and optimization. In recent years, the concept of convex functions and its variant forms have been extended and generalized using innovative techniques to study complicated problems. Convex functions are related to integral inequalities. Related to the arithmetic means, we have harmonic means. The harmonic means have applications in electrical circuit theory and other branches of sciences. For example, the total resistance of a set of parallel resistors is obtained by adding up the reciprocal of the individual resistance value and then considering the reciprocal of their total. The authors introduce and investigate a new class of harmonically convex functions, which is called harmonically convex function. It is shown that this class unifies several new and known classes of harmonically convex functions. They derive some new Hermite-Hadamard like inequalities for harmonically convex functions. In this paper, a mapping $M(t)$ is considered to obtain some preliminary results and a new trapezoidal form of Fejer inequality related to the harmonically functions.

Key Words: Harmonically convex, Fejer inequality, Trapezoidal form.

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Two-Weighted Inequalities for High Order Riesz-Bessel Transforms in Weighted Lebesgue Spaces

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ABSTRACT

Singular integral operators have a central role in Harmonic Analysis, theory of functions and partial differential equations. Since one of the most important typical examples of singular integral operators are Riesz transforms, these operators have been extensively studied by many mathematicians. While most of these works on Riesz transforms focus on the boundedness of these operators on the various function spaces. Further, this singular integral operators have many interesting properties. For example, they are the simplest, non-trivial and invariant operators in $\mathbb{R}^n$ and they also constitute typical and important examples of Fourier multipliers.

In this talk, we shall obtain the boundedness of high order Riesz-Bessel transforms generated by generalized translation operator T^y, associated with the Bessel differential operator in weighted Lebesgue spaces with general weights. This translation operator was first introduced by B.M. Levitan in [7] and has studied by many mathematicians in [4-6, 8]. Also, in this talk, we gave some sufficient conditions on weighted functions ω and v so we showed that this singular integral operator is bounded from the weighted $L_{p,\omega,\gamma}(\mathbb{R}^n_+)$ spaces into the weighted $L_{p,v,\gamma}(\mathbb{R}^n_+)$ spaces. However, the generalized Hardy inequalities have an important role in proofs of our main results. Our proofs are based on a modification of technique due to [4, 6].

This talk based on the joint work with İsmail Ekincioğlu.

Key Words: Bessel differential operator, generalized translation operator, Riesz Bessel transforms, weighted Lebesgue spaces.

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APPLIED MATHEMATICS

A Hermite Collocation Method For The Approximate Solutions Of High-Order Linear Volterra Integro-Differential Equations

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ABSTRACT

In recent years there has been a growing interest in the integro-differential equations (IDEs), which are combinations of differential and integral equations. These are an important branch of modern mathematics and arise frequently in many applied areas which include engineering, mechanics, potential theory, electrostatics, etc. [1-3]. These equations are usually difficult to solve analytically; and hence a numerical method is required.

The concepts of IDEs have motivated a huge size of research work in recent years. Several numerical methods were used such as the successive approximation method, the Adomian decomposition method, the Chebyshev and Taylor collocation method, Haar Wavelet method, Wavelet-Galerkin method, the monotone iterative technique, the Tau method, the Walsh series method [1-7], Hybrid function method [8] etc

In this study, a Hermite matrix method is presented to solve high-order Linear Volterra integro-differential equations (VIDEs) with variable coefficients under the mixed conditions in terms of Hermite polynomials. The proposed method converts the equation and its conditions to matrix equations, which correspond to a system of linear algebraic equations with unknown Hermite coefficients, by means of collocation points on a finite interval. Then, by solving the matrix equation, Hermite coefficients and polynomial approach are obtained. Also, examples that illustrate the pertinent features of the method are presented; accuracy of the solutions and error analysis are performed.

Key Words: Hermite polynomials and series, Volterra integro-differential equations, Collocation points

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A New Decision Making Model Based on Soft Set Theory

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ABSTRACT

In 1999, Molodtsov initiated the soft set theory as an effective mathematical principle for dealing with the difficulties based on the uncertainties or unknown data. Immediately after the first step to this theory, in the papers [1,5,7], almost all of the operations in the classical set theory were derived for the soft sets. Also, it was successfully applied in various directions such as the operations research, decision making, game theory, Riemann integration, Perron integration, measurement theory, probability and so on.

In this work, we deal with the analysis of parameters in a soft set. We argue that the status of parameters in the soft sets is effective in decision making. Therefore, we propose a new decision making model based on the parameter of soft set for decision making, which is one of the application areas of soft sets. While the present soft decision making models are inadequate for some decision making problems, we emphasize that our decision making model has the solutions for these problems. To demonstrate this, we present a comparison of our decision making model with some decision making models obtained using the soft set. Lastly, we give an example illustrating the practicability of the proposed decision making model.

Key Words: Soft sets, operations of soft sets, decision making.

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A Novel Hybrid Method for Singularly Perturbed Delay Differential Equations

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ABSTRACT

Our aim of this study is to solve singularly perturbed second order linear delay differential equations which have boundary and interval conditions by combining the flexibility of differential transform and the efficiency of Taylor series expansion method. For this, we use two-term Taylor series expansion method for delayed parameter linearization and then apply differential transform method. Two examples are presented to demonstrate the efficiency, rapidly and reliability of the proposed hybrid method. This method give the desired accurate results only in a few terms and in a series form of the solution and demonstrate more simple performance than known methods.

Singulary perturbed problems have a small parameter which exists as multipling the highest order derivative terms. Standard numerical methods for solving singularly perturbed problems are fail to give accurate results and unstable due to the perturbation parameter ε. Therefore, there are some fitted numerical methods to solve equations, such as finite difference method, Adomian decomposition method, differential transform method, finite elements method, reprocuding kernel method etc. These kinds of problems are ubiquitous in mathematical problems in engineering and science. Some of these astrophysics, nonlinear dynamical systems, probability theory on algebraic structures, chemical and biochemical reactions, electro dynamics, quantum mechanics and cell growth, number theory, economics, financial mathematics, mixing problems, population models, etc.

Key Words: Delay Differential Equation, Taylor series expansion, differential transform method, approximate solution.

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A Numerical Method for Singularly Perturbed Nonlocal Boundary Value Problem on Bakhvalov Mesh

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ABSTRACT

This study deals with the singularly perturbed nonlocal boundary value problem and finite difference method.

The method analysis the appropriate singularly perturbed nonlocal boundary value problem theoretically and experimentally; That is, some properties of the exact solution of the problem is investigated. Finite difference schemes on Bakhvalov mesh for the problem are constructed. These schemes are based on the method of integral identities with the use of exponential basis functions and interpolating quadrature rules with the weight and remainder terms in integral form. The error analysis for the difference scheme is performed. Uniform convergence is obtained in the discrete maximum norm. We formulate the iterative algorithm for solving the discrete problem and a numerical example is presented to find the solution of approximation. It is shown that the method displays uniform convergence with respect to the perturbation parameter. Then we have applied the present method on a test problem. In table and figures, when N takes increasing values, it is seen that the convergence rate of the smooth convergence speed p^N is first-order. Thus, the obtained results show that the proposed scheme is working very well. These results emerge on the validity of the theoretical analysis of our method.

We believe that our study improves academic understanding of the singularly perturbed problems with nonlocal condition.

Key Words: Singular perturbation, finite difference scheme, Bakhvalov mesh, uniformly convergence, nonlocal condition.

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A Study on Hermite-Hadamard Type Inequalities via New Fractional Conformable Integrals

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ABSTRACT

The following inequality is the classical Hermite Hadamard inequality for convex functions.

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \left(\frac{f(a)+f(b)}{2}\right)$$

where $f : I \subseteq \mathrm{R} \to \mathrm{R}$ is a convex function and $a, b \in I$ with $a < b$.

It is a known fact that the theory of convex functions is closely related to the theory of inequalities. Recently, many researches have studied and investigated this theory. Especially, after 1983, Hermite-Hadamard's inequality has been considered as one of the most useful inequality in mathematical analysis. A number of the papers have been written on this inequality and lots of new results were established by using several fractional integral operators for some kinds of convex functions.

In this present paper, firstly, some necessary definitions and some results related to Riemann-Liouville fractional and new fractional conformable integral operators defined by Jarad et al. [1] are given. As a second, a new identity has been proved. By using this identity and some well-known inequalities such that triangle inequality, Hölder inequality and power mean inequality, several new Hermite-Hadamard type inequalities has obtained involving fractional conformable integral operators. Also, it is shown that, in some special cases, the results presented in this paper are reduce to some results given before in the literature.

Key Words: Gamma function, Beta function, convex functions,Hermite—Hadamard type inequalities, Riemann-Liouville fractional integrals, fractional conformable integral operators.

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A Study On Reliability Bounds

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ABSTRACT

The mathematical theory of reliability date back to World War II and its getting far more complex according to the needs arising from those days until now. To put it all in simple terms like as in the first days of the theory of reliability; for a system consisting of n independent components and repair and/or replacement of its components are unpermitted, the system reliability is the probability of that the system is working adequately at from beginning until time t and at the time t. In reliability literature and real life, generally, coherent systems are the most used systems. If a system consisting of relevant components and the system's structure function $\phi(x)$ is a monotone non-decreasing function for all independent $x_i(i \in [n] = \{1, 2, ..., n\})$ components such a system is called a coherent system. In here; state of the i th. component in a system consisting of n components is defined by $x_i = 1$ if it is working at time t otherwise $x_i = 0$. The probability of $x_i = 1$ is represented as $P(T_i > t) = P(x_i = 1) = p_i$ where T_i is the life span of the system .For a coherent system it is always hard to determine exact reliability even under the circumstances the structure function of the system is developed and knowing the distribution of the component life times. Due to this, with the help of the minimal paths and minimal cuts several methods have been developed to determine the useful bounds of reliability. In this paper, lower and upper bounds approaches of a structure function known system are estimated by using some methods in the literature and these methods advantage of each other are compared.

Key Words: Coherent systems, minimal paths, minimal cuts.

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A View on Intuitionistic Fuzzy Soft Near Rings

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ABSTRACT

In 1999, Molodtsov initiated a novel concept of soft set theory, which is a completely new approach for modelling vagueness and uncertainty. By a soft set we mean a pair (F,E) , where E is a set interpreted as the set of parameters and the mapping F: E → P(X) is referred to as the soft structure on X. The concept of soft set draw attention both of specialists working in the field of pure mathematics and applied mathematics. This concept is well coordinated with such modern mathematical concepts as a fuzzy set and more general, a many valued set. After Molodtsov' s work, some different applications on soft sets were studied. Maji et. al. combined the advantage of soft set and Attanassov' s intuitionistic fuzzy set and presented the concept of intuitionistic fuzzy soft set. The algebraic structure of set theories dealing with uncertainties had been studied by some authors. In this work, we examine the near ring structure of intuitionistic fuzzy soft sets. Near rings are generalized rings and addition needs not be commutative and only one distributive law is approved.

The aim of this study is to introduce a basic version of intuitionistic fuzzy soft near ring theory which extends the concept of soft near rings. We define intuitionistic fuzzy soft near ring, intuitionistic fuzzy soft near ideals of near rings and investigate some basic properties and some characterizations of these structures. Furthermore, we give the theorems of homomorphic image and homomorphic preimage.

Key Words: intuitionistic fuzzy set, soft set, intuitionistic fuzzy soft set, soft near ring, intuitionistic fuzzy soft near ring

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An Application of Fuzzy Differential Equations in Economics

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ABSTRACT

In the modelling of real world circumstances, generally some of the information in datas is uncertain. To handle this problem, fuzzy logic is used as a naturel way to model such real world events. In 1965, fuzzy set theory was introduced by Lotfi A. Zadeh as an extension of classical (crisp) set theory. In classical set theory every element can be assigned with characteristic function to 0 or 1. However in fuzzy set theory, every element is accompanied with a function $\mu: X \to [0, 1]$, which is called as membership function. This point of view enables us to use data with uncertainty in modelling real world problems.

Recently fuzzy logic is extensively used in differential equations. The study of fuzzy differential equations (FDEs) establishes a more realistic approach for modelling the real world event including uncertainty and vagueness. Hence, the study of fuzzy differential equations (FDEs) has been interested by many researchers in different fields of science, engineering and economics.

Especially in economics, a variety of fuzzy phenomena exist. So it is of great importance to build a model with fuzzy variables.

In this work we study on the extension of the classical Solow growth model [4], which has important applications in economics, to fuzzy environment by using fuzzy differentiability concept.

Key Words: Solow model, fuzzy differentiability, fuzzy differential equations.

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An Application of the Modified Expansion Method to Nonlinear Partial Differential Equation

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ABSTRACT

In this article, the travelling wave solutions of the medium equal width equation are obtained using the modified expansion method. The solution functions were obtained by selecting the appropriate parameters. It has been checked that these functions provide the MEW equation. Two and three dimensional graphics of the obtained solutions and other mathematical operations were found with the Mathematica software program. When the resulting solution functions are examined, it is determined that they include trigonometric, topological and singular soliton properties. Using the modified expansion function method, the nonlinear partial differential equation with travelling wave transformation takes the form of nonlinear ordinary differential equation. To find the solution of the nonlinear differential equation found, the solution function which contains the exponential function in the rational form is used. It is used to determine the boundaries of the solution function using the balancing procedure. In this way, a model of the solution function to be used for solution of the equation emerges. By using this obtained solution function, the necessary derivatives of nonlinear differential equations are found and replaced in the equation. The resulting algebraic equation system The coefficients found in the solution function are obtained using the Mathematica package program. The solution functions of nonlinear partial differential equations are written by using MEFM instead of the coefficients found.

Key Words: The modified expansion function method (MEFM), The medium equal width equation, singular soliton solution.

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An Investigation on Linear Differential Equations with Fuzzy Coefficients

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ABSTRACT

Fuzzy sets are an effective tool for modelling uncertainties [1]. Therefore, many problems with uncertainties are described by differential equations with fuzzy inputs [2-3]. In this study, we investigate such a fuzzy differential equation. If specify in details, we examine the homogeneous linear ordinary differential equation, where the coefficients are constant triangular fuzzy numbers. In addition, the initial values are triangular fuzzy numbers too. The same Initial Value Problem (IVP), but with interval inputs, we considered in [4]. (A similar Interval *Boundary* Value Problem was investigated in [5]). Therefore, in the present paper, we generalize the results of [4] for the fuzzy environment.

More formally, we investigate the following Fuzzy IVP:

$$\begin{cases} Y'' + PY' + QY = 0 \\ Y(0) = A \\ Y'(0) = B \end{cases} \tag{1}$$

where $P = (p_a, p_c, p_b)$, $Q = (q_a, q_c, q_b)$, $A = (a_a, a_c, a_b)$ and $B = (b_a, b_c, b_b)$ are given constant fuzzy triangular numbers. We define the α-cut problem (where $0 \le \alpha \le 1$) as

$$\begin{cases} Y'' + P_\alpha Y' + Q_\alpha Y = 0 \\ Y(0) = A_\alpha \\ Y'(0) = B_\alpha \end{cases} \tag{2}$$

where $P_\alpha = [\underline{p}(\alpha), \overline{p}(\alpha)]$, $Q_\alpha = [\underline{q}(\alpha), \overline{q}(\alpha)]$, $A_\alpha = [\underline{a}(\alpha), \overline{a}(\alpha)]$ and $B_\alpha = [\underline{b}(\alpha), \overline{b}(\alpha)]$ are the α-cuts of P, Q, A and B, respectively.

(2) is an Interval IVP. If use the results of [3-4], the following statement can be established.

If (1) $p_a \geq 0$, or $q_b \leq 0$;

(2) $p^2 > q$, for all $p_a \leq p \leq p_b$ and $q_a \leq q \leq q_b$;

(3) $a_a \geq 0$, $b_a \geq 0$, and $a_a p_a + b_a \geq 0$;

(4) $\max\{ a_a q_b + b_a p_b,\ a_b q_a + b_b p_b\} \leq 0$,

then the solution is found by analytical formula, for each problem (2). Consequently, the Fuzzy IVP (1) is solved analytically:

$$Y_\alpha(t) = \left[y_{\overline{p}(\alpha)\overline{q}(\alpha)\underline{a}(\alpha)\underline{b}(\alpha)}(t),\ y_{\underline{p}(\alpha)\underline{q}(\alpha)\overline{a}(\alpha)\overline{b}(\alpha)}(t) \right],$$

where $y_{pqab}(t) = e^{-pt}\left(a\cosh t\sqrt{p^2-q} + (ap+b)\dfrac{\sinh t\sqrt{p^2-q}}{\sqrt{p^2-q}} \right)$.

We demonstrate the obtained result on a numerical example. An interesting issue is that, in general, at a time t, the value of the solution is not a triangular fuzzy number, despite that the problem is linear and the inputs are triangular fuzzy numbers.

Key Words: Fuzzy differential equation; Linear differential equation; Triangular fuzzy number.

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Analysis of Surface Wave on a Coated Half-Space Caused by In-Plane Surface Loading

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ABSTRACT

The problem of the surface wave, first investigated by Rayleigh in his famous work [1], is an important research area of the applied mathematics due to its applicability to acoustic, seismology, electromagnetism, etc. Among the works on the surface waves, ground vibrations arising from the wind turbines and development on the interfacial waves have also attracted attention to in plane surface loading, see [2], [3].

This study is concerned with the dynamic response of a three-dimensional isotropic coated half-space subject to in plane surface loading. In order to extract the Rayleigh wave contribution to the overall dynamic response, a long wave model developed for the surface elastic wave in a coated half space in [4] and an asymptotic model for an elastic half-space subject to tangential surface load in [5] are employed. By using these two asymptotic models the problem is reduced to three dimensional elliptic equations over the interior of the half-space and two dimensional hyperbolic equations singularly perturbated by a pseudo differential operator on the surface of the half-space. After formulation of the problem, Rayleigh wave field is derived for a plane strain problem in case a point force acting along one of the in-plane axes. The vertical displacement may be expressed by applying integral transforms e.g. Fourier transform, Laplace transform. The obtained integral solution for the vertical displacement can be evaluated numerically and illustrated for the particular values of the material parameters.

Key Words: Rayleigh wave, coated half-space, tangential load.

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Applications of Differential Transform Method to Some Random Component Time-Fractional Partial Differential Equations

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ABSTRACT

In this study, the solutions of the random component time-fractional partial differential equations are researched by using Differential Transform Method (DTM). The parameters and the initial conditions of random component time-fractional partial differential equations are examined by Beta distribution. The fractional derivatives are defined in the Caputo sense. A few examples are specified to illustrate the influence of the solutions acquired with Differential Transform Method (DTM). Functions for the expected values and variances of the approximate analytical solutions of the random component time-fractional partial differential equations are acquired. Differential Transform Method for analyzing the solutions of these partial differential equations is applied and MAPLE software is used to find the solutions and to draw the figures. The results for the random component time-fractional partial differential equations with Beta distribution are analyzed to examine effects of this distribution on the results. The results of the deterministic partial differential equations are compared with random characteristics of these partial differential equations. The influence of this method for the random component time-fractional partial differential equations is analyzed by comparing the formulas for the expected values and variances with results from the simulations of the random component time-fractional partial differential equations.

Key Words: Random Component Time-Fractional Partial Differential Equation, Expected Value, Differential Transform Method, Caputo Derivative

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Approximation Properties of Mittag-Leffler Operators via GBS Operators

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ABSTRACT

Positive linear operators and their Korovkin-type approximation properties and applications have been studied by many mathematicians to date and continue to be studied. There are many different types of positive linear operators which have been investigated. Mittag-Leffler operator is the one of them.

Mittag-Leffler operator which contains the modified Szasz-Mirakjan operator was introduced in [3] and it was acquired the transformation properties and investigated the rate of convergence via modulus of continuity.

The concepts of *B- continuous* and *B-differentiable* functions introduced by Bögel in [2]. For the uniform approximation of *B-continuous* function, the GBS (Generalized Boolean Sum) operators are used. This term GBS operators was defined by Badea et al in [1].

In this presentation, we first mention the notions of *B-continuity, B-differentiability, B-bounded function* and the following function sets,

$B(I\times J)=\{f\,|\,f:I\times J\to\mathbb{R}, f\ is\ bounded\ on\ I\times J\}$ with the usual sup-norm $\|.\|_\infty$

$B_b(I\times J)=\{f\,|\,f:I\times J\to\mathbb{R}, f\ is\ B-bounded\ on\ I\times J\}$

$C_b(I\times J)=\{f\,|\,f:I\times J\to\mathbb{R}, f\ is\ B-continuous\ on\ I\times J\}$

and

$D_b(I\times J)=\{f\,|\,f:I\times J\to\mathbb{R}, f\ is\ B-differentiable\ on\ I\times J\}$

where, I and J are compact real intervals. Furthermore, we recall the concepts of Mittag-Leffler operators, GBS operators and the rate of convergence. Secondly, we introduce the bivariate Mittag-Leffler operator and the GBS operator associated with the operator. Finally, we obtain the approximation properties and give the results of the rate of convergence.

Key Words: Mittag-Leffler operators, GBS operators, rate of convergence.

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Asymptotic Investigation of The Singular Differential Equations and Error Analysis

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ABSTRACT

Deriving a relation between the solutions of a singular differential equation, whose highest derivative is multiplied by either a small (ε) or equivalently large (1/ε) large parameter, with the small or a large parameter is the key point of the perturbation analysis. The difficulty mathematicians face in the literature is that the series solutions of the equations are mostly divergent. The series solution approximates the exact solution for first terms, then the it begins to diverge. Henri Poincaré in 1886 *[Poincaré, 1886]* introduced the technique of asymptotic expansion and his definition is accepted general formula in 20th century and today. His method is still used and gives a good approximation only if the first a couple terms of the expansion are sufficient. The main problem of Poincaré's method is that it assumes asymptotic power series solution of the equations are valid for all the terms of the expansion. His method, unfortunately, fails to approximate the exact solution for divergent series. For the series in this form, mathematicians truncate the series where it starts diverging to infinity, and ignore the remainder. In this work, we apply the beyond all orders approach via optimally approximating the resultant remainder and obtain a better approximation than the expansion methods existing in the literature. In particular, the associated error becomes smaller which is why the accuracy of the approximate solution increases.

Key Words: Asymptotics, perturbation expansion, differential equation, integral representation, special functions.

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Behaivours of Random Effected Volterra and Fredholm Integral Equation

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ABSTRACT

In this study, the probability characteristics of random volterra and fredholm integral equation obtained when volterra and fredholm integral equations are taken as a random variable with coefficient beta distribution are calculated. Also, the approximate analytical solutions are obtained by applying random differential transformation method(RDTM) to random volterra and fredholm integral equation. The method is further extended with a formulation to treat Volterra and Fredholm integrals. Approximate correlation for expected value and variance were found using these solutions from obtaibed random differential transformation method(RDTM). Also obtained the approximate expected value and variance formulas converging to a wider region by applying the modified DTM. To demonstrate this capability and robustness, some volterra and integro-differential equation are solved as numerical examples. MAPLE software is used for the finding the solutions and drawing the figures. The results show that random DTM is an effective and accurate technique for finding exact and approximate solutions. Finally, these solutions are compared.

Key Words: Volterra and Fredholm Integral Equation, Expected Value, Variance, Differential Transformation Method, Modified DTM.

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Boundedness And Convergence Properties For A Class Of Kantorovich-Type Operators In Variable Exponent Lebesgue Spaces

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ABSTRACT

The classical Bernstein polynomials $B_n f$ defined by

$$(B_n f)(x) := \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right), \qquad f \in C([0,1]), x \in [0,1]$$

are known to give the most elegant proof of Weierstrass approximation theorem by algebraic polynomials in the space of all the continuous functions in [0,1], (see, e.g., [3]).

Let $1 \leq p < \infty$ and define an operator $K_n : L^p([0,1]) \to L^p([0,1])$ by

$$(K_n f)(x) := \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} (n+1) \int_{k/(n+1)}^{(k+1)/(n+1)} f(y) dy,$$

for every $n \geq 1$ and $0 \leq k \leq n$, $f \in L^p([0,1])$, $x \in [0,1]$. Each $K_n f$ is a polynomial of degree not greater than n and every K_n is a positive linear operator from $L^p([0,1])$ (and, in particular, from $C([0,1])$ into $C([0,1])$). For additional information on these operators, see [3]. In 1953 Lorentz [3, Theorem 2.1.2] proved that for $f \in L^p([0,1]), p \geq 1$

$$lim_{n\to\infty} \|K_n f - f\|_p = 0.$$

Some other authors have dealt with the degree of L^p-approximation of the Kantorovich operators, for instance cf. [4].

In this present we deal with Kantorovich–type operators

$$(S_n f)(x) = \sum_{k=0}^{r(n)} K_n\left(x, v_{n,k}\right) \frac{1}{\lambda_{n,k}} \int_{v_{n,k}}^{v_{n,k+1}} f(y) dy, \ n \in \mathbb{N}, \ x \in I = [0,1],$$

where for every fixed $n \in \mathbb{N}$ a finite sequence of points $\Gamma_n = \left(v_{n,k}\right)_{k=0,1,\dots,r(n)+1} \subset I$, $\left(r(n)\right)_{n\in\mathbb{N}}$ be an increasing sequence of natural numbers and a_n, b_n are positive real numbers that satisfying the following assumption

$0 < a_n \leq v_{n,k+1} - v_{n,k} := \lambda_{n,k} \leq b_n, k = 0,1,\dots,r(n),$

$\lim_{n\to\infty} b_n = 0,$

and $f: I \to \mathbb{R}$, $(K_n)_{n\in\mathbb{N}}: I \times \Gamma_n \to \mathbb{R}$ is a sequence of nonnegative kernel functions (see [1]). We construct boundedness and convergence of a sequence of Kantorovich-type operators in variable exponent Lebesgue spaces $L^{p(\cdot)}([0,1])$ ([2,5]).

Key Words: Variable exponent Lebesgue spaces; Kantorovich-type operators; boundedness and convergence.

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Comparison Theorems For One Sturm-Liouville Problem With Nonlocal Boundary Conditions

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ABSTRACT

In this study we present a new approach for investigation of some Sturm-Liouville systems with nonlocal boundary conditions. In the theory of boundary value problems for two-order differential equations the basic concepts and methods have been formulated studying the problems of classical mathematical physics. However, many modern problems, which arise as the mathematical modeling of some systems and processes in the fields of physics, such as the vibration of strings, the interaction of atomic particles motivate to formulate and investigate the new ones, for example, a class of Sturm-Liouville problems with nonlocal boundary conditions. Such conditions arise when we cannot measure data directly at the boundary. In this case, the problem is formulated, where the value of the solution and its derivative is linked to interior points of the considered interval. Sturm-Liouville problems together with transmission conditions at some interior points is very important for solving many problems of mathematical physics. In this study we present a new approach for investigation of boundary value problems consisting of the two interval Sturm-Liouville equations

This kind of boundary value transmission problems are connected with various physical transfer problems(for example, heat and mass transfer problems). We define a new Hilbert space and linear differential operator in it such a way that the considered nonlocal problem can be interpreted as an spectral problem. We investigate the main spectral properties of the problem under consideration. Particularly we present a new criteria for Sturm-Comparison theorems. Our main

result generalizes the classical comparison theorem for regular Sturm-Liouville problems.

Key Words: Boundary value problems, Comparison theorems, transmission conditions.

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Computational Method for a Singularly Perturbed Mixed Type Delay Differential Problem

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ABSTRACT

This paper deals with the singularly perturbed boundary-value problem (SPBVP) for a linear second order differential difference equation with delay as well as advance. These problems arise frequently in science and engineering field. For instance, they occur in the study of human pupil light reflex, first-exit problems in neurobiology, models of physiological processes and diseases, optimal control theory, models of climate systems, optically bistable devices and signal transmission, quantum photonic systems [1,3-7].

On the other hand, for small values of perturbation parameter ε, standard numerical methods for solving SPBVPs are unstable and do not give accurate results. Therefore, it is important to develop suitable numerical methods for solving these problems, whose accuracy does not depend on the parameter value ε, i.e., methods that are convergent ε -uniformly [2,8].

Firstly, we examine some properties of exact solution of the problem. Next, in order to numerical solution of this problem, we use a fitted difference scheme on a piecewise uniform mesh of Shishkin type which is accomplished by the method of integral identities with the use of linear basis functions and interpolating quadrature rules with weight and remainder term in integral form. It have shown that it gives essentially first order uniform convergence in the discrete maximum norm, independently of the perturbation parameter. Furthermore, a numerical experiment illustrate in practice the result of convergence proved.

Key Words: Boundary layer, delay differential equation, finite difference method, uniform convergence.

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Determination of The Relationship Between Students' Official Test and Pilot Test In The Transition To High School Using The Intuitionistic Fuzzy Logic

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ABSTRACT

In this paper, we have proposed an application of intuitionistic fuzzy set in high school determination using distance measures in intuitionistic fuzzy sets. The purpose of this paper is to interpret the relationship between students' official test and the pilot test by means of distance measures in intuitionistic fuzzy sets. Distance measures have been compared in this study. The data have been calculated with all distance measures and the most appropriate distance measure for this paper has been determined. This application of intuitionistic fuzzy set in high school determination is very useful; because by calculating distance between each student and each school, the most proper school for each student has been determined. Available evaluation system could be renewed by using this application of intuitionistic fuzzy logic. Using this application in evaluation and determination system will have very beneficial results. The contribution of this new system are more stress-free, less anxious for testing, easier to implement, more economical and more advantageous in many ways. Thanks to this system; the enroll of students in high schools will not depend on a single test. Moreover; this is the first study to guide the students in the direction of students preferences in evaluation of success in education.

Key Words: intuitionistic fuzzy sets, distance measure, decision making.

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Dirac System for Hahn Difference Operator

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ABSTRACT

The quantum calculus is known as the calculus without limits (see [1]). Quantum difference operators have been applied in many different fields such as orthogonal polynomials, basic hypergeometric functions, combinatorics, the calculus of variations and the theory of relativity. In recent years, Annaby et. al. have investigated q-(basic) Sturm-Liouville problems and q,ω-Sturm-Liouville problems in [2] and [3], respectively. Allahverdiev and Tuna have investigated q-Dirac system in [4]. They have proved that the existence and uniqueness of the solution, all eigenvalues of this system are real and the eigenfunctions satisy on orthogonality relation and they have also constructed Green function of q-Dirac system. Some spectral properties and asymptotic formulas for the eigenvalues and the eigenfunctions for the same q-Dirac system have been given in [5].

In this paper, we consider q,ω- Dirac system when the differential operator is replaced by the q,ω-Hahn difference operator $D_{q,\omega}$. The q,ω-Hahn difference operator is defined by $D_{q,\omega}f(t) := \left(f(qt+\omega)-f(t)\right)/\left((qt+\omega)-t\right)$, where $0<q<1$ and $\omega>0$ are fixed real numbers (see [4]). This operator unifies the classical difference operator $\Delta_{\omega}f(t)=\left(f(t+\omega)-f(t)\right)/\omega$ and the Jackson q-difference operator $D_q f(t) := \left(f(qt)-f(t)\right)/\left(t(q-1)\right)$. We present an existence and uniqueness theorem by using the method of successive approximations and discuss the properties of the eigenvalues and the eigenfunctions for this q,ω- Dirac system.

Key Words: q,ω- Dirac system, Hahn difference operator.

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Discrete Fractional Solution of a Nonhomogeneous Non-Fuchsian Linear Differential Equations

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ABSTRACT

One of the most popular research interests of science and engineering is the Discrete fractional calculus theory in recent times. In this work, we acquire new discrete fractional solutions of the nonhomogeneous linear ordinary differential equation by using discrete fractional nabla operator.

Fractional and discrete fractional calculus are popular works that have been studied in recent years. The fractional calculus theory, which spans a wide field, has contributed many science fields and has been gained a lot of scientific publications to the literatüre [1,2].

Granger and Joyeux [3] and Hosking [4], defined notion of the fractional difference as follows

$$\nabla^{\alpha}\varphi(t)=(1-q)^{\alpha}\varphi(t)=\sum_{n=0}^{\infty}(-1)^{n}\frac{\Gamma(\alpha+1)}{\Gamma(n+1)\Gamma(\alpha-n+1)}q^{n}\varphi(t)$$

where α is any real number and $q^{n}\varphi(t)=\varphi(t-n)$ is the standard backward-shift operator. Gray and Zhang [5] studied on a new definition of the fractional difference. Many properties based on this definition are established including an extensive exponential law and the important Leibniz rule.

Lately, many papers have appeared on the discrete fractional calculus that helped to construct some of the basic theory of this field. For example, Atici and Eloe [6] studied on a family of finite fractional difference equations and use the transform method of solution. Atici and Sengul [7] developed the Leibniz rule in discrete fractional calculus. Yilmazer, R., Inc, M., Tchier, F. and Baleanu, D. [8] obtained particular solutions of the confluent hypergeometric differential equation by using the nabla fractional calculus operator.

Key Words: Fractional Calculus, Discrete Fractional Calculus, Nabla Operator.

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Explicit Solutions of the Homogeneous Second Order Differential Equation by means of Fractional Calculus

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ABSTRACT

Fractional calculus is a very useful and simple means in obtaining particular solutions to certain nonhomogeneous linear differential equations. Our aim in this work is to obtain fractional solutions of the second order homogeneous differential equation with Nishimoto's operator.

Claim of the derivatives and integrals with any arbitrary order was born in 1965 and, this new and remarkable subject has intensive work fields such as mathematics, physics, chemistry, biology, medicine and engineering. The widely investigated subject of fractional calculus has gained considerable importance and popularity during the past three decades or so, due chiefly to its demonstrated applications in numerous seemingly diverse fields of science and engineering (see, for details [1-4]). We can mention that the fractional differential equations are playing an important role in fluid dynamics, traffic model with fractional derivative, measurement of viscoelastic material properties, modeling of viscoplasticity, control theory, economy, nuclear magnetic resonance, geometric mechanics, mechanics, optics, signal processing and so on.

The differintegration operators and their generalizations [5-8] have been used to solve some classes of differential equations and fractional differential equations.

Some of most obvious formulations based on the fundamental definitions of Riemann-Liouville fractional integration and fractional differentiation are, respectively,

$$ {}_aD_t^{-\alpha}\varphi(t)=\frac{1}{\Gamma(\alpha)}\int_a^t\varphi(\tau)(t-\tau)^{\alpha-1}d\tau \quad (t>a,\alpha>0), $$

and

$$ {}_aD_t^{\alpha}\varphi(t)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^n\int_a^t\varphi(\tau)(t-\tau)^{n-\alpha-1}d\tau \quad (n-1\le\alpha<n) $$

where $n\in N$, N being the set of positive integers, Γ stands for Euler's function gamma.

Key Words: Fractional calculus; Generalized Leibniz rule; Ordinary differential equation.

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Finding single-wave solutions using polynomial methods for analogs of the nonlinear Boussinesq equation

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ABSTRACT

The equation

$$\lambda^2 \Delta u_{tt} - u_{tt} + \sigma^2 \Delta u = 0$$

is called Boussinesq equation. Here Δ is the Laplace operator with respect to the space variables. Each $\lambda \, and \, \sigma$ are real numbers. Similar equations express the propagation of waves that form on the surface of water and mixtures, and sometimes they are expressed in terms of the longitudinal waves propagating in a wire [1-2].

Many articles have investigated the solvability of boundary-value problems for these and similar equations, resolution robustness, and in particular some soliton-type solutions and their properties. As an example, it is possible to show the research studies [3]. On the other hand, we are encountering Boussinesq equations in nonlinear equations, where the nonlinearity is usually in the space derivative of the sought function. As an example of such research studies, we can show these references [4].

$$\mu u_{xxxxxx} + u_{ttxx} + \alpha^2 u_{xx} - \beta^2 \left(q(u)\right)_{tt} = f(x,t)$$

Such equations are called analogues of the Boussinesq equation. The main purpose of this work is to investigate single-wave solutions of such and similar equations. In this study differs from the others in that the function sought in the nonlinear equation is found in the term derivative with respect to time. In the study, we will find single-wave solutions specifically for the following Boussinesq equation:

$$(q(u))_{tt} - u_{xx} + u_{xxtt} - \mu u_{xxxxxx} = 0 \qquad (1)$$

Here, $q(\xi)$ is a sufficiently differentiable function. In this study, for the case $q(\xi) = \xi^2$, the single-wave solutions for equation (1) will be studied for $\mu = 0 \, and \, \mu = 1$ using polynomial function method.

In this work, single-wave solutions have been investigated in some special cases for nonlinear Boussinesq equation analogues. The Boussinesq equation studied up to this time and the nonlinearity in the analogue are found in the function sought, or in terms differentiated by the space variables of this function [5]. But the importance of this study is that the presence of the time derivative of the nonlinearity function is a term.

Key Words: Boussinesq equation,nonlinear Boussinesq equation,solitary solutions.

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Finite difference method for delay pseudo-parabolic equations

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ABSTRACT

Pseudo-parabolic equations (PPEs) appear in many areas of mechanics and physics. They are used to study homogeneous fluid flow in fissured rocks, heat conduction involving thermodynamic temperature and conductive temperature, quasistationary processes in semiconductors and other physical models. The significant characteristic of these equations is that they state the conservation of a certain quantity (mass, momentum, heat, etc.) in any sub-domain. Various numerical schemes have been constructed to treat PPEs in [1-4].

The present study is concerned with the one dimensional initial–boundary value problem for DPPEs. Our aim is to construct higher order difference method for approximation to the considered problem when the coefficients are independent of spatial variable. Based on the method of energy estimates and difference analogue of the Gronwall's inequality with delay, the fully discrete scheme is shown to be convergent of order four in space and of order two in time. Numerical example supporting the theory is presented.

In this work, one dimensional initial-boundary delay pseudo-parabolic problem is being considered. We solve this problem numerically. We construct higher order difference method for approximation to the considered problem and obtain the error estimate for its solution. Based on the method of energy estimate the fully discrete scheme is shown to be convergent of order four in space and of order two in time. Numerical example is presented.

Key Words: Pseudo-parabolic equation, Delay difference scheme, Error estimate

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Fractional Variational Iteration Method for the Random Component Time-Fractional Fornberg-Whitham Equation

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ABSTRACT

In this work, the solutions of the random component time-fractional partial differential equations are analyzed by using Fractional Variational Iteration Method (FVIM). The parameters and the initial conditions of random component time-fractional partial differential equations are studied with Beta distribution. The fractional derivatives are defined in the Riemann-Liouville sense. A few examples are indicated to illustrate the effect of the solutions obtained with Fractional Variational Iteration Method (FVIM). Functions for the expected values and variances of the approximate analytical solutions of the random component time-fractional partial differential equations are acquired. Fractional Variational Iteration Method is applied to analyze the solutions of these partial differential equations and MAPLE software is used for the finding the solutions and drawing the figures. The results for the random component time-fractional partial differential equations with Beta distribution are analyzed to examine effects of this distribution on the results. Random characteristics of the partial differential equations are compared with the results of the deterministic partial differential equations. The efficiency of this method for the random component time-fractional partial differential equations is examined by comparing the formulas for the expected values and variances with results from the simulations of the random component time-fractional partial differential equations.

Key Words: Random Component Time-Fractional Partial Differential Equation, Expected Value, Fractional Variational Iteration Method

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Fuzzy Eigenvalue Problem with Eigenvalue Parameter Contained in the Boundary Condition

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ABSTRACT

The study of fuzzy differential equations forms a suitable setting for the mathematical modelling and physical systems of real world problems in which uncertainty or vagueness spread. Most of researchers assume the derivative in the differential equation as a derivative of a fuzzy function in some approach. In earlier researchers the derivative was considered as Hukuhara derivative for fuzzy functions. When Hukuhara derivative is used, then uncertainty of the solution may increase infinitely with time. In order to overcome this difficulty Bede and Stefanini (2013) developed the generalized Hukuhara derivative concept. Hence, we continue the research of fuzzy differential equation using this differentiability concept. So investigate the solutions such that the solutions satisfy real world models becomes an interesting and important problem on the field of fuzzy differential equations.

In this study we consider two-point fuzzy boundary value problem with eigenparameter dependent boundary conditions under the approach generalized Hukuhara differentiability (gH-differentiability). We research the solution method of the fuzzy boundary problem with the basic solutions $\hat{\phi}(t,\lambda)$ and $\hat{\chi}(t,\lambda)$ which are defined by the special procuder. We give operator-theoretical formulation, construct fundamental solutions and investigate some properties of the eigenvalues and corresponding eigenfunctions of the considered fuzzy problem. Then results of the propesed method are illustrated with a numerical example.

Key Words: Fuzzy eigenvalue, Fuzzy eigenfunction, Generalized Hukuhara derivative, Fuzzy coefficient.

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Fuzzy Eigenvalue Problem with Fuzzy Coefficients of Boundary Conditions

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ABSTRACT

In this study two point fuzzy boundary value problem is defined under the approach generalized Hukuhara differentiability. It is well known that the Hukuhara derivative with Hukuhara difference exits only under very restrictive conditions such as the diameter of the solutions is unbounded as time t increases and Hukuhara difference between two fuzzy numbers exists under restrictive circumstances. However generalized Hukuhara differentiability with generalized Hukuhara difference exists under much less restrictive conditions. So generalized Hukuhara differentiability can be of a great help in the study of the fuzzy eigenvalue problem.

The solution method of the fuzzy boundary problem is examined with the aid of the fuzzy boundary problems for boundary values. To do this, an initial value problem is created for each fuzzy boundary value. Then, the Wronskian for this fuzzy initial value problems of solution functions is obtained with fuzzy arithmetic. So eigenvalues of two point fuzzy boundary value problem are found this method. If these eigenvalues obtained are written in the fuzzy initial value problem, then the eigenfunctions of the two point fuzzy boundary problem are found. Later, whether these eigenfunction are fuzzy or not is checked by known fuzzy conditions. So results of the proposed method are illustrated with a numerical example.

Key Words: Fuzzy eigenvalue, Fuzzy eigenfunction, Generalized Hukuhara derivative.

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Fuzzy Revenue for Linear and Quadratic Demand Functions Using Polygonal Fuzzy Number

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ABSTRACT

Let $P(x) = \mathrm{a}_0 - \mathrm{b}_0\mathrm{x}$ be the linear demand function where $0 \leq \mathrm{x} \leq \mathrm{a}_0/\mathrm{b}_0$ and $a_0 > 0, b_0 > 0$ are fixed numbers and $P(x)$ is the unit price with respect to the demand quantity x. Hence revenue function is $R(x) = \mathrm{a}_0\mathrm{x} - \mathrm{b}_0 x^2$ where $0 \leq \mathrm{x} \leq \mathrm{a}_0/\mathrm{b}_0$. Chang and Yao [4] fuzzfied this revenue function using the triangular fuzzy numbers. They obtained more optimum results than the crisp case.

In this talk, fuzzifying the constants a_0 and b_0 (instead of quantity) of linear demand function and quadratic demand function we estimate the optimal revenue. We use polygonal fuzzy numbers for fuzzification. Polygonal fuzzy number is generalization of triangular fuzzy number. This type of fuzzy number was introduced by Baez et al [1]. For a fixed partition $P_m: 0 = \alpha_0 < \alpha_1 < \alpha_2 < \cdots < \alpha_m = 1$ of the interval $[0,1]$ the fuzzy number A is called polygonal fuzzy set associated with partition P_m. $\forall \alpha \in (0,1]$ we have that the α-level set A_α satisfies $A_\alpha = (1-\lambda)A_{\alpha_i} + \lambda A_{\alpha_{i+1}}$ where $0 \leq \alpha_i < \alpha \leq \alpha_{i+1} \leq 1$ for some $i = 0, \ldots, m-1$ and $\lambda = \lambda(\alpha) = (\alpha - \alpha_i)/(\alpha_{i+1} - \alpha_i)$.

Then we use the signed distance and graded mean defuzzification method for defuzzification. Here signed distance for fuzzy price and signed distance for fuzzy revenue are calculated. Similarly graded mean for fuzzy price and graded mean for fuzzy revenue are calculated.

In the second step, we extend the linear fuzzy price to the quadratic form $\tilde{P}(x) = \tilde{a} - \tilde{b}x + \tilde{c}x^2$ to obtain the fuzzy revenue. We repeated similar calculations for this quadratic form. For each step, we give numerical examples which shows that our model gives more optimum results than not only crisp model but also Chang and Yao's [4] model. Here we used Chang and Yao's example to compare with our model.

Key Words: Polygonal fuzzy number, Signed distance defuzzification method, Graded mean defuzzification method, Fuzzy Revenue, Demand.

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Generalized Fractional-Order Mathematical Model for Bacterial Infectious

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ABSTRACT

In this study, a mathematical model with a fractional order in case of bacterial infection has proposed. This model examines the interaction of the following variables: the population sizes of sensitive bacteria and resistant bacteria to multiple antibiotics and host's immune system cells to the bacterial infection. Because the differential equations with fractional-order are as stable as the least integer differential equations, the system of fractional differential equations have used in this model.

Let us consider the model in the study. The population sizes of sensitive and resistant bacteria to multiple antibiotics at time t is denoted by $S(t)$ and $R(t)$, respectively. The population sizes of host's immune system cells at time t is denoted by $B(t)$ and the concentration of the i-th antibiotic for $i = 1,2,\dots,n$ is showed by $A_i(t)$. Therefore, it is obtained the following system with fractional-order

$$\begin{aligned}
&D^\alpha S = \beta_S S\left(1-\frac{S+R}{K}\right) - \omega_S S - S\sum_{i=1}^{n}\alpha_i A_i - S\sum_{i=1}^{n} d_i A_i - \gamma BS \\
&D^\alpha R = \beta_R R\left(1-\frac{S+R}{K}\right) - \omega_R R + S\sum_{i=1}^{n}\alpha_i A_i - \gamma BR \qquad (1)\\
&D^\alpha B = H - v(S+R) \\
&D^\alpha A_i = \delta_i - \mu_i A_i, i = 1,2,\dots,n
\end{aligned}$$

where $\alpha \in (0,1]$. Additionally, $\beta_S, \beta_R, \omega_S, \omega_R, H, \gamma, K, v, d_i, \alpha_i, \delta_i$ and μ_i parameters for $i = 1,2,\dots,n$ are positive, it is $S = S(t), R = R(t), B = B(t), A_i = A_i(t)$ and the initial conditions are $S(t_0) = S_0, R(t_0) = R_0, B(t_0) = B_0$ ve $A_i(t_0) = A_{i_0}$ for $i = 1,2,\dots,n$.

The parameters used in the model (1) are shown below. It is assumed that the bacteria growth is in accordance with the logistic rule. Therefore, the parameters β_S and β_r are the growth rate of susceptible and resistance bacteria, respectively. The parameter K indicates carrying capacity of the bacteria. The sensitive and resistant bacteria to multiple antibiotics have natural death rates ω_S and ω_R, respectively. It is presumed that the immune system cells of the host are produced in a constant amount of H. The elimination rate of bacteria by host's immune system cells is γ. During administration of the i-th antibiotic, depending on the mutations of susceptible bacteria exposed to this antibiotic, a number of resistance bacteria may arise. It is modelled this situation by the term $\alpha_i A_i S$ where α_i is the mutation rate of sensitive bacteria due to exposure to i-th antibiotic. Sensitive bacteria also die due to the action of the antibiotics and it is shown that this situation in model is by the term $d_i A_i S$, where d_i is the death rate of sensitive bacteria due to exposure to i-th antibiotic. Also, immune system cells are destroyed at a rate of v, proportional to the size of the current bacteria. Finally, the i-th antibiotic concentration is supplied at a constant rate δ_i and removed from the host's body at rate per capita μ_i.

According to the qualitative analysis of the aboved model; the stability conditions has made according to the parameters used in the model. In the numerical analysis of the model, the effects of the mycobacterium tuberculosis and the antibiotic used for it have examined.

Key Words: Keyword Fractional-Order Differential Equation, Qualitative Analysis, Stability, Numerical Simulation. one, keyword two, keyword three.

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Green's Functions For Two-Interval Sturm-Liouville Problems In Direct Sum Space

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ABSTRACT

The theme of the development of the theory and applications of Green's Functions is skilfully used to motivate and connect clear accounts of the theory of distributions, Fourier series and transforms, Hilbert spaces, linear integral equations.

In this work we analyze the Green's functions of boundary value problems defined on two interval and associated with Schrödinger operators with interaction conditions. We have constructed some special eigensolutions of to this problem and presented a formula and the existence condition of Green's function in terms of the general solution of a corresponding homogeneous equation is known. We have obtained the relation between two Green's functions of two nonhomogeneous problems. It allows us to find Green's function for the same equation but with different additional conditions. These problems include the cases in which the boundary has two, one or none vertices. In each case, the Green's functions, the eigenvalues and the eigenfunctions are given in terms of asymptotic formulas. A preliminary study of two–point regular boundary value problems on with additional transmission conditions was developed by the authors of this study under the denomination of two–point transmission boundary value problems. In each case, it is essential to describe the solutions of the Schröndinger equation on the interior nodes of the path. As an consequence of this property, we can characterize those boundary value problems that are regular and then we obtain their corresponding Green's function, as well as the eigenvalues and the eigenfunctions for the regular case, in terms of eigenfunctions.

Key Words: Schrödinger operators, Green Functions**,** transmission conditions.

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Hyers-Ulam stability for Caputo-Fabrizio type fractional differential equations

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ABSTRACT

Fractional differential equations are the area of concentration of recent investigations and significant progress has been recorded in this area. However, the concept of fractional derivatives is not new and is very much as old as differential equations. Firstly, in 1965, L Hospital raised the question about fractional derivative in a letter written to Leibniz and related his generalization of differentiation. In recent years, many researchers have focused on the study of Hyers-Ulam stability of fractional differential equations, and gained a series of results.

Hyers-Ulam stability is one of the main topics in the theory of functional equations. Firstly, in 1940, S.M. Ulam posed a problem concerning the stability of functional equations to give conditions in order for a linear mapping near an approximately linear mapping to exist in the talk at the University of Wisconsin. In 1941, Hyers [2] solved it.

Recently, by applying Laplace transform method, Rezaei et al. [3] and Alqi_ary and Jung discussed Hyers-Ulam stability and generalized Hyers-Ulam stability of linear differential equations, respectively. In 2011, Wang et al. [5] studied the Ulam stability for the following fractional differential equation

$$c_{D^{\alpha}}\, x(t) = f(t, x(t)),\ t \in (a, b),\ b = +\infty,$$

where $c_{D^{\alpha}}$ is the Caputo fractional derivative of order $\alpha \in (0,1)$ and the function f satisfies some onditions.

In 2012, Wang and Zhou [7] presented four types of Mittag-Leffler-Ulam stability for the following fractional evolution equation in a Banach space:

$$c_{D^{q}}\, x(t) = -Ax(t) + f(t, x(t)),\ q \in (0,1), t \in I \subset \mathbb{R},$$

where ${}^{c}D^{q}$ is the Caputo fractional derivative of order q.

In 2015 Wang and Xu [6] studied the following two types of fractional linear differential equations:

$$\left({}^{c}D_{0+}^{\alpha} y\right)(x) - \lambda y(x) = f(x)$$

and

$$\left({}^{c}D_{0+}^{\alpha} y\right)(x) - \lambda\left({}^{c}D_{0+}^{\beta} y\right)(x) = g(x)$$

where $x > 0$, $\lambda \in \mathbb{R}, n-1 < \alpha \leq n, m-1 < \beta \leq m, 0 < \alpha < \beta, m, n \in \mathbb{N}$, $m \leq n$, $f(x)$ and $g(x)$ are real functions defined on R+, and ${}^{c}D_{0+}^{\alpha}$ is the Caputo fractional derivative of order α defined by

$$\left({}^{c}D_{0+}^{\alpha} y\right)(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} y^{(n)}(t)dt.$$

They obtained Hyers-Ulam stability of the two types of fractional linear differential
equations by applying the Laplace transform method.

In 2016, Wang and Li [4] obtain the Ulam-Hyers stability of the following fractional order linear differential equation using Laplace transform method and the properties of Mittag-Leffler functions:

$${}^{c}D_{t}^{\alpha}\, y(t) + \lambda y(t) = f(t), t \in J = [0, T), 0 < T \leq \infty, \lambda \geq 0,$$

where ${}^{c}D_{t}^{\alpha}$ is the Caputo fractional derivative of order $\alpha \in (m-1, m), m \in \mathbb{N}^{+}$ with the lower limit zero and f is a m -times continuously differentiable and of exponential order.

Motivated by the ideas of Wang and Li [4] and Wang and Xu [24], in this paper we establish the stability in the sense of Hyers-Ulam for the fractional differential equations

$$({CF}_{D}{}^{\alpha} y)(x) = f(x)$$

and

$$({CF}_{D}{}^{\alpha} y)(x) - \lambda y(x) = f(x),$$

where the new Caputo-Fabrizio fractional derivative is defined by

$$({CF}_{D}{}^{\alpha} y)(x) = \frac{(2-\alpha)M(\alpha)}{2(1-\alpha)} \int_0^x \exp\left(-\frac{\alpha}{1-\alpha}(x-t)\right) y'(t)dt, x > 0,$$

and $M(\alpha)$ is a normalization constant depending on α.

Key Words: Differential equations, Ftractional differential equations, Hyers-Ulam satability.

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Importance of Stokes rays of the singular differential equations

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ABSTRACT

In 1864, George Stokes *[Stokes, 1864]* realized that the coefficients of the divergent series are discontinues along the complex plane. However, he was unable to prove it at that time and stated that as if the solution of the equation gets into the mist in the complex plane in *[Stokes, 1902]*. Unfortunately, the theories of Stokes neither were understood or solved by the mathematicians for decades; hence, they were ignored and Henry Poincaré's method is used *[Poincaré, 1886]*. However, Poincaré's definition of expansion, unfortunately, cannot deliver the importance of exponential order terms, as they are hidden behind the algebraic order terms. These small terms have a plethora applications including to but not limited to applications in mathematics, engineering, electromagnetic wave theory, ecological modelling, fluid dynamics, and atomic interactions. In this work, we investigate the importance of the exponential order terms and extract them thorough local analysis near the singular points. We identify the singular points, if exist, and the late order terms, where we emphasise whether the remainder is exponentially small via the method of asymptotics in all orders *[Berry, 1991, Boyd, 1999]*. In some certain regions of the complex plane, these ignored quantities take big values and manipulate the solution. Particularly, across some certain rays, they turn on and off. In fact, this happens via error function *[Berry, 1988, Berry, 1989]*. Therefore, ignoring them misleads the solution. We show wherein the complex plane these small terms occur like solving a puzzle.

Key Words: Asymptotics, special differential equations, integral representation, special functions, Stokes rays, local analysis.

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Inverse Nodal Problems For Dirac Differential Operators With Discontinuity Conditions Inside The Interval

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ABSTRACT

Consider the following boundary value problem, generated by the system of discontinuous Dirac differential equations

$$BY'(x)+\Omega(x)Y(x)=\lambda Y(x),\ x\epsilon(0,\pi) \qquad (1)$$

with boundary conditions

$$\begin{cases} y_1(0)sin\theta+y_2(0)cos\theta=0 \\ y_1(\pi)sin\beta+y_2(\pi)cos\beta=0 \end{cases} \qquad (2)$$

and discontinuity conditions

$$\begin{cases} y_1(\pi/2+0)=\sigma y_1(\pi/2-0) \\ y_2(\pi/2+0)=\sigma^{-1}y_2(\pi/2-0) \end{cases} \qquad (3)$$

here, $B=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $Y=\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$, $\Omega(x)=\begin{pmatrix} V(x)+m & 0 \\ 0 & V(x)-m \end{pmatrix}$, $V(x)$ is a real valued functions in $W_2^1(0,\pi)$, m and $0\le\theta,\beta<\pi$ are real numbers, $\sigma\in\mathbb{R}^+$ and λ is the spectral parameter.

The inverse spectral analysis is very important research area in mathematics and mathematical physics. Discontinuous boundary value problems often appear in mathematics, mechanics, physics, geophysics and other branches of natural properties and play a important role in these areas. The basic and comprehensive results about Dirac operators were given in [7]. Classical inverse problems for Dirac operators have been studied fairly completely. Inverse nodal problems were first proposed and solved for Sturm--Liouville operator by McLaughlin [4] in 1988. In this study, it has been shown that a dense subset of zeros of eigenfunctions ,called nodal points, uniquelly determines the potential of the Sturm Liouville operator. In 1989, Hald and McLaughlin consider inverse nodal Sturm--Liouville problem with more general boundary conditions and give some numerical schemes for the reconstruction of the potential from nodal points in [5]. Yang [8] proposed an algorithm to solve an inverse nodal problem for Sturm--Liouville operator in 1997 .

Such problems have been extensively addressed by various researchers in several papers.The inverse nodal problems for Dirac operators with various boundary conditions have been studied and shown that the dense subsets of nodal points which are the first components of the eigenfunctions determines the coefficients of discussed operator in for example [6] and [1] and the references therein. Since, there are not sufficiently good asymptotic expressions for the integral equations of the solutions of discontinuous Dirac system, inverse nodal problems for the Dirac system with discontinuity condition inside the interval has not been considered before. The aim of this work is to study an inverse nodal problem of reconstructing the discontinuous Dirac system. Firstly, I have obtained a new approach for asymptotic expressions of the integral equations of the solutions of such discussed problem. Secondly, more accurate estimates of eigenvalues and nodal points have been calculated with the help of these asymptotics. Lastly, I have proved that the operator can be reconstructed by nodal points.

Key Words: Dirac differential operator, discontinuity conditions inside the interval, inverse nodal problem, uniqueness theorem,.

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Inverse Nodal Problems for Dirac-Type Integro-Differential System with Boundary Conditions Polynomially Dependent on the Spectral Parameter

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ABSTRACT

Consider the following boundary value problem generated by the system of discontinuous Dirac differential equations

$$BY'(x) + \Omega(x)Y(x) + \int_0^x M(x,t)Y(t)dt = \lambda Y(t)\,, x\epsilon(0,\pi)$$

with boundary conditions

$$a_1(\lambda)y_1(0) + a_2(\lambda)y_2(0) = 0$$
$$b_1(\lambda)y_1(\pi) + b_2(\lambda)y_2(\pi) = 0,$$

where,

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \Omega(x) = \begin{pmatrix} V(x)+m & 0 \\ 0 & V(x)-m \end{pmatrix}, M(x,t) = \begin{pmatrix} M_{11}(x,t) & M_{11}(x,t) \\ M_{11}(x,t) & M_{11}(x,t) \end{pmatrix},$$

$Y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$, $V(x)$ and $M(x,t)$ are real valued functions in $W_2^1(0,\pi)$, $a_1(\lambda)$, $a_2(\lambda)$, $b_1(\lambda)$ and $b_2(\lambda)$ are monic polynomials, m is a real number and λ is the spectral parameter.

Inverse nodal problems were first proposed and solved for Sturm--Liouville operator by McLaughlin in 1988 [3]. In this study, it has been shown that a dense subset of zeros of eigenfunctions, called nodal points, uniquelly determines the potential of the Sturm Liouville operator..In 1989, Hald and McLaughlin gave some numerical schemes for reconstructing potential from nodal points for more general boundary conditions. [4]. In 1997 Yang gave an algorithm to determine the coefficients of operator for the inverse nodal Sturm-Liouville problem [8]. Inverse nodal problems have been addressed by various researchers in several papers for different operators. The inverse nodal problems for Dirac operators with various boundary conditions have been studied and shown that the dense subsets of nodal points which are the first components of the eigenfunctions determines the coefficients of discussed operator in [1] ,[5] and the references therein.

In recent years, perturbation of a differential operator by a Volterra type integral operator, namely the integro-differential operator have begun to take significant place in the literature for example [6] and [7]. Integro-differential operators are nonlocal, and therefore they are more difficult for investigation, than local ones. New methods for solution of these problems are being developed. For Sturm-Liouville type integro-differential operators, there exist some studies about inverse problems but there is very little study for Dirac type integro-differential operators. The inverse nodal problem for Dirac type integro-differential operators was first studied by [2]. In their study, it is shown that the coefficients of the differential part of the operator can be determined by using nodal points and nodal points also gives the partial information about integral part. In our study, we deal with an inverse nodal problem of reconstructing the Dirac type integro-differential operators with the spectral parameter in the boundary conditions. Firstly, i have obtained a new approach for asymptotic expressions of the integral equations of the solutions of such discussed problem. Secondly, more accurate estimates of eigenvalues and nodal points have been calculated with the help of these asymptotics. Lastly, i have proved that the operator can be reconstructed by nodal points.

Key Words: Dirac operator, integro-differential operators, inverse nodal problem

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Investigation of solutions of the Pade-II equation by MEFM

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ABSTRACT

In this study, singular soliton, topological, nontopological and rational solutions of the Pade-II equation, which is a type of nonlinear partial differential equations, are obtained. By selecting the appropriate parameters, solutions were obtained which provided the equations. Two and three dimensional graphics of the solutions were drawn. It appears that the resulting solution functions and graphics have similar features. Hyperbolic and trigonometric functions were also seen to have the same properties in their graphs since they are periodic functions at the same time. The obtained solutions were found by using the modified expansion method. All solutions using this method are controlled by the Mathematica software program which provides the Pade-II equation. Using the modified expansion function method, the nonlinear partial differential equation with travelling wave transformation takes the form of nonlinear ordinary differential equation. To find the solution of this differential equation obtained, the solution function containing the exponential function is used. However, the balancing procedure is used to determine the solution boundaries of this solution. In this way, the solution function model to be used for solving the equation is prepared. Nonlinear differential equations are arranged in the form of the necessary derivative of the equation is replaced by a system of written algebraic equations. In this system of equations, our aim is to find the coefficients found in the solution function. The coefficients are mathematically obtained by the help of the packet program and written in the solution function.

Key Words: The modified expansion function method (MEFM), The Pade-II equation, topological solution.

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Investigation of the Sturm-Liouville Problem with Eigenvalue Parameter in the Boundary Condition

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ABSTRACT

Sturm-Liouville theory is important for solving many mathematical physics problems. Boundary value problems investigated in relation to heat conduction and similar problems arising from further generalization of these problems are called Sturm-Liouville problems. Sturm-Liouville problems were first researched by Charles Sturm and Joseph Liouville in 1836. Sturm-Liouville theory applied to heat transfer problems is an effective method for researching many physical problems. In general, this theory is applied to examine the solutions of partial differential equations in the $u_{xx}+u_{yy}=0$, $u_{tt}=c^2u_{xx}$, $u_t=c^2u_{xx}$ form at different boundary and initial conditions. The first equation is the equation showing the temperature is not changed over time, that is, showing the determined temperature. The second equation is the wave equation for small vibrations in the homogeneous rod. Third equation is the heat flow equation in a homogeneous metal rod with heat isolation. Here, c is a constant number connected to the density, the heat intensity and the thermal conductivity of the material that is made of the rod. $u(x,t)$ is the temperature in the x point and at the time t.

The main purpose of the Sturm-Liouville theory is to research the solution satisfying the differential equation and the boundary conditions.

In this paper, we investigate the eigenvalues and the eigenfunctions of the Sturm-Liouville problem with eigenvalue parameter in the boundary condition. Here, the classical Sturm-Liouville theory and methods, the solution methods of the eigenvalue problem, the known methods from functional and real analysis, the integral equations theory in the differential equations are used.

Key Words: Sturm-Liouville boundary value problem, eigenvalue, eigenfunction.

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Korovkin type approximation via statistical equal convergence of the deferred Nörlund summability

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ABSTRACT

Srivastava et al. (RACSAM, DOI10.1007/s13398-017-0442-3) have introduced the generalized equi-statistical convergence, statistical pointwise and statistical uniform convergence with respect to the deferred Nörlund mean. The definition of equal convergence for real functions was introduced by Császár and Laczkovich and they improved their investigation on this convergence (Acta. Math. Acad. Sci. Hungar. 33 No:1-2, (1979), 51-70). Also, Das, Dutta and Pal introduced the ideas of *I* and *I**-equal with the help of ideals by extending the equal convergence (Mat. Vesnik, 66(2) (2014), 165-177). In this work, using these convergence methods, we define a new and different statistical convergence which named by statistical equal convergence with respect to the deferred Nörlund mean. Hence, we obtain an interesting convergence method which is stronger than statistical uniform convergence with respect to the deferred Nörlund mean with the help of statistical equal convergence and statistical convergence with respect to the deferred Nörlund mean. We prove Korovkin type approximation theorem via this new convergence method. So, we get stronger result than Korovkin type approximation theorem given via statistical uniform convergence with respect to the deferred Nörlund mean. Also, we study the rate of statistical equal convergence with respect to the deferred Nörlund mean by means of the modulus of continuity.

Key Words: Nörlund summability, Korovkin theorem, statistical equal convergence.

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Korovkin type theorem for double sequences via power series method in weighted spaces

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ABSTRACT

In approximation theory, the Korovkin theory has big importance. This theory is connected with the approximation to continuous functions by means of positive linear operators. The Korovkin-type approximation theorems have investigated by many mathematicians for a sequence of positive linear operators defined on different spaces by using various types of convergence. The weighted Korovkin type theorems, first proved by A. D. Gadjiev (Math. Zamet., 20 (1976) 781-786 (in Russian)). Then these theorems are studied by many authors for different convergence methods. In this paper, we obtain a Korovkin type approximation theorem for double sequences of real valued functions by using the power series method in weighted spaces. The power series method includes both Abel and Borel methods and is also member of the class of all continuous summability method. It is well known that the main purpose of using summability theory has always been to make a non-convergent sequence to converge. Hence, we get interesting result by applying this method. We also study the rate of convergence by using the weighted modulus of continuity and present an application satisfies our new Korovkin type approximation theorem for a double sequences of positive linear operators in weighted spaces but not satisfies classical one.

Key Words: Korovkin theorem, weighted space, double sequence, power series method.

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Laguerre Collocation Method for Solutions of Systems of Linear Differential equations

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ABSTRACT

In this study, a collocation method based on Laguerre polynomials is presented to solve systems of linear differential equations. The method contains the following steps. The Laguerre polynomials are written in the matrix form. So, the solution forms can be expressed in the matrix forms. The derivatives of the Laguerre polynomials are expressed in the matrix forms. A new matrix form is constructed from the solutions forms of the derivatives of the unknowns in system. The system of differential equations is written in matrix form. By using the constructed matrix forms, the collocation points and matrix operations, the system of linear differential equations is transformed into a system of linear algebraic equations. By writing values of the conditions in the matrix forms of solution forms, the matrix forms of conditions are obtained. A new system of algebraic equations is gained by previous linear algebraic system and the matrix forms of conditions. The solution of this linear algebraic system gives the coefficients of the solutions forms. So, the solution based on the Laguerre polynomials is found. Also, error estimation is made by using residual functions. In addition, the solutions are improved by the estimated error functions. The method is explained by solving numerical examples and it is shown that the presented method is effective by making comparisons with other methods.

Key Words: System of differential equations, Laguerre polynomials, Laguerre collocation method.

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Lazhar Type Inequalities For p-Convex Functions

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ABSTRACT

Mathematical inequalities are one of the most significant instruments in many branches of mathematics such as functional analysis, theory of differential and integral equations, interpolation theory, harmonic analysis, probability theory, etc. They have very useful application areas in mechanics, physics and other sciences. The first comprehensive study in inequalities was made in the first half of the 20th century and in the second half of the century inequalities attracted the attention of many researchers. Especially, in recent years, so many study have been published that inequalities may be considered as a different area of mathematics.

Convex functions are one of the most important topic of the inequalities. Many important inequalities have been established for the class of the convex functions such as Jensen inequality and Hermite-Hadamard inequality. Also in [1], Lazhar gave Lazhar inequality which is generalization of Popoviciu's inequality for convex functions.

This study concerned with p-convex functions which are generalization of convex and harmonic convex functions.In [4] L. Azócar et all. gave some inequalities for harmonic convex function. The aim of this study is to establish some new Lazhar and Jensen type inequalities for p-convex function which, in some special cases, reduces to convexity and harmonic convexity that are already in the literature

Key Words: p-convex function, Lazhar inequality, Jensen inequality

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Lyapunov Matrix Equations Solution with Iterative Decreasing Dimension Method

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ABSTRACT

System of first order linear differential equation and system of first order difference equation has considered. The systems' coefficient matrix is $A \in \boldsymbol{R}^{N\times N}$**.** The existence of a solution of continuous and discrete time Lyapunov matrix equations has been studied. Continuous - time Lyapunov matrix equation $A^T H + HA = -I$ that has a unique solution under $H = H^T > 0$ condition. Here I_N is identity $_{N\times N}$ matrix. There exists and unique positive definite $H = H^T$ matrix satisfying the discrete - time Lyapunov matrix equation $A^T HA - H = -I$. Both Lyapunov matrix equations are transformed into a matrix vector equation $Gh = z$. When Kronecker product of C and D matrices denoted by $C \otimes D$ and Kronecker sum by $C \oplus D$. For continuous – time Lyapunov matrix equation $G = A^T \oplus A^T$, for discrete – time Lyapunov matrix equation $G = A^T \otimes A^T - I_{N^2}$ as obtain. It is determined whether the solutions of the obtained new systems are positively defined. Lyapunov matrix equations have unique solutions if and only if G is non – singular. The equation $Gh = z$ may be solved by varied methods. The iterative decreasing dimension method (IDDM) [4, 6] has implemented for solving the generated matrix vector equation. This method has arranged for equation $Gh = z$ and prepared for computer aided computation. At last, based on the calculated solutions of the Lyapunov matrix equations, it was examined whether the systems are asymptotically stable. Computer computations have done with MAPLE procedures that run the constituted algorithms.

Key Words: Linear difference equation systems, linear differential equation systems, Lyapunov matrix equations, Hurwitz stability, Schur stability.

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Mathematical theory of entangled states of spontaneous parametric down conversion (SPDC)

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ABSTRACT

The nonlocal behaviour of the entangled quantum system has troubled us since 1935. We determined two arguments, “ghost interference” and “quantum correlation”. Our theory demonstrates the astonishing quantum correlation behaviour of an entangled photon pair. The state of the signal-idler photon pair of SPDC is a typical entangled Einstein-Podolsky-Rosen (EPR) state. SPDC is a nonlinear optical process from which a pair of signal-idler photon is generated when a pump laser beam is incident onto an optical nonlinear crystal (NLC) according to the phase maching conditions. In the case of type-II SPDC, the quantum correlated two-photon beams leaving the nonlinear crystal are orthogonally polarized. The process of type-II SPDC is formalized in analogy with the classical Hamiltonian. In the interaction picture, our Schrödinger’s equation solution provided us a signal-idler system, suggested by Einstein-Podolsky-Rosen. As a result, the two-photon entanglement can be calculated by the first order perturbation theory. Mathematical theory of two-photon entanglement is unique and bizarre state in quantum mechanics. Besides, quantum entanglement prompts some of the more philosophically oriented discussions concerning quantum information theory.

Key Words: Entanglement, quantum system tomography, EPR bizarre state.

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Measures Of Some Systems Using System Signature

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ABSTRACT

System signature is a tool for comparing coherent systems. Let X_i's be iid n-components lifetimes of a system with T lifetime. The signature of the system is $\mathbf{s} = (s_1, s_2, \dots, s_n)$ where s_i is equal to probability of event and $X_{i:n}$ is the ith order statistics of X_i's. Also, dynamic system signature is the truncated form of system signature when exactly i components of the system have failed at time t. A system of n-components that consisting of any components which does not affect system performance of the system and when a failed component is replaced by a working component the system keeps working is called to be a coherent system. Let X_i's be independent and identically distributed n-components' lifetimes of a coherent system with T lifetime where $i = 1,2, \dots, n$. While it is easy to calculate the system signature of systems with a small number of components, it is difficult to calculate the system signature of multidimensional coherent systems. In addition, since system signatures are calculated with the information of each component that constitutes the system, this makes it difficult to calculate the system signature of some systems. Due to these circumstances, the scope and limitations of this work are based on the examination of 3, 4 and 5 dimensional coherent systems. In this study, comparison of new better than used and uniformly new better than used properties of aging system with dynamic system signature.

Key Words: Order statistics, system signature, dynamic system signature.

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Multiple Small Solutions For p-Schrödinger- Kirchhoff Equations Via Genus Theory

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ABSTRACT

In this present we investigate the existence of multiple solutions for the nonhomogeneous fractional p-Schrödinger problem:

$$M\left(\iint_{\mathbb{R}^{2N}}\frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}}dxdy+\int_{\mathbb{R}^N}V(x)|u|^p dx\right)\left[(-\Delta)_p^s u+V(x)|u|^{p-2}u\right]=f(x,u)\text{ in }\mathbb{R}^N,$$

where $(-\Delta)_p^s$ is the fractional p-Laplacian operator which (up to normalisation factors) may be defined along any $\varphi\in C_0(\mathbb{R}^N)$ as

$$(-\Delta)_p^s\varphi(x)=2lim_{\varepsilon\to 0^+}\int_{\mathbb{R}^N\setminus B_\varepsilon(x)}\frac{|\varphi(x)-\varphi(y)|^{p-2}(\varphi(x)-\varphi(y))}{|x-y|^{N+ps}}dy$$

for $x\in\mathbb{R}^N$, where $B_\varepsilon(x):=\{y\in\mathbb{R}^N:|x-y|<\varepsilon\}$ (see [3,4]) and the references therein for further details on the fractional p-Laplacian, with $0<s<1,\ p\geq 2,\ N\geq 2,\ ps<N$ and $M:\mathbb{R}_0^+\to\mathbb{R}_0^+$ is a continuous and positive Kirchhoff function, the nonlinearity $f:\mathbb{R}^N\times\mathbb{R}\to\mathbb{R}$ is a Carathéodory function obeys following conditions;

(M_1) (Polynomial growth condition): $M:\mathbb{R}_0^+\to\mathbb{R}_0^+$ is a nonnegative continuous Kirchhoff function, $c_*|t|^{\beta-1}\leq M(t)\leq c^*|t|^{\alpha-1}$ for all $t>0$ with $1\leq\beta\leq\alpha$ and $c_*,\ c^*$ real numbers such that $0\leq c_*\leq c^*$;

(V_1): $V\in C(\mathbb{R}^N), inf_{x\in\mathbb{R}^N}V(x)>0$ and there are constants $r>0$ and $\sigma>\frac{N}{p-1},\ p\geq 2$ such that for any $b>0,\ lim_{|y|\to+\infty}\mu(\{x\in B_r(y):\frac{V(x)}{|x|^\sigma}\leq b\})=0$, where $B_r(y)=\{x\in\mathbb{R}^N:|x-y|<r\}$ and $\mu(.)$ denotes the Lebesgue measure;

(f_1): $f: \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a continuous function such that $C_*|t|^{m-1} \leq |f(x,t)| \leq C^*|t|^{q-1}$ for all $t \geq 0$ and $x \in \mathbb{R}^N$, where C_*, C^* are positive constants and $2 \leq p < m < q < p\beta < p_s^* = Np/(N-sp)$;

(f_2): f is an odd function according to t, that is, $f(x,t) = -f(x,-t)$ for all $t \in \mathbb{R}$, $x \in \mathbb{R}^N$.

We first establish Batsch-Wang type compact embedding theorem for the fractional Sobolev spaces (see [1]). Then multiplicity results are obtained by using the variational method and Krasnoselskii's genus theory (see [2]).

Key Words: p-Laplace operator; Schrödinger equation; variational method; Krasnoselskii's genus theory.

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Numerical Analysis of Backward-Euler Method for Simplified MagnetoHydroDynamics with Linear Time Relaxation

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ABSTRACT

In this study the solutions of the Simplified Magnetohydrodynamics (SMHD) equations by using finite element methods are examined. Magnetohydrodynamics (MHD) equations are obtained from the combination of fluid dynamics, Navier-Stokes equations and Maxwell's equations of electromagnetism. MHD equations are frequently observed in engineering area such as plasma confinement, controlling thermonuclear fusion, liquid-metal cooling of nuclear reactors, electromagnetic casting of metals and also sea water propulsion. SMHD equations which are examined, is a particular form of Magnetohydrodynamics equations. The MHD equations are defined as follows;

$$N^{-1}(u_t + u.\nabla u) = f + M^{-2}\Delta u - \nabla p + j \times B, \qquad \nabla .u = 0,$$

$$-\nabla \phi + u \times B = j, \qquad \nabla . j = 0,$$

$$\nabla \times B = R_m j, \qquad \nabla .B = 0.$$

If $R_m \ll 1$ is selected on purpose in the equation given above, MHD equations are reduced to SMHD equations. So that R_m is a magnetic Reynolds number [1]. The term $\kappa(u - \bar{u})$ is added to Navier-Stokes equations by Layton and Neda (2007) to reduce power of exponential waving and the following equation systems are obtained.

$$\begin{cases} N^{-1}\left(u_t + u \cdot \nabla u\right) - M^{-2}\Delta u + \nabla p - B \times \nabla \phi - (u \times B) \times B + \kappa(u - \bar{u}) = f, \\ \nabla \cdot u = 0, \\ -\Delta \phi + \nabla \cdot (u \times B) = 0. \end{cases}$$

In these equations $\bar{u}$ is a differential filter of u, and it is given as [2]

$$\begin{cases} u_t = \dfrac{(u-\overline{u})}{\delta}, t > 0 \\ \overline{u}(x,0) = u(x,0). \end{cases}$$

First the linear differential filter $\kappa(u-\overline{u})$ is added to SMHD equations and then the SMHD Linear Time Relaxation Model (SMHDLTRM) is obtained. For the numerical solution of the model in the time dimension, an algorithm that is depending on finite element methods is provided by using discretization of Backward-Euler (BE). At first it is proved that the method is unconditionally stable, then the error analysis of the algorithm is examined and the convergence is investigated. The results obtained from the sample problems by using the algorithm of SMHDTRM are compared with BE and Crank Nicolson (CN) methods, and it is showed that the theoretical results are coincide with samples. Moreover, it is observed that for a problem whose exact solution is not known, the algorithm of SMHDTRM equilibrium errors are small. All these calculations and computations are performed by using FreeFem++ programme [3].

Key Words: Magnetohydrodynamics, Backward-Euler, Finite Element Method, Time Relaxation Model

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Numerical Solution of Volterra and Fredholm İntegral Equations by Homotopy Perturbation Method

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ABSTRACT

In this study, an application of the homotopy perturbation method for Volterra and Fredholm integral equations is presented. The proposed method is based on the homotopy perturbation method, which consists in constructing the series whose sum is the solution of the problem considered. The homotopy perturbation method couples a homotopy technique of topology and a perturbation technique. A homotopy with an embedding parameter $p \in [0, 1]$ is constructed, and the impeding parameter p is considered a small parameter. The method was derived and illustrated in , and several differential equations were examined. The coupling of the perturbation method and the homotopy method has eliminated the limitations of the traditional perturbation technique. In what follows we illustrate the homotopy perturbation method to handle Fredholm integral equations of the second kind.

$$u(x) = f(x) + \int_a^b K(x,t)u(t)dt \quad (1)$$

We now define the operator

$$F(u) = u(x) - f(x)$$

$$L(u) = u(x) - f(x) - \int_a^b K(x,t)u(t)dt = 0, \quad (2)$$

Next we define the homotopy $H(u, p)$, $p \in [0, 1]$ by

$$H(u,0) = F(u), \qquad H(u,1) = L(u) \quad (3)$$

where $F(u)$ is a functional operator. We construct a convex homotopy of the form

$$H(u,p) = (1-p)F(u) + pL(u) = 0 \quad (4)$$

This homotopy satisfies (3) for $p = 0$ and $p = 1$ respectively. The embedding parameter p monotonically increases from 0 to 1 as the trivial problem $F(u) = 0$ continuously deformed to the original problem $L(u) = 0$. The Homotopy Perturbation Method admits the use of the expansion

$$u = \sum_{n=0}^{\infty} p^n u_n \qquad (5)$$

And consequently ,

$$u = \lim_{p \to 1} \sum_{n=0}^{\infty} p^n u_n \qquad (6)$$

The series (6) converges to the exact solution if such a solution exists. Substituting (5) into (4), using $F(x) = u(x) - f(x)$, and equating the terms with like powers of the embedding parameter p we obtain the recurrence relation

$$p^0 \;:\; u_0(x) = f(x),$$

$$p^{n+1}: \;\; u_{n+1}(x) = \int_a^b K(x,t) u_n(t) dt \,, \;\; n \gg 0.$$

Furthermore, this integral equations are solved by Adomian Decomposition Method, Variational Iteration Method and Successive Approximations Method. The effectiveness and practicality of the Homotopy Perturbation Method is evaluated according to analytical and numerical results.

Key Words: Integral equations, Homotopy Perturbation Method.

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Numerical Solutions Of Nonlinear Pantograph Type Delay Differential Equations

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ABSTRACT

In this paper Adomian Decomposition (ADM), Differential Transformation (DTM) and Daftardar-Gejji and Jafari (DGJ) methods are introduced and applied to three nonlinear pantograph type delay differential IVP s. ADM, DTM and DGJ are iterative methods that consider solutions in series forms and have been shown to meet the exact solutions in a radius of convergence. In most of the cases it is very difficult to solve nonlinear differential equations with analytical methods. In general, perturbation, linearization, Newton`s or Picard`s methods kind of methods are implemented to rescue from the nonlinearity of the problems but these simplifications inevitably cause the solutions of simplified problems not to be in accordance with the solutions of the original nonlinear problems. Delay differential equations (DDE) have many applications in science, engineering, biology etc for the phenomena where the rate of change of the unknown function depends not only in the current state but also in the history of the system. In DDE delay terms may be constant or variable. In this paper we consider pantograph type DDE where the delays are proportional to the current time of the system. DDE have different special properties compared with Ordinary Differential Equations (ODE) and are very difficult to solve with analytical methods. ODE techniques are not applicable to solve DDE. Therfore numerical methods need to be used to solve DDE. Here we solved three nonlinear pantograph type DDE with the ADM, DTM and DGJ with same iteration steps and compared efficiencies of the methods by comparing their results with the exact solutions. Numerical computations are performed using Mathematica.

Keywords: Adomian decomposition, Differential transformation, Daftardar-Gejji and Jafari, Delay differential

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On Fuzzy Contractibility

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ABSTRACT

The concept of a fuzzy set was discovered by Zadeh [7] and one of its earliest branches, the theory of fuzzy topology, was developed by Chang [3] and others. Recently, Zheng [4] and Wuyts [8] introduced the concept of a fuzzy path. Using this concept, Salleh and Md Tap [1] constructed the fundamental group of a fuzzy topological space.

Let $f, g\colon (X,\tau) \to (Y,\sigma)$ be two fuzzy continuous mappings. If there exists a fuzzy continuous mapping

$$F\colon (X,\tau) \times (J,\varepsilon_J) \to (Y,\sigma)$$

such that $F(x_\lambda, 0) = f(x_\lambda)$ and $F(x_\lambda, 1) = g(x_\lambda)$ for every fuzzy point x_λ in (X,τ), then we say f is fuzzy homotopic to g and we write $f \simeq g$. The mapping F is called a fuzzy homotopy between f and g.

Let $1_X\colon (X,\tau) \to (X,\tau)$ be an identity mapping. If 1_X is fuzzy homotopic to a constant then, (X,τ) is called a fuzzy contractible space.

Let $f\colon (X,\tau) \to (Y,\sigma)$ be a fuzzy continuous function. If there is a fuzzy continuous function f' which satisfies the following conditions:

(i) $ff' \simeq 1_Y$

(ii) $f'f \simeq 1_X$

then, f is called a fuzzy homotopy equivalence. Fuzzy topological spaces are called fuzzy homotopic equivalent and denoted by $X \simeq Y$.

In this talk, we will give some characterizations of fuzzy contractible spaces and we will show that X is a fuzzy contractible space if and only if X is fuzzy homotopic equivalent with a fuzzy single-point space.

Key Words: fuzzy path,fuzzy homotopy, fundamental group of a fuzzy topological space,fuzzy contractibility.

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On Fuzzy Sub-H-Groups

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ABSTRACT

Zadeh introduced the concepts of fuzzy sets and fuzzy set operations in [5]. In 1968, Chang developed a theory of fuzzy topological spaces[1]. After that, basic concepts from homotopy theory were discussed in fuzzy settings. In this direction, Zheng [2] introduced the concept of fuzzy paths. Then the fundamental group of a fuzzy topological space was developed. Also in [4], fuzzy homotopy concepts in fuzzy topological spaces were conceived. Later many topics of algebraic topology were extended to fuzzy topology. For example, the concept of H-spaces and H-groups have been introduced by Demiralp and Guner in [3].

An H space consists of a pointed topological space X together with a continuous multiplication $\mu: X \times X \to X$ for which the constant map $c: X \to X$ is a homotopy identity, i.e. $\mu o(1_x, c) \simeq 1_x$ and $\mu o(c, 1_x) \simeq 1_x$. An H-group is an H-space whose multiplication is homotopy associative and has a homotopy inverse.

In [6] Park defined the concept of sub-H-group; A pointed subspace Y of an H-group X with the same base point x_0 is called a sub-H-group if X' is itself an H-group such that the inclusion map is an H-homomorphism.

Pakdaman[7] defined homotopically belonging; For each $y \in X$ we say that y homotopically belongs to Y which is sub-H-group of X if and only if there exists a path $\alpha: I \to X$ such that $\alpha(0) = y$ and $\alpha(1) \in Y$. (denoted by $y \,\widetilde{\in}\, Y$)

In this paper we introduce fuzzy H-isomorphism and give some examples. Then we defined fuzzy sub-H-group. Also we defined fuzzy right coset and fuzzy left coset of a fuzzy sub-H-group via the concept of fuzzy homotopically belonging.

Key Words: Fuzzy H-Space, Fuzzy sub-H-group, Fuzzy H-homomorphism, Fuzzy H-isomorphism.

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On Hyers-Ulam-Rassias Stability For First-Order Linear Differential Equations And Bernoulli Differential Equations

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ABSTRACT

In this paper, we present the stability in the sense of Hyers-Ulam and Hyers-Ulam-Rassias for the first-order linear differential equation which including Bernoulli's differential equations with the given initial condition.

Hyers-Ulam stability is one of the main topics in the theory of functional equations. Firstly, in 1940, Dr. Ulam posed a problem concerning the stability of functional equations to give conditions in order for a linear mapping near an approximately linear mapping to exist in the talk at the Mathematics Club of the University of Wisconsin.

This problem was first answered in the following way by Hyers one year later:

Let E and E′ be Banach spaces and let f(x) be a δ-linear transformation of E into E′. Then the limit for $n\to\infty$

$$\ell(x)=\lim((f(2^n x))/(2^n))$$

exists for each x in E, $\ell(x)$ is a linear transformation and the inequality

$$\|f(x)-\ell(x)\|\leq\delta$$

is true for all x in E. Moreover $\ell(x)$ is only linear transformation satisfying this inequality.

The above result of Hyers was extended by Aoki, Bourgin and Rassias. The first result on Hyers-Ulam stability of differential equations was given by Obloza. Thereafter, in 1998, Alsina and Ger investigated the Hyers-Ulam stability for the linear differential equation $y'(t)=y(t)$.

They proved that if a differentiable function $y: I\to\mathbb{R}$ satisfies

$$|y'(t)-y(t)|\leq\varepsilon$$

for all $t\in I$, then there exists a differentiable function $f: I\to\mathbb{R}$ such that $f'(t)=f(t)$ and

$$|y(t)-f(t)|\leq 3\varepsilon$$

for all $t\in I$. Here, I is an open interval and $\varepsilon>0$.

Furthermore, Miura et al., Miura, Takahasi et al. and Miura et al. generalized the above result of Alsina and Ger. Indeed, they proved the Hyers-Ulam stability of the differential equation

$$y'(t)=\lambda y(t)$$

where λ is a complex number.

In 2017, Onitsuka and Shoji studied the Hyers-Ulam stability of the first order linear differential equation

$$y'(t)-ay(t)=0$$

where a is a nonzero reel number.

In 2004, Jung investigated the Hyers-Ulam stability of the differential equation $\phi(t)y'(t)=y(t)$. More later, the result of the Hyers-Ulam stability for first-order linear differential equations has been generalized by Miura et al., Takahasi et al. and Jung. They studied the following nonhomogeneous linear differential equation of first-order:

$$y'(t)+p(t)y(t)+q(t)=0$$

Our aim in this study to investigate stability in the sense of Hyers-Ulam-Rassias for Bernoulli's differential equation:

$$y'(t)+p(t)y(t)=G(t,y)=q(t)y^{n}\ ,\ n\neq 0,1$$

Key Words: Hyers-Ulam stability; Hyers-Ulam-Rassias stability.

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On Linear Combination of a Quadratic or a Cubic Matrix and an Arbitrary Matrix

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ABSTRACT

Quadratic matrices are a wide class of matrices containing idempotent, involutive and several other types of matrices. It may be find deeper properties of quadratic matrix in [2]. Idempotent, tripotent, involutive, and quadratic matrices, special cases of cubic matrices, have been extensively studied in the literature (for example [1, 3-7]).

Let us consider a linear combination of the form

$$\mathbf{K} = a\mathbf{A} + b\mathbf{B} \tag{1}$$

where a, b are complex numbers and $\mathbf{A}$, $\mathbf{B}$ are $n \times n$ complex matrices.

Recently, under some conditions, it has been studied some problems of characterizing all situations where a linear combinations of the form (1) is a special type of matrix when $\mathbf{A}$ and $\mathbf{B}$ are special types of matrices (see, for example, [3-7]). Liu et al. characterize the involutiveness of the form (1) when $\mathbf{A}$ is a quadratic or tripotent matrix and $\mathbf{B}$ is an arbitrary matrix [4].

Now, let $\mathbf{A}$ is a quadratic or a cubic matrix and $\mathbf{B}$ is an arbitrary matrix. In this study, we establish the necessary and sufficient conditions for $\mathbf{K} = a\mathbf{A} + b\mathbf{B}$ to be idempotent matrix, where $\mathbf{A}$ is a quadratic or a cubic matrix and $\mathbf{B}$ is an arbitrary matrix with some certain conditions.

Quadratic forms with idempotent matrices are used extensively in statistical theory. So it is remarkable to stress and spread these kinds of results. Eigenvalues are used in the theory of diagonalization, difference equations, Fibonacci numbers and Marcow processes (see, e.g, [8]).

Key Words: Quadratic matrix, cubic matrix, linear combination, diagonalization, direct sum of matrices; eigenvalues.

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On Minkowski And Inverse Minkowski Inequalities Via Generalized Fractional Integrals

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ABSTRACT

Recently, a number of scientist in the field of mathematics have introduced different results about the fractional derivatives and integrals. Several fractional derivatives and integrals have been introduced by a number of scientist such as Riemann-Liouville fractional derivative, Riemann-Liouville fractional integral operator, Hadamard integral operator, Saigo fractional integral operator and some other, and applied them to some well-know inequalities with applications [1]-[22]. In this paper the authors will provide the some Minkowski's inequalities and the reverse Minkowski's inequalities by means of the generalized fractional integral operators whic provide some fractional integral. The schema of the this paper consist of three sections including introduction. The remaining part of the paper proceeds as follows: In Section 2, we obtain reverse Minkowski's inequalities by means of the generalized fractional integral operators which provide some fractional integral. In section 3, we provide the main results involving the Minkowski's inequality with the help of the generalized fractional integral operators which provide some fractional integral.

Yildirim et al. defined the following generalization of the fractional integral;

Defination: Let $f \in L_1[a,b]$ and h(x) be an increasing and positive monotone function on (a,b] and also derivative h′(x) is continuous on (a,b).The left and right sided Generalized Riemann-Liouville type fractional integrals of a function f with respect to h(x) on [a,b] are defined by

$$J_{a^+,h}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)}{\left(h(x)-h(t)\right)^{1-\alpha}} h'(t) dt$$

and

$$J_{b^-,h}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b \frac{f(t)}{\left(h(t)-h(x)\right)^{\frac{\alpha}{k}-n+11-\alpha}} h'(t)\,dt$$

respectively.

Furthermore, we noticed that these generalized fractional integrals can obtain other types of fractional integral operators with different choices of h such as Riemann-Liouville fractional integral, k-Riemann-Liouville fractional integral, Katugampola fractional integral, conformable fractional integral, etc., with some special choices.

The aim of this article is to obtain new integral inequalities using generalized Riemann-Liouville h-fractional integrals.

Key Words: Integral Inequalities, Special Functions, Fractional Calculus.

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On Stability and Bifurcation Analysis in a Discrete Time Predator-Prey System

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ABSTRACT

In recent years, a remarkable number of papers have been on mathematical system in biology. The predator-prey systems which can show relationships between two predator-prey species have an important role in biological field. Mathematical models on predator-prey system have created a significant during the last few decades.

On the other hand, bifurcation theory is a research field that anlyzes the changes of dynamical models with respect to a control parameter. The change of qualitative structure of dynamical systems associated with varying parameter values is known as the bifurcation. There are many different works in both difference and differential equations. Furthermore, many scholars have taken attention to the discrete time population models by forward Euler method.

This work deals with a discrete predator-prey system obtained by the forward Euler method. The existence and local asymptotic stability conditions of only fixed point of the dynamic system has been investigated. Then it has been shown that the system undergoes Neimark-Sacker bifurcation by using center manifold and bifurcation theory. Also, we present the direction of bifurcation. Moreover, bifurcation diagrams and phase portraits of the considered system are given. Numerical simulations are performed to the support of our analytical findings. Because of consistency with the biological facts, the parameter values are taken from literature.

Key Words: Discrete predator-prey system, Local Stability Analysis, Neimark-Sacker bifurcation.

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On Surrogate Dual Search Method for Minimum-Cost Flow Problems

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ABSTRACT

In this paper, we study on surrogate dual formulations which generate relaxations by assembling multiple constraints into a single surrogate constraint. Similar to the Lagrangian dual search methods for integer programming, the conventional surrogate dual method utilizes an auxiliary linear programming problem for updating the multiplier vector. The technique enlarges the feasible region of the original (primal) problem and provides a lower bound for the optimal objective value. This bound is tighter than the Lagrangian lower bound. In case there exists a duality gap, the conventional surrogate dual search method fails to find the optimal solutions of the primal problem. In order to eliminate this issue, nonlinear *p*-norm surrogate constraint methods can be used. To illustrate how we choose the initial multiplier vector or the parameter *p*, we argue on minimum-cost flow problems, in which we find the feasible flow from the source nodes to the sink nodes with minimum cost. Some integer programming problems, such as transportation problems, transshipment problems, assignment problems, shortest path problems (with or without time windows), and maximal flow problems can be seen those type of problems. Furthermore, we consider arrangements to solve those network problems which cannot be solved with the conventional surrogate dual method.

Key Words: Surrogate dual search, Lagrangian dual search, Duality gap, Integer programming, Minimum-cost flow problems.

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On The K_A-Continuity of Real Functions

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ABSTRACT

In 1946 Robbins(Amer. Math. Monthly, 53 (1946), 470-471) proposed a problem. Solution by R. C. Buck(Amer. Math. Monthly 55 (1948) 36) was published in 1948 (the problem was also solved by five others). Since then, different type of continuities defined and studied by authors. Antoni and Salat(Acta Math. Univ. Comenian. 39 (1980), 159-164) defined the concept of A-continuity for real functions based on A-summability. After that the notion of F-continuity based on almost convergence (F-convergence) was introduced in the paper (Comm. Fac. Sci. Univ. Ankara, Ser. A, 32 (1983), 25-30) by Öztürk. This method studied by Borsik and Salat(Tatra Mountains Math. Publ., 2 (1993), 37-42) and they remark that almost convergence and A-summability are not equivalent. Also some authors studied different concepts of continuity. The aim of the present paper is to define K_a-continuity is associated to the number sequence a=(a_n) and based on K_a-convergence which is an interesting convergence method. Also, we give the relations between concepts of continuity. It is now naturel to ask: Is the K_a-continuity a special case of A-continuity or do K_a-continuity and F-continuity contain each other? In general the answer is no. Simple examples show that these continuity methods do not contain each other. Namely, these methods are overlap.

Key Words: K_a-continuity, F- continuity, A- continuity.

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On The Solutions Of The Singular Differential Equations

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ABSTRACT

The spectral theory of differential operators has an important place in applied sciences. Especially in quantum theory there are many applications of singular Shrödinger operator. For example, the problem of finding the hydrogen atom and corresponding wave functions and these levels is reduced to the problem of learning the behaviour of the eigenvalues of the Shrödinger operator with Columb potential and the like and of the corresponding eigenfunctions. Therefore, some of the characteristics of the Shrödinger operator with a special type of potential have been examined in our study. Differential operators with singularity and discontinuity conditions at the interval have been studied by Amirov and Yurko (2001). In this study, for Sturm-Liouville operator with non-self adjoint Bessel potential with singularities at (x=0), we have investigated the case where the solution of the end point of the finite interval has discontinuity. The spectral properties of the given operator and location of inverse problem with respect to these spectral properties and the uniqueness theorem for the solution have been proven.

We will investigates equation of Sturm-Liouville

$$-y'' + q(x)y(x) = \lambda^2 y(x) \tag{1}$$

on the finite interval $(0,\pi)$ with real potential q(x), which have nonintegrable singularity at the x=0, satisfying to the following condition

$$\int_0^\pi t|q(t)|dt < +\infty \tag{2}$$

and to separate boundary conditions of the form

$$y(0) = 0 = y(\pi) \tag{3}$$

By virtue of singularity of potential, generally, there doesn't exist finite values for the derivative of solution of equation (1) at the point x=0 of interval. All considered problem self-adjoint, it eigenvalues are real. In this study, properties of solutions, properties of spectral characteristics for Sturm-Liouville differential operators with nonintegrable singularity.

Key Words: Sturm-Liouville, Singular potential,differential equation.

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On The Stability of $y'(x) = f(x, y(x))$ in The Sense of Ulam

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ABSTRACT

The study of data dependence in the theory of differential equations grows by means of different concepts such as monotonity, continuity and differentiability of solutions with respect to parameters, asymptotic behavior and G-convergences of solutions, Liapunov stability and structuralstability of solutions; analiticity and regularity of solutions, etc..

In this paper we study the data dependence of differential equations on a relatively new concept, called Ulam Stability, which has been of an increasing interest in the last decades. On a talk given at Wisconsin University in 1940, S. M. Ulam posed the following problem: "Under what conditions does there exists an homomorphism near an approximately homomorphism of a complete metric group?" More precisely: Given a metric group (G, . , d), a number $\varepsilon > 0$ and a mapping $f : G \to G$ satisfying the inequality $d(f(xy), f(x)f(y)) < \varepsilon$ for all $x, y \in G$, does there exist a homomorphism g of G and a constant K, depending only on G, such that $d(f(x), g(x)) < K\varepsilon$ for all $x \in G$? One year later, Hyers [7] gave an answer to this problem for linear functional equations on Banach spaces: Let E_1, E_2 be real Banach spaces and $\varepsilon > 0$. Then, for each mapping f: $E_1 \to E_2$satisfying $\|f(x+y) - f(x) - f(y)\| < \varepsilon$ " for all $x, y \in E_1$ there exists a unique additive mapping g: $E_1 \to E_2$ such that $\|f(x) - g(x)\| < \varepsilon$ holds for all $x \in E_1$. After Hyers' answer, a new concept of stability for functional equations
established, called today Hyers-Ulam stability, and many papers devoted to this topic. In 1978, Rassias [6] provided a remarkable generalization, which known as Hyers-Ulam-Rassias stability today, by considering the constant as a variable in Ulam's problem.

Stability problem of differential equations in the sense of Hyers-Ulam was initiated by the papers of Obloza [5]. Later Alsina and Ger [7] proved that, with assuming I is an open interval of reals, every differentiable mapping $y : I \to \mathbb{R}$ satisfying $|y'(x) - y(x)| \leq \varepsilon$ for all $x \in I$ and for a given $\varepsilon > 0$, there exists a solution y_0 of the differential equation $y_0'(x) = y_0(x)$ such that $|y(x) - y_0$ (x)| $\leq 3\varepsilon$ for all $x \in I$. This result was later extended by Takahasi, Miura and Miyajima [9] to the equation $y'(x) = \lambda y(x)$ in Banach spaces, and [10] to higher order linear differential equations with constant coefficients. Recently Jung [8] proved Hyers-Ulam stability as well as Hyers-Ulam-Rassias stability of the equation $y'(x) = f(x, y(x))$ which extends the above mentioned results to nonlinear case. Later Bojor, modified Jung's [8] technique for the linear equation $y'(x) + f(x)y(x) = g(x)$ and proved a stability result with some different assumptions.

In this paper, we will extend and improve these result by proving the stability results for differential equations with less assumptions.

Key Words: Differential equations, Hyers-Ulam satability.

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Ostrowski Type Inequalities via New Fractional Conformable Integrals

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ABSTRACT

The classical Ostrowski integral inequality was proved by A. Ostrowski in 1938. It gives an upper bound for the approximation of the integral average $\frac{1}{b-a}\int_a^b f(t)dt$ by the value $f(t)$ at point $t \in [a,b]$. It is defined as follows: Let $f: I \to \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, be a mapping differentiable in the interior I° of I and let $a,b \in I^\circ$ with $a<b$. If $|f'(x)| \le M$ for all $x \in [a,b]$, then the following inequality holds:

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)dt \right| \le M(b-a)\left[\frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b-a)^2}\right]$$

for all $x \in [a,b]$. The constant $1/4$ is the best possible in sense that it cannot be replaced by a smaller one. In this present study, firstly, some necessary definitions and some results related to Riemann-Liouville fractional and new fractional conformable integral operators defined by Jarad et al. [1] are given. As a second, a new identity has been proved. By using this identity and some well-known inequalities such that triangle inequality, Hölder inequality and power mean inequality, several new Ostrowski type inequalities has obtained involving fractional conformable integral operators. Also, some new inequalities has established for AG-convex functions via fractional conformable integrals in this study. Relevant connections of the results presented here with those earlier ones are also pointed out.

Key Words: Gamma function, Beta function, incomplete gama function, AA-convex function, AG-convex functions, Ostrowski type inequalities, Riemann-Liouville fractional integrals, fractional conformable integral operators

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Parameter-Dependent Sturm-Liouville Problems on Time Scales

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ABSTRACT

The time scale theory was introduced by Hilger in order to unify continuous and discrete analysis [6]. Later on, this approach has received a lot of attention and has applied quickly to various areas in mathematics. It can be referred to [3] and [8] for the basic notation and terminology of the time scale theory.

The Sturm-Liouville theory on time scales was studied first by Erbe and Hilger [4] in 1993. Important results on the properties of eigenvalues and eigenfunctions of a Sturm–Liouville problem on time scales were given in various publications (see [1, 2, 5, 7] and the references therein).

We consider the more general second order dynamic equation and study on the following boundary value problem:

$$-y^{\Delta\Delta}(t) + q(t,\lambda)y\big(\sigma(t)\big) = 0,\ t \in \mathbb{T}^{k^2} \tag{1}$$

$$a(\lambda)y(\alpha) - b(\lambda)y^{\Delta}(\alpha) = c(\lambda)y(\beta) - d(\lambda)y^{\Delta}(\beta) = 0 \tag{2}$$

where $\mathbb{T}$ is a bounded time scale; $q(t,\lambda) = \sum_{j=0}^{r} q_j(t)\lambda^j$, $q_j(t)$ are real valued continuous functions on $\mathbb{T}$; λ is the complex parameter; $\alpha = inf\mathbb{T}$, $\beta = \rho(sup\mathbb{T})$; $a(\lambda)$, $b(\lambda)$, $c(\lambda)$ and $d(\lambda)$ are polynomials with real coefficients.

We prove that if $q(t,\lambda)$, $a(\lambda)$, $b(\lambda)$, $c(\lambda)$ and $d(\lambda)$ satisfy some special conditions, then all eigenvalues of (1)-(2) are real numbers and two different eigenfunctions are orthogonal. Additionally, we obtain a formulation of the number of eigenvalues on a discrete time scale.

Key Words: Time scale, Sturm-Liouville dynamic equation, parameter-dependent boundary conditions.

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Persistence and Stability of Synchronization in a System with a Diffusive-Time-Lag Coupling

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ABSTRACT

We study synchronization in the framework of invariant manifold theory for systems with a time lag. Normal hyperbolicity and its persistence in infinite dimensional dynamical systems in Banach spaces is applied to give general results on synchronization and its stability.

Key Words: Normal hyperbolicity; Synchronization; Robustness; Lyapunov numbers

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Picone Identity and Some Applications

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ABSTRACT

In the qualitative theory of ordinary differential equations (ODEs), celebrated Sturm–Picone theorem plays a crucial role. In 1836, the first important comparison theorem was established by C.Sturm [1], which deals with a pair of linear ODEs

$$l(u)=(r(t)u(t\,)')'+q_1(t)u(t)=0 \qquad (1)$$

$$L(v)=(R(t)v(t)')'+q_2\,(t)v(t)=0 \qquad (2)$$

on a bounded interval $j \subset R$, where r, q_1, q_2, R are real valued and continuous functions defined in $j \subset R$. Let us consider two linear, second order and ODE of under assumption classic Sturm comparison theorem $r(t) > R(t)$ and $q_2(t) > q_1(t),\ t \in J$. Existing a nontrivial $u(t)$ solution of equation (1) in classic Sturm Theorem which J two zeros t_1 and t_2 $(t_1 < t_2)$ guarantees being a zero on an interval (t_1, t_2) of $v(t)$ linearly independent and non-trivial equation (2).

M. Picone [2] enhanced the original demonstration of the theorem in 1909. This proof is based on

$$\frac{d}{dt}\{(uru' - uRv')\} = (r-R)(u')^2 + (q_2 - q_2)(u)^2$$

$$+R\left(u' - \tfrac{u}{v}u'\right)' + \tfrac{u}{v}\{ul[u] - vL[v]\} \qquad (3)$$

knows as picone identity. This identity is valid, if u, v, ru', Rv' functions are differentiable on the J and $v(t) \neq 0$ for every $t \in J$.

Classical proof of Sturm-picone's theorem is seen in the variable lemma of the Leighton. Also Sturm Picone theorem can be generalized with various ways. The theorem is extended for system by S. Ahmad, for non-self adjoint differential equation by E. Müller-Pfeiffer, for degenerate elliptic equations by W. Allegretto [3], for time skew linear equations by C. Zhang and S. Sun. Jaros and Kusano [4] showed that semi-linear differential operators in the forms

$$l[u] = (r(t)|u'|^{a-1}v')' + q_1(t)|u|^{a-1}u$$
$$L_a[v] = (R(t)|u'|^{a-1}v')' + q_2(t)|v|^{a-1}v$$

in conclusion many studies have been carried out on Sturm picone's theorem which is very important for oscillation theory, and these studies are continuing.

There is also a good amount of interest in the qualitative theory of PDEs to determine whether the given equation is oscillatory or not. In this direction, Sturm–Picone theorem plays an important role.

The recently established Picone identity for the so-called *p*-Laplacian

$$\Delta_{p(t)}[u] \coloneqq div(\|\nabla u\|^{p(t)-2}\, \nabla u), \qquad p > 1$$

turned out to be a very useful tool in the extension of various important properties of solutions of equations associated with the classical Laplacian (which correspond to *p* = 1 in (1)) to equations associated with *p*-Laplacian $\Delta_{p(t)}$. In this work we consider the two terms half-linear partial differential equation with p-Laplacian in the form

$$div(\|\nabla u\|^{p(t)-2}\, \nabla u) + c(t)\Phi(t) = 0,$$

where $\Phi(s) = |u|^{p-2}\, u, p > 1$ and by using the Picone identity for PDEs we derive some useful comparision results about oscillation theory.

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School Success Ranking in Multi Criteria Decision Making with Intuitionistic Fuzzy Logic

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ABSTRACT

In this paper; success ranking of schools has done by multi criteria method. Also the most successful school has been determined among these ranked schools. The aim of this paper is to proposed an application multi criteria decision making in intuitionistic fuzzy sets. For this paper have been benefitted from similarity measure for intuitionistic fuzzy sets in multi criteria decision making problem. The advantage of this method; options have been compared to the positive-ideal solution and negative-ideal solution. The best considered option should be as close as possible to the positive-ideal solution and as far as possible to the negative-ideal solution. Because; the best option taking into account only positive-ideal solution can be misleading. Many institutions make decisions based on a single criterion in the selection of staff. But a single criterion may not always give accurate results. This application could be used in situations that are not dependent on a single criterion. This method is suitable in order to achieve more sensible results. Applications could be made in different areas with this method. In this paper; the options are schools. Criteria that determine the success of school are lessons. The criteria in this study have been determined as the basic lessons in school. School base points have been determined as criteria. Each criterion represents a lesson.

Key Words: intuitionistic fuzzy sets, distance measure, decision making, multi criteria.

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Soft Encryption And Aes

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ABSTRACT

In 1999, the flexible set theory introduced by Molodtsov appears to be an effective mathematical tool to deal with uncertainty [1]. This theory was applied to many areas of uncertainty such as information systems, decision making problems, optimization theory, algebraic structures and mathematical analysis [2,3]. Çağman and Enginoğlu [8] defined and studied soft matrix. Atagün and Sezgin [6] studied on soft set operations with many corresponding examples as well. Sezgin, Atagün and Aygün [7] studied soft near-rings and idealistic soft near-rings. In [4], the same authors introduced two new operations on soft sets, called inverse production and characteristic production depending on the relation forms of soft sets and obtained two isomorphic abelian groups called "the inverse group of soft sets" and "the characteristic group of soft sets". In this study, we redefine the operations inverse and characteristic products on the set of soft matrices and we define soft encryption. On October 2, 2000, after a long and complex evaluation process, NIST announced that it has selected Rijndael [5] to propose for the Advanced Encryption Standard. Rijndael algorithm with 128, 192, 256-bit key length options developed by cryptographers Joan Daemen and Vincent Rijmen has been introduced as a data encryption standard in order and the number of rounds for these versions are 10, 12 and 14 respectively. The AES algorithm is a symmetric block cipher that can process data blocks of 128 bits. It may be used with three different key lengths (128, 192, and 256 bits), and these different “flavors” are referred to as “AES-128”, “AES-192”, and “AES-256”. There are three main operations used in AES: the s-box substitution, shift row, and mix column operations. AES still maintains its reliability today and is used for security in the world of information.

Key Words: Aes encryption,soft sets, inverse product, characteristic product

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Solutıons Of Lınear Parabolıc Type Equatıons Wıth Homotopy Perturbatıon Method

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ABSTRACT

In this study, numerical solutions of linear partial differential equations of parabolic type are investigated. Although there are many studies on analytical solutions of partial differential equations of linear parabolic species in the literature, there are very few studies about numerical solutions. Such numerical solutions have an important place in such literature. In this study, Crank-Nicolson, Homotopy Perturbation and Adomian decomposition methods were used to numerically solve linear partial differential equations of parabolic species. Stability properties of these methods have been examined along with appropriate starting and boundary conditions.

From these methods, the Homotopy Perturbation method is a new and effective method for solving various differential equations. This technique was introduced by Ji-Huan He in 1998. He combines the perturbation technique with the homotopy concept while creating the method and transforms non-linear problems into easy linear problems [1]. The Adomian decomposition method was introduced and developed by George Adomian in 1980s [2–4]. This method is a serial solution method that can be applied to simpler and more complex equations than other methods used for solving linear and nonlinear equations.

A numerical and analytical solutions of the heat equation discussed in this study were solved with Crank-Nicolson, Homotopy Perturbation and Adomian Decomposition methods, and the numerical results obtained were compared in a table. As a result, the Homotopy Perturbation and Adomian Decomposition methods are more stable than the Crank-Nicolson method. Similar studies can be applied to other parabolic equations.

Key Words: Parabolic Equation, Homotopy Perturbation Method, Adomian Decomposition Method, Crank-Nicolson Method

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Solvability And Coerciveness of A Sturm-Liouville Problem With Multipoint-Transmission Conditions

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ABSTRACT

We have investigated the following boundary value problem with discontinuous boundary conditions

$$L(\lambda)u := -p(x)u''(x) + \lambda u(x) = f(x) \ , \quad x \in [-1,0) \cup (0,1]$$

$$L_k u := \alpha_k u^{(m_k)}(-1) + \beta_k u^{(m_k)}(-0) + \eta_k u^{(m_k)}(+0) + \gamma_k u^{(m_k)}(1)$$

$$+\sum_{i=1}^{n_k} \delta_{ki} u^{(m_k)}(x_{ki}) = f_k \, , \quad k = 1,2,3,4 \, .$$

where $p(x)$ is piecewise constant functions, $p(x) = p_1$ at $x \in [-1,0)$, $p(x) = p_2$ at $x \in (0,1]$; λ - complex parameter; $p_i\,(i = 1,2),\ \alpha_k,\ \beta_k,\ \eta_k,\ \gamma_k,\ \delta_{ki}\ (k = 1,2,3,4;\ i = 1,2,...,n_k)$ are complex coefficients; $x_{ki} \in (-1,0) \cup (0,1)$ are internal points; $m_k \geq 0\ (k = 1,2,3,4)$ are any integers. Naturally, we shall assume that, $p_1 \neq 0,\ p_2 \neq 0$ and $|\alpha_k| + |\beta_k| + |\eta_k| + |\gamma_k| \neq 0\ (k = 1,2,3,4)$. Note that we do not assume the classical restriction that $m_k \leq 1$. In particular, we establish such properties as Fredholmness, topological isomorphism and coerciveness with respect to the spectral parameter for this problem.

In the recent past various generalizations of two-order boundary value problems have attached a lot of attention for its wide application in mechanics, physics, electronics and other branches of naturel sciences. An important special case of the generalized boundary value problems are so-called multipoint (more than two points) boundary value problems.

In this paper, we consider a discontinuous boundary value problem with so-called multipoint-transmission conditions in direct sum of Sobolev spaces. We establish such properties as isomorphism, Fredholmness and coerciveness with respect to spectral parameter for consider problem. Obtained results in the article are new even in case of boundary conditions without internal points.

The purpose of this paper is to investigate the second order ordinary linear differential equation on two disjoint intervals involving linear spectral parameter together with multipoint-transmission conditions.

Boundary-value problems with supplementary transmission conditions arise after on application of the method of separation of variables to the varied assortment of physical problems, namely in heat and mass transfer problems, in diffraction problems, in vibrating string problems, when the string loaded additionally with point masses and etc. For example, in electrostatics and magnetostatics the model problem which describes the heat transfer through an infinitely conductive layer is a transmission problem. Also, some problems with transmission conditions which arise in mechanics (thermal conduction for a thin laminated plate) were studied in.

Key Words: Boundary-value problem, multipoint-transmission conditions, isomorphism, coerciveness, solvability.

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Some Bounds On The Topological Matrix and The Topological Indices

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ABSTRACT

The Topological index and The Topological matrix are very useful in the study of Graph Theory in Mathematics, Physics and Chemistry. Some bounds are reported for The Topological index and The Topological matrix. These bounds are obtained in a connected graph. Considering that these studies can be made for weighted graph, the question comes to mind that how can The Topological index and The Topological matrix of a weighted graph be possible. We found the answer to our question. This topic deals with the definition of The Topological index and The Topological matrix in weighted graph and accordingly some conclusions and examples.

The Topological matrix M of a graph G is defined by

$$M_{ij} = \begin{Bmatrix} 1 & i,j \in L \\ 0 & otherwise \end{Bmatrix}$$

where L is link. Firstly, we obtain an upper bound for this topological matrix in this presentation. These bounds contain various properties and applications. Also, these bounds related to the eigenvalues of this topological matrix. Then, we define the weighted topological matrix and we obtain a bound for the spectral radius of the weighted topological matrix. Besides, we establish Nordhaus- Gaddum type results.

The topological energy and The Laplacian topological energy of a graph is defined similar to graph energy and laplacian graph energy, respectively in trhis presentation. We give some upper bounds for the topological energy of M(G) and the Laplacian topological energy of the weighted topological matrix. For these bounds, we use some known lemmas and inequalities.

The Topological index TI(G) of a graph G is defined by TI=|A+D| . In this definition, A is the adjacency matrix and D is the distance matrix. We found basic mathematical operations of Topological index of G. Also, we define the Atom- Bond

Connectivity index, the Randic index, the Harmonic index and the Forgotten topological index Using these indices, we admit some inequalities.

Key Words: Topological matrix, Topological indices, Bounds.

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Some Results On Hausdorff Metric For Intuitionistic Fuzzy Sets

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ABSTRACT

Fuzzy set theory was frstly introduced by L. A. Zadeh in 1965 [1]. In fuzzy sets, every element in the set is accompanied with a function μ(x): X → [0, 1], called membership function. The membership function may have uncertainty in some applications because of the subjectivity of the expert or the missing information in the model. Hence some extensions of fuzzy set theory were proposed [2-4]. One of these extensions is Atanassov's intuitionistic fuzzy set (IFS) theory [2].

In 1986, Atanassov [2] introduced the concept of intuitionistic fuzzy sets and carried out rigorous researches to develop the theory. In this set concept, he introduced a new degree v(x) : X → [0, 1], called non-membership function, such that the sum μ+v is less than or equal to 1. Hence the diﬀerence 1 −(μ+v) is regarded as degree of hesitation. Since intuitionistic fuzzy set theory contains membership function, non-membership function and the degree of hesitation, it can be regarded as a tool which is more flexible and closer to human reasoning in handling uncertainty due to imprecise knowledge or data.

The Hausdorff metric distance between the alpha cuts of fuzzy numbers is one of the most used metric on fuzzy set theory. It measures how far fuzzy numbers are [5]. In this work we will give some results based on the maximum metric involving Hausdorff metric to measure the distance between intuitionistic fuzzy numbers. We will show that intuitionistic fuzzy numbers are complete under the maximum metric based on Hausdorff metric. As a result we will prove that that space of continuous intuitionistic fuzzy number valued functions are complete under this metric.

Key Words: Fuzzy sets, İntuitionistic fuzzy sets, Hausdorff metric, Completeness

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Stability Criterion For Fractional Difference Equations with Delay Term

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ABSTRACT

In this study we consider the asymptotic stability of nonlinear fractional difference equations of the form

$\Delta\big(p(t)\Delta_*^{v}x(t)\big) = f\big(t+v, x(t+v-\sigma)\big), t \in \mathbb{N}_{1-v}, 0 < v \leq 1$

$x(t) = \varphi(t), t = -\sigma, \sigma + 1, \dots, 0.$

Here $\mathbb{N}_v = \{v, v+1, v+2, \dots\}$, Δ_*^{v} is Caputo type dicrete fractional difference operator, $f: [0,\infty) \times \mathbb{R} \to \mathbb{R}$ and $\Delta^{-1} f\big(t+v, x(t+v-\sigma)\big)$ are continuous with respect to both of t and x, the delay term σ is positive integer, $p: \mathbb{N}_{1-v} \to \mathbb{R}$ and $p(t) \neq 0$ for $t \in \mathbb{N}_{1-v}$.

The fractional calculus is the generalization of ordinary differentiation and integration to the arbitrary non-integer order and the history of fractional calculus begins with the classical calculus. Fractional differential equations gives powerful tools in engineering, economics, physics, chemistry and mechanics, see the monographs [16,20,2218,17,24] and the references therein. Fractional differential equations have various applications in viscoelasticity, eloctrochemistry, electromagnetics and control theory, see [13,14,19,512,15,21].

In classcical calculus the derivative of a function can be interpreted geometrically,unfortunately, in fractional calculus, fractional derivatives have no such geometric interpretation. So to investigate the stability of fractional difference equations, instead of using the Liapunov method, fixed point theorems are more useful. See the papers [6,7,23].

In recent years there is an increasing interest on fractional difference eqautions. In [2] Atıcı and Eloe extended the discrete Laplace transform to develope discrete transform method and defined a family of finite fractional difference equations and employed the transform method to obtain solutions. In [3] Atıcı and

Eloe developed commutativity properties of fractional sum and fractional difference operators. In [1] Anastassiou defined Caputo like discrete fractional difference and compared it to the earlier defined Riemann-Liouville fractional discrete analog and derived related discrete Ostrowski, Poincare and Sobolev type inequalities. In [4] Atıcı and Şengül developed some basics of discrete fractional calculus such as Leibniz rule and summation by part formula. In [9] Chen et al. concerned with the initial value problem to a nonlinear fractional difference equation with the Caputo like difference operator and obtained some global and local existence results using some fixed point theorems. In [8] Chen obtained some asymptotic stability results for some nonlinear fractional difference equations employing Schauder fixed point theorem and discrete Arzela-Ascoli theorem. In [10] Chen and Liu presented some results for the asymptotic stability of solutions for nonlinear fractional di¤erence equations involving Reimann-Liouville-like difference operator.

In this paper motivating with [8] we investigate the asymptotic stability of nonlinear fractional difference equation (1) by using Schauder fixed point theorem and discrete Arzela-Ascoli's theorem.

Key Words: Fractional Difference Equations, Stability.

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Stability of Nonlinear Time Relaxation Model for Simplified MagnetoHydroDynamics and Comparison of Solutions

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ABSTRACT

In this study, Simplified MagnetoHydroDynamics (SMHD) equations and solutions of linear and nonlinear time relaxation models by using finite element method are examined. MHD equations are combination of fluid dynamics, Navier-Stokes equations and Maxwell's equations of electromagnetism. MHD equations are used in many areas such as, MHD power generators, cooling of nuclear reactors, liquid metal flow control, micro MHD pumps, high temperature plasmas, biological transportation, drying processes and solidification of binary alloy.

When R_m Reynolds Number in MHD equations are taken as a special condition ($R_m \ll 1$) then MHD equations are reduced to SMHD equations [1].

In the current study, Nonlinear Time Relation Model is examined. This model is obtained by adding the term $\kappa|u-\bar{u}|(u-\bar{u})$ [2] where Layton and Neda (2007) stated that it is a special form of nonlinear time relaxation of SMHD equations. The model is defined as follows,

$$\begin{cases} N^{-1}\left(u_t + u\cdot\nabla u\right) - M^{-2}\Delta u + \nabla p - B\times\nabla\phi - (u\times B)\times B + \kappa\left|u-\bar{u}\right|(u-\bar{u}) = f, \\ \nabla\cdot u = 0, \\ -\Delta\phi + \nabla\cdot(u\times B) = 0. \end{cases}$$

In these equations $\bar{u}$ is a differential filter of u, and it is given as [2]

$$\begin{cases} u_t = \dfrac{(u-\bar{u})}{\delta}, t>0 \\ \bar{u}(x,0) = u(x,0). \end{cases}$$

An algorithm depending on finite element method is constructed to obtain the numerical solution of this model by using Backward-Euler (BE) time step method The stability analysis of this algorithm is investigated and as a result the unconditional

stability of the algorithm is proved. The sample problems are solved by the algorithm which are added both linear and nonlinear filter. The obtained results are compared with the BE and CN solutions of SMHD equations.

Moreover, it is investigated that which situations, the solutions of model obtained nonlinear filter is better. All this computations are conducted by using FreeFem++ programme [3].

Key Words: Magnetohydrodynamics, Backward-Euler, Finite Element Method, Time Relaxation Model

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Statistical Equal Convergence Of Bögel Type Continuous Functions

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ABSTRACT

Recall that the concept of Bögel-type continuity (or, simply, *B*-continuity) was first introduced by K. Bögel in 1934 (J. Reine Angew. Math. 170 (1934), 197-217). The classical Korovkin theory which is a very effective role in approximation theory is mainly connected with the approximation to continuous functions by means of positive linear operators. In 1986, this theory was generalized by using the notion of Bögel-type continuity instead of the ordinary continuity since Bögel-type continuity is more general than the ordinary continuity which was first introduced by Badea et. al. (Bull. Austral. Math. Soc. 34 (1986), 53-64) Recently, Dirik et. al. give statistical version of this theorem (Studia Sci. Math. Hungar. 47 (3) (2010), 289-298) and more recently Okçu Şahin and Dirik introduce statistical equal convergence for double sequences and prove Korovkin type theorem by using this new convergence method (Applied Mathematics E-Notes (accepted)). In the present work, using the statistical equal convergence for double sequences, we obtain a Korovkin type approximation theorem of positive linear operators defined on the space of all real valued Bögel type continuous functions. Hence, we get stronger result than given before. After then, we present an example in support of our definition and result presented in this paper.

Key Words: Statistical equal convergence, B-continuity, Korovkin theorem.

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Structural Stability For The Benard Problem With Voight Regularization

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ABSTRACT

In this talk we consider the Bénard problem involving Voight regularizing terms. We study continuous dependence of solutions on the coefficients μ and ϰ.

Our main aim here to study the structural stability for the following system of equations in $\Omega = (0,L_1) \times (0,L_2) \times (0,1.)$

(1.1) $\frac{\partial u}{\partial t} - \nu\Delta u - \mu\Delta u_t + (u.\nabla)u + \nabla p' = e_3\theta \qquad \text{in } \Omega \times (0,\tau)$

(1.2) $\frac{\partial \theta}{\partial t} - \kappa\Delta\theta - \varkappa\Delta\theta_t + (u.\nabla)\theta - u_3 = 0 \qquad \text{in } \Omega \times (0,\tau)$

(1.3) $\nabla.u = 0 \qquad \text{in } \Omega$

where $\tau > 0$, e_3 is the third component of the canonical basis of $\mathbb{R}^3$, u_3 is the third component of u. $u(x,t)$ the fluid velocity, $p' = p'(x,t)$ is modified pressure given by $p' = p - (x_3 + \frac{x_3^2}{2})$ where $p(x,t)$ is the pressure of the fluid in the box Ω. $\theta(x,t)$ is the scaled fluctuation of the temperature around the steady state background temperature profile $(T_1 - T_0)x_3 + T_0$ and it is given by $\theta = T - \left(\frac{T_0}{T_0 - T_1} - x_3\right)$ where $T = T(x,t)$ is the temperature of the fluid inside the box Ω, T_0 is the temperature of the fluid at the bottom and T_1 is the temperature of the fluid at the top. ν, μ, κ and $\varkappa$ are positive constants. Now we state boundary conditions for (1.1)-(1.3).

(1.4) $u = 0, \ \theta = 0$ at $x_3 = 0, \ x_3 = 1,$

(1.5) $p, u, \theta, \frac{\partial u}{\partial x_i}, \frac{\partial \theta}{\partial x_i}$ $(i = 1,2)$ are periodic in the x_i directions which means that

$$\varphi(x,t) = \varphi(x + L_i e_i, t) \quad i = 1,2 \quad \forall x \in \mathbb{R}^3, \ \forall t > 0$$

for a generic function φ.

The initial conditions are given by

(1.6) $u(x,0) = u_0(x),\ \theta(x,0) = \theta_0(x).$

The existence- uniqueness of this problem was given in [5]. In this study the continuous dependence of solutions of the given problem on the coefficients of the Voight regularizing terms is established. Continuous dependence of solutions is a type of structural stability, which represents the effect of small changes in coefficients of equations on the solutions. Structural stability results can be found for linear and nonlinear partial differential equations (see e.g. [1,2-4,6]).

Key Words: Bénard Problem, structural stability, continuous dependence

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Symmetric Difference Operation in Soft Matrix Theory

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ABSTRACT

It is very difficult to apply some soft operations on the soft sets containing many parameters and objects. Also, in such cases, it is needed considerable time due to the complexity of the operations. In order to overcome these, the soft matrix which is a matrix representation of the soft set was introduced in [2]. Also, various soft matrix operations such as complement, union, intersection, And product and Or product were depicted. The main objective of this study based on soft matrices is to introduce the symmetric difference operation of soft matrices, and to give the several properties and results related to it.

On the other hand, the similarity is a frequently discussed matter in every scientific field. For mathematical tools such as the fuzzy set, rough set and vague set, this concept has been researched in detail and is still being investigated. The similarity in soft set theory which is a mathematical tool has been extensively studied in recent years. Relatedly, this study aims to find the similarity of soft matrices, which are formally equal to the soft sets, by using the symmetric difference operation. Thus, it presents an alternative notion, which is more practical with help of matrices, to obtain the similarity of soft sets.

Key Words: Soft set, soft matrix, symmetric difference.

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Tauberian Theorems For Statistical Convergence And Statistical Summability Of Fuzzy Number Valued Functions

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ABSTRACT

The idea of statistical limit of a measurable function at infinity is introduced by Moricz [3]. We adapt this idea for continuous fuzzy number valued functions. Let E^1 be space of all fuzzy numbers on the set of real numbers and $f:[0,\infty)\to E^1$ be a continuous fuzzy number valued function. We say that f has a statistical limit at infinity if there exists a $\mu \in E^1$ such that for every $\varepsilon > 0$

$$\lim_{a\to\infty}\frac{1}{a}\left|\{x\in[0,a]: D(f(x),\mu)\geq\varepsilon\}\right| = 0,$$

where the vertical bars denote the Lebesgue measure of the inner set and D is the metric on E^1. The limit of a fuzzy number valued function at infinity implies its statistical limit at infinity but the inverse statement is not true in general. By using a similar condition involved in [2], we prove a Tauberian theorem under which the classical limit of a fuzzy number valued functions at infinity follows from its statistical limit at infinity.

We also deal with the concept of statistical weighted mean summability method of improper Riemann integrals of fuzzy number valued functions with the help of notions used in [1]. Let $0 \neq p:[0,\infty)\to[0,\infty)$ be a nondecreasing function such that $p(0)=0$ and $st-\liminf_{t\to\infty}\frac{p(\lambda t)}{p(t)}>1$ for every $\lambda>1$, and $f:[0,\infty)\to E^1$ be a continuous fuzzy number valued function. Define

$$s(t)=\int_0^t f(x)dx \quad \text{and} \quad \sigma(t)=\frac{1}{p(t)}\int_0^t s(x)dp(x),$$

where the second integral exists in fuzzy Riemann-Stieltjes sense (see [4]). Then we find necessary and sufficient conditions under which the existence of statistical limit

of $s(t)$ follows from the existence of statistical limit of $\sigma(t)$.

Key Words: Tauberian theorems, Statistical convergence, Fuzzy number valued functions.

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The Generalized Hermite-Hadamard type inequalities for Caputo k-fractional Derivatives

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ABSTRACT

Fractional calculus was born in 1695. In the past three hundred years, fractional calculus developed in diverse fields from physical sciences and engineering to biological sciences and economics. Fractional Hermite-Hadamard inequalities involving all kinds of fractional integrals have attracted by many researches.

Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

is known in the literature as Hermite-Hadamard inequality for convex mappings. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the mapping f. Both inequalities hold in the reversed direction if f is concave.

The convexity property of a given function plays an important role in obtaining integral inequalities. Proving inequalities for convex functions has a long and rich history in mathamatics

Definition: A function $f: I \to \mathbb{R}$, is said to be convex if for every $x, y \in I$ and $t \in [0,1]$, given by

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y).$$

We say that f is concave if $(-f)$ is convex.

Fractional Hermite-Hadamard inequalities involving all kinds of fractional integrals have attracted by many researchers. Many authors carried out the study of

fractional Hermite-Hadamard inequalities according to the first-order integral equalities and convex with functions of different classes.

Very recently, the authors <cite>farid</cite> introduced the new definition of fractional derivatives and established some interesting Hadamard-type inequalities.

Definition: Let $\alpha>0$, $k>1$ $\alpha\notin\{1,2,3,...\}$, $n=[\alpha]+1$, $f\in AC^n[a,b]$. The Caputo k-fractional derivatives of order α are defined as follows:

$$ {}^{c}D_{a^+}^{\alpha,k} f(x)=\frac{1}{k\Gamma_k\left(n-\frac{\alpha}{k}\right)}\int_a^x \frac{f^{(n)}(t)}{(x-t)^{\frac{\alpha}{k}-n+1}}dt $$

and

$$ {}^{c}D_{b^-}^{\alpha,k} f(x)=\frac{(-1)^n}{k\Gamma_k\left(n-\frac{\alpha}{k}\right)}\int_x^b \frac{f^{(n)}(t)}{(t-x)^{\frac{\alpha}{k}-n+1}}dt $$

were $\Gamma_k(\alpha)$ is the k-Gamma function defined as

$$ \Gamma_k(\alpha)=\int_a^\infty t^{\alpha-1}e^{-\frac{t^k}{k}}dt $$

also

$$ \Gamma_k(\alpha+k)=\alpha\Gamma_k(\alpha). $$

In this paper, we establish the Hermite-Hadamard's type inequalities for Caputo k-fractional derivatives. The results presented here would provide extensions of those given in earlier works.

Key Words: Integral Inequalities, Special Functions, Fractional Calculus.

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The Left (Right) Rough Approximations In A Group

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ABSTRACT

Rough sets were introduced by Z. Pawlak [6] in 1982. One of the basic concepts of Rough sets is the notion of an approximation space. In the original approach an approximation space is the ordered pair (U;R), where U is a finite nonempty set of objects, called the universe, and R is an equivalence relation on U, called the indiscernibility relation. This relation defines a partition of U into non-empty, pairwise disjoint subsets that are equivalence classes. The equivalence classes are the building blocks for the construction of the lower and upper approximations. Some authors have studied the algebraic structures of rough sets. Kuroki and Wang [5] presented some properties of the lower and upper approximations with respect to the normal subgroups. In addition, some properties of the lower and the upper approximations with respect to the normal subgroups were studied in [3,7,8]. On the other hand, Biswas and Nanda [2] introduced the notion of rough group and rough subgroups that their notion depends on the upper approximation and does not depend on the lower approximation. Miao et al. [4] improve de notions of rough group and rough subgroup, and prove their new properties. In addition, Bağırmaz and Özcan [1] introduced the notion of rough semigroups on approximation spaces.

In this study, we define the left (righ) lower and upper approximations with respect to a subgroup of a group, and give some properties of them.

Key Words: Rough sets, rough approximations, rough subgroups.

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To The Application Gauss-Hermite Approximate Method For Initial-Boundary Value Problems

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ABSTRACT

In the work [5] we considered the class of schemes corresponding to Gauss-Hermite method for approximate solution of Cauchy problem for ordinary differential equations (ODE). This method is concrete effective realization by high order accuracy of the "Extrapolation method" [4] which suppose some clear extensions for investigation of initial-boundary value problems. In this connection our report contains two parts:

I. For ODE would be investigated the problem of stability and convergence of the same processes which is essentially connecting to the property of normality by Fejer sense of matrices of nodes. Then for study problems connected with convergence preliminary remark that in case when we used Runge-Kutta-Buthcher type methods the property (the consistency condition) of Lax equivalence Theorem [3] for some examples of scheme of Runge-Kutta is broken; in case using Adams-Bashforth method (when the number of nodes tends to infinite) is true Kuzmin effect [2] about unbounded of corresponding weights of Newton-Kotes quadrture formulas.

II. In the second one if we consider the problems connected with an extension (enlarge) of initial data for constructing by evident scheme to finding the approximate solution of evolutionary equations by high order accuracy than Resolvent methods (or semigroup operators theory) [see, for example 1] or Courant, von Neumann, Lax direct methods for approximate solution some problems of mathematical physics [see, for example [3]. As it is known for Resolvent methods for solving by high order accuracy lies in the best approximation of corresponding kerners while for Difference methods difficulties represent incorrectness of multipointing (high order accuracy) schemes. In the report we construct the explicit

schemes giving the approximate solution of some initial-boundary value problems by arbitrary order accuracy depending only of order of smoothness of desired solution.

Key Words: Approximate solution of Cauchy problem, normality of nodes, Resolvent and Difference methods

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COMPUTER

A Survey On Dynamical Behaviors Of Systems Of Nonlinear Difference Equations

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ABSTRACT

Difference equation or discrete dynamical system is a diverse field which impact almost every branch of pure and applied mathematics. The theory of difference equations occupies a central position in applicable analysis. There is no doubt that the theory of difference equations will continue to play an important role in mathematics as a whole. Nonlinear difference eqautions of order greater than one are of paramount importance in applications. Such equations also appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations which model various diverse phenomena in biology, ecology, psychology, physics, engineering, and economics.

Recently, there has been great interest in studying the systems of difference equations. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economics, probability theory, genetics, psychology and so forth. It is very interesting to investigate the behavior of solutions of a system of nonlinear difference equations and to discuss the asymptotic stability of their equilibrium points.

The study of properties of nonlinear difference equations and systems of rational difference equations, systems of max-type difference equations and systems of exponential type difference equations has been an area of interest in recent years. There are many papers in which systems of difference equations have been studied in the literature.

This presentation deals with the dynamical behavior of the positive solutions of the systems of difference equations. This is review of recent studies in systems of difference equations. We concentrate on papers dealing with two-dimensional, third-dimensional and multi-dimensional systems of nonlinear difference equations.

Key Words: Difference equations, solution, stability, boundedness, equilibrium point.

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Central Measurement of the Value of a Vertex in Graphs

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ABSTRACT

In real-world network analysis, one of the fundamental aims is to find the most significant component of the network. The value of the significance of these components is generally expressed within central invariants. Until now, measurements related to many central invariants have been defined [3, 4]. One of them is betweenness centrality. The concept of betweenness centrality was first introduced by Bavelas in 1948 [1].

Betweenness centrality is quite prevalent in the analysis of many network data models, physical computer networks, social networks, etc. [2, 6]. When we deal with communication networks, a vertex has two important characteristics. First, distance to other vertices. The other is that which vertices are located on the shortest paths between the other vertex pairs. The latter gains more importance. Because these vertices have control over the information flow in the network.

Betweenness centrality $C_B(v)$ for a vertex v is defined as

$$C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is the number of shortest paths with vertices s and t as their end vertices, while $\sigma_{st}(v)$ is the number of those shortest paths that include vertex v.

The betweenness centrality of graph G is defined as

$$C_B(G) = \frac{2\sum_{i=1}^{n}[C_B(v^*) - C_B(v_i)]}{(n-1)^2(n-2)}$$

where $C_B(v^*)$ is the largest value of $C_B(v_i)$ for any vertex v_i in the graph G [5].

Betweenness centrality specifies how important a vertex is in the shortest paths between the other vertices pairs. Let's take the information flows between each pair of vertices. Also, assume that these streams of information are the shortest path

for every pair of vertices. The betweenness centrality measure is the value of the effect of a vertex over these information flows. In this article, we consider betweenness centrality of some graphs.

Key Words: Graph theory, network design and communication, betweenness centrality.

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Disjunctive Total Domination of Corona Graphs

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ABSTRACT

Domination and total domination are well studied topics in graph theory [1,3]. Many domination type structures are expensive to implement in networks. Some restrictions are added to existing domination and total domination parameters to enable greater flexibility. But, this does not reduce their implementation costs. Henning and Naicker [4] was introduced disjunctive total domination as a relexation of total domination in 2016. A set $S \subseteq V(G)$ is called a disjunctive total dominating set if every vertex is adjacent to a vertex of *S* or has at least two vertices in *S* at distance 2 from it. The disjunctive total domination number of *G* is the minimum cardinality of such a set.

Disjunctive total domination number is studied on some graphs such as trees [4,5], claw-free graphs [4], grids [6] and permutation graphs [7]. We study on disjunctive total domination number of corona graphs. The concept of corona of two graphs was introduced by Frucht and Harary [2]. In this work, we give some results about disjunctive total domination of corona of two connected graphs and compute disjunctive total domination number of corona $G \circ H$, where *G* is a path, cycle, star, complete graph or wheel graph and *H* is any connected graph

Key Words: Graph theory, disjunctive total domination, corona graph.

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Fuchsian Groups and Continued Fractions

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ABSTRACT

Continued fractions were studied by the great mathematicians of seventeenth and eighteenth centuries and are a subject of active investigation today. Continued fractions are of great service in solving many interesting problems. They constitute a most important tool for new discoveries in the theory of numbers and in the field of Diophantine approximations. There are the important generalization continued fractions called the analytic theory of continued fractions, an extensive area for present and future research. In the computer field, continued fractions are used to give approximations to various complicated functions, and once coded for the electronic machines, give rapid numerical results valuable to scientists and to those working in applied mathematical fields[1,2].

On the other hand, Fuchsian groups are discrete groups of isometries of the hyperbolic plane. Their very concrete nature allows us to illustrate their many features that have far-reaching generalizations in geometry and number theory[3,4]. The most interesting and important ones are the so-called arithmetic Fuchsian groups, i.e. discrete subgroups of $PSL(2,\mathbb{R}) = \left\{ z \to \frac{az+b}{cz+d} \,\middle|\, a,b,c,d \in \mathbb{R}, ad-bc=1 \right\}$ obtained by some "arithmetic" constructions. In here, we focus on some arithmetic Fuchsian groups and their group action on extended rational numbers. We give some number theoretical results arise from these group actions in terms of continued fractions.

Key Words: Fuchsian groups, continued fractions, discrete groups.

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Generalized Farey Squences

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ABSTRACT

A rational number is one that is expressible as the quotient of two integers. Real numbers that are not rational are said to be irrational. First, we mention about Farey fractions; they give a useful classification of the rational numbers. The sequence of all reduced fractions with denominators not exceeding n-natural number is called the Farey sequence of order n. Any reduced fraction with positive denominator $\leq n$ is a member of the Farey sequence of order n and can be called a Farey fraction of order n. Let a/b, c/d, e/f be any three consecutive fractions in the Farey sequence of order n. It is known that c/d=(a+e)/(b+f). This equation is a method that Farey uses to show that the set of rational numbers is countable[1].

On the other hand,it is known that continued fractions offer a useful means of expressing numbers and functions. The process of expressing a rational number as a continued fraction is essentially identical to the process of applying the Euclidean algorithm to the pair of integers given by its numerator and denominator in lowest terms. Two well-known result from number theory are as follows: The continued fraction expression of a real number is finite if and only if the real number is rational. If a real number is irrational, then its continued fraction expression is infinite[3]. As a result, one can say that Farey sequences and continuous fractions are two useful tools for examining real numbers. In this talk, it will be mentioned how the group action of the Modular group is a bridge that combines these two concepts[2].

Key Words: Farey sequences, continued fractions, modular group.

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Global Attractivity of Some Rational Difference Equations

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ABSTRACT

Difference equations have been investigated by many researchers in the last few decades. We can see many papers and books concerning theory and applications of these difference equations. Although difference equations are very simple in form, it is extremely difficult to understand thoroughly the behaviors of their solutions.Recently it is very interesting to investigate the asymptotic behavior of solutions of rational difference equations and there has been a lot of work concerning the global asymptotic behavior of solutions of rational difference equations. The theory of discrete dynamical systems and difference equations developed greatly during the last twenty five years of the twentieth century.There is no doubt that the theory of difference equations occupies a central position in applicable analysis and willcontinue to play an important role in mathematics as a whole.One of the reasons is that difference equations have been applied in several mathematical models in biology, economics, genetics, psychology, ecology, physics, population dynamics, medicine, physiology and so forth. See, for example, [2], [3], [4], [5], [6], [7].

In this work, we investigate the asymptotic behavior of the solutions and the global attractivity of the equilibrium point of the following rational difference equation

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1}}{C x_{n-1} + D x_{n-2}}$$

with positive parametres α, β, γ, C, D and with arbitrary positive initial conditions x_{-2}, x_{-1}, x_0. , see [1].

Key Words: Difference equation,equilibrium point, locally asymptotically stable, global attractor.

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Group Action and Orbits for Modular group

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ABSTRACT

Many problems in mathematics have remained unsolved because of missing links between mathematical disciplines, such as algebra, geometry, analysis, or number theory. Here, we mention a recently discovered result concerning quadratic polynomials, which uses a bridge between algebra and analysis. The iterations of quadratic polynomials are obtained by computing the value of a polynomial for a given number and feeding the outcome into the exact same polynomial again. These iterations of polynomials have interesting applications, such as in fractal theory[1].

Another similarity of the above idea is that the vertices of the graphs arising from the group action of the modular group on the extended rational numbers can be examined in terms of number theory[2]. Modular group which is possible the most well-known discrete group consists of all linear transformations

$$g(z) = \frac{az+b}{cz+d}$$

where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$. Congruence subgroups of modular group are also very important in number theory, they all have finite index in Γ, but not every subgroup of finite index is a congruence subgroup. Using the notion of the imprimitive action for an invariant equivalence relation $\mathbb{Q}$ by the congruence subgroup $\Gamma_0(n)$, Jones, Singerman and Wicks obtained suborbital graphs of Γ and showed that these graphs are the generalization of the well-known Farey graph. In this talk, we shared some interesting results on them[2,4,5].

Key Words: Quadratic polynomials, Möbiüs transformation, Farey graph.

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Hierarchical Bucket Queue System for Priority Programming in GPU's development

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ABSTRACT

The modern graphics processing unit (GPU) is severely limited in its ability to influence the programming of these enormous resources, while offering great parallel computing power. For this reason, the GPU is considered an externally controlled processor for homogeneous workloads that severely restrict the potential applications of GPU computation. To remove this problem, the hierarchical bucket queue system, which is structured with priority time scheduling, is presented as a new method. This system is revealed using visualization application programming interface. The aim of the developed application is to design a priority queue system discipline scheme that can integrate and develop existing applications. Various scheduling strategies have been provided by carefully distributing the workload among multiple queues and deciding from which queue to start the next job. These strategies include fair scheduling, earliest-dead-first scheduling, and user-defined dynamic priority scheduling. Compared to previous studies, advantages of the hierarchical bucket queue system have been revealed. This system has been tried to be interpreted under the framework of queuing systems, which is a statistical approach, and it is aimed to give the theoretical contrast of the system. Finally, it has been shown to what extent the programming developed in real world applications improves path monitoring and micropolygon generation.

Key Words: Stochastic processes, Markov chains, Hidden Markov models, Probability distribution.

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Nonoscillation And Oscillation Criteria For A Class Of Higher - Order Difference Equations Involving Generalized Difference Operator

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ABSTRACT

Due to its numerous applications in fields such as economics and mathematical biology, the asypmtotic behaviour of solutions of difference equations has been receiving intensive attention in the last few decades; we refer the reader to the monographs [1], [2].

In 1987, Popenda [3] obtained sufficient conditions for nonoscillation / oscillation of solutions of a class of nonlinear nonhomogeneous second order difference equations involving generalized difference of the form

$$\Delta_a^2 x_n = F(n, x_n, \Delta_b x_n).$$

In [7], Tan and Yang generalized and improved the result of Popenda by considering the equation

$$\Delta_a\left(p_n \Delta_a x_n\right) + q_n \Delta_a x_n = F(n, x_n, \Delta_b x_n).$$

In [8], Parhi and Panda obtained sufficient conditions for nonoscillation / oscillation of all solutions of a class of nonlinear third order difference equations of the form

$$\Delta_a\left(p_n \Delta_a^2 y_n\right) + q_n \Delta_a^2 y_n = f(n, y_n, \Delta_a y_n, \Delta_a^2 y_n).$$

In this work, we generalize and improve the result of Popenda and Panda by considering the higher - order difference equations involving the generalized difference operator of the form

$$\Delta_a^k(p_n \Delta_a^2 y_n) = f(n, y_n, \Delta_a y_n, \ldots, \Delta_a^{k+1} y_n), n \in \mathbb{N}$$

and give sufficient conditions to be nonoscillatory / oscillatory its all solutions, where Δ_a is generalized difference operator and defined as in the form

$$\Delta_a y_n = y_{n+1} - a y_n,\ n \in \mathbb{N}, a \in \mathbb{R}$$

$a \in \mathbb{R}\backslash\{0\}$, $\{p_n\}$ is a real sequence with $p_n \neq 0$ for $n \in \mathbb{N}$, $f: \mathbb{N}x\mathbb{R}^{k+2} \to \mathbb{R}$, $\mathbb{N}$ is the set of natural numbers, $\mathbb{R}$ is the set of real numbers.

Key Words: Difference Equations, Generalized Difference Operator, Oscillation, Nonoscillation.

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On Boundedness Of Solutions Of The Rational Difference Equations

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ABSTRACT

The goal of this presentation is to review the boundedness character of the solutions of the rational difference equations. We concentrate on boundedness behaviors of the rational difference equations. Thus, we present a few technique used to find the boundedness of the solutions of rational difference equations which studied in the literature.

We classify the techniques used in five sections. We name these technique as follows contradiction methods, invariant interval methods, min-max methods, invariant product methods and initial conditions methods. Then we give detailed examples of each method in each section. Also, we investigate the global behaviors of the equilibrium points of the second and third order rational difference equations.

We consider the difference equation of order $(k+1)$ is an equation of the form

$$x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-k}) \quad n = 0,1,\dots. (1.1)$$

where I is some interval of real numbers and the function $f: I^{k+1} \to I$ is a continuously differentiable function. For every set of initial conditions $x_{-k}, x_{-(k-1)}, \dots, x_0 \in I$, the difference equation (1.1) has a unique solution $\{x_n\}_{n=-k}^{\infty}$.

A solution $\{x_n\}_{n=-k}^{\infty}$.of Eq. (1.1) is called bounded if, for all $n \geq -k$, there exist m and M positive numbers such that

$$m \leq x_n \leq M.$$

For the last few years the boundedness behaviors of solutions of rational difference equations are being extensively investigated [1-8].

Key Words: Difference equations, solution, boundedness, equilibrium point.

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On Stability Of A Class Of The Nonlinear Difference Equation Systems With Variable Delayed

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ABSTRACT

Difference equations usually describe the evolution of certain phenomena over the course of time. These equations and their stabilities are the appropriate mathematical representation for discrete processes different from differential equations, which have special importance in areas such as economy, physics, mathematical biology and population models. Many of these models are encountered as nonlinear difference equations in difference equations. This studying gives us some results related to stability theory for difference equations. Let us consider the system

$$x(n+1) = ax(n) + B(n)F\left(x_{n-m_n}\right), \qquad n = 0,1,2,... \tag{1}$$

where $a \in (-1,1)$, F is the real valued vector function, the function m maps the set of integers to the set of positive integers, which is a bounded function and maximum value of m is k and $B(n)$ is a $k \times k$ variable coefficient matrix. Taking into account this one, we investigate the asymptotic stability of zero solution, and particularly obtain necessary and sufficient conditions for the asymptotically stability of the equation (1). This study has a different significance from similar ones such as the studies of D. C. Huong and N. V. Mau (2013). Because $B(n)$ is taken as a $k \times k$ variable coefficient matrix in this work. Therefore, our results generalize some of the previous results given in this area.

Key Words: Difference equations, characteristic equation, asymptotic stability.

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Solving Cross-Matching Puzzle Using Heuristic Approximation with Probabilities

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ABSTRACT

Nowadays, games take more and more an important place in human life. The games have a vital status, especially for children as such in eating, drinking, sleeping and taking the air. The games that reach such a crucial level are necessary not only for children's fun but also in mental training for children and has a vital importance for future generations. Cross-matching puzzle improved for this purpose can help children to evaluate their leisure time. In addition to this, an exercise booklet was printed to increase the popularity of the puzzle. Then, cross-matching puzzle contests were organized in elementary schools.

Cross-matching puzzles are logic-based games being played with numbers, letters, symbols, flags or color blokes. A cross-matching puzzle consists of three tables: solution table, detection table, and control table. In order to solve the puzzle, the detection and control tables can be superposed. Previously, the depth first search and intelligent genetic algorithm is used to solve the puzzle. However, this study proposes a heuristic approximation which is more proper and faster method when the size of puzzle is small. The heuristic approximation is based mainly on the probabilities that the elements in the matching matrix are in the desired position.

Key Words: Cross-matching puzzle, Heuristic search.

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The Normalizer of Modular Group in Picard Group

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ABSTRACT

In all analytic functions the first order rational functions have the simplest mapping properties, for they define mappings of the extended plane onto itself which are at the same time conformal and topological. The linear transformations have also very remarkable geometric properties, and for that reason their importance goes far beyond serving as simple examples of conformal mappings. The most general linear transformation is composed by a translation, an inversion, a rotation, and a homotetic transformation. In here, we focus on the three kind of general linear transformations as follows[3].

The Picard group $\mathbb{P}= PSL\left(2,\mathbb{Z}(i)\right)$ is the group of linear transformations

$$t(z) = \frac{az+b}{cz+d}\,, ad-bc=1$$

with $a,b,c,d \in \mathbb{Z}(i)$. Here an element of $\mathbb{Z}(i)$ has the form $m+ni$ with $m,n \in \mathbb{Z}$. Modular group which is possible the most well-known discrete group consists of all linear transformations

$$g(z) = \frac{az+b}{cz+d}$$

where $a,b,c,d \in \mathbb{Z}$ and $ad-bc=1$. The structure of principal congruence subgroups of $\mathbb{P}$ shows similarities to the structure of principal congruence subgroups of Γ. Pointing out this fact given as a reason to study the connection between Γ and $\mathbb{P}$, the normalizer of Γ in $\mathbb{P}$ was studied [1,2]. In here, we mention about some properties of this normalizer in terms of combinatorics.

Key Words: Picard group, Modular group, Normalizer.

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GEOMETRY

A Note On A Paraholomorphic Riemannian Extension

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ABSTRACT

The cotangent bundles of differentiable manifolds are very important in many areas of mathematics and physics. Cotangent bundle is dual of the tangent bundle. Because of this duality, some of the geometric results are similar to each other. The most significant difference between them is construction of lifts (see [5] for more details).

Let M be an n-dimensional C^{∞}- manifold with torsion free connection ∇, $T^{*}M$ be its cotangent bundle and π the bundle projection $T^{*}M \to M$. The Riemannian extension ${}^{R}\nabla \in \Im_{2}^{0}\left(T^{*}M\right)$ which is a pseudo-Riemannian metric is defined by

$$ {}^{R}\nabla\left({}^{C}X, {}^{C}Y\right) = -\gamma\left(\nabla_{X}Y + \nabla_{Y}X\right) $$

for any $X, Y \in \Im_{0}^{1}(M)$, where $\gamma\left(\nabla_{X}Y + \nabla_{Y}X\right) = p_{f}\left(X^{t}\nabla_{t}Y^{f} + Y^{i}\nabla_{i}X^{f}\right)$(see [5, p. 268]). The components of the Riemannian extension is given by the form

$$ {}^{R}\nabla = \left({}^{R}\nabla_{IJ}\right) = \begin{pmatrix} -2p_{k}\Gamma_{ij}^{k} & \delta_{i}^{j} \\ \delta_{j}^{i} & 0 \end{pmatrix} $$

with respect to the natural frame $\left\{\frac{\partial}{\partial x^{j}}, \frac{\partial}{\partial x^{\bar{j}}}\right\}$, where δ_{i}^{j} denote the Kronecker delta. The Riemannian extensions were defined by Patterson and Walker [3] and intensively studied for the cotangent bundle.

An almost product structure $F \in \Im_{1}^{1}\left(M^{2m}\right)$ is defined by $F^{2} = I$. The pair $\left(M^{2m}, F\right)$ is called an almost product manifold . An almost paracomplex manifold is an almost product manifold $\left(M^{2m}, F\right)$, such that the two eigenbundles $T^{+}M^{2m}$ and $T^{-}M^{2m}$ associated to the two eigenvalues +1 and -1 of F , respectively, have the

same rank. Note that the dimension of an almost paracomplex manifold is necessarily even. Considering the paracomplex structure F, we obtain the set $\{I,F\}$ on *M*, which is an isomorphic representation of the algebra of order 2, which is called the algebra of paracomplex (or double) numbers and is denoted by $R(j),\ j^2=1$ [2].

Using paracomplex structure F and the pure tensor field ϑ, the operator ϕ_F defined by

$$(\phi_F\vartheta)\left(X,Y_1,...,Y_q\right)=\left(FX\right)(\vartheta\left(Y_1,...,Y_q\right))-X(\vartheta\left(FY_1,Y_2,...,Y_q\right))$$
$$+\vartheta\left(\left(L_{Y_1}F\right)X,Y_2,...,Y_q\right)+...+\vartheta\left(Y_1,Y_2,...,\left(L_{Y_q}F\right)X\right),$$

where L_X denotes the Lie derivative with respect to X and $\phi_F\vartheta\in\Im^0_{q+1}\left(M^{2m}\right)$.

If $\phi_F\vartheta=0$, then ϑ is said to be almost paraholomorphic with respect to the paracomplex algebra $R(j)$ [4].

In this paper, we investigate paraholomorphy property of the Riemannian extension by using compatible almost paracomplex structure on the cotangent bundle.

Key Words: Riemannian extension, cotangent bundle, paracomplex structure.

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A Note on the Generalized Myers Theorem for Riemannian Manifolds

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ABSTRACT

One of the most fundamental topics in Riemannian geometry is to examine the relation between topology and geometric structure on Riemannian manifolds. To give nice compactness criteria for complete Riemannian manifolds is one of the most natural and interesting problems in Riemannian geometry. The well-known Bonnet–Myers theorem says that if M is a complete Riemannian manifold with Ricci curvature bounded below by a (strictly) positive constant, then M must be compact and hence has finite volume measure. The proof of this theorem also permits one to get sharp upper diameter estimates. Since then, this result has been widely extended and improved in several directions. The first generalization was given by Ambrose (1957), where the positive lower bound for the Ricci curvature was replaced by a condition on its integral along some geodesics. On the other hand, motivated by relativistic cosmology, Galloway (1979) proved a compactness theorem by perturbing the positive lower bound on the Ricci curvature by the derivative in the radial direction of some bounded function. One of the most important features of the two generalizations above is that the Ricci curvature is not required to be everywhere nonnegative. In this paper, we shall give a generalization of Ambrose compactness theorem for the Bakry-Emery Ricci tensor on M. To obtain this result, we utilize the Riccati inequality for the mean curvature of distance spheres instead of the classical second variation of geodesics. After that, using the index form of a minimizing unit speed geodesic segment, we will obtain a new upper diameter estimate for complete Riemannian manifolds in the case that the Ricci curvature has a lower bound and the norm of the potential function has an upper bound. Our diameter estimate improves previous one obtained by Galloway.

Key Words: Bakry-Emery Ricci tensor, Index form, Myers-type theorem, Riccati inequality.

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Ambrose Type Results for Weighted Ricci Curvature in Finsler Manifolds

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ABSTRACT

Finsler geometry, a natural generalization of Riemannian geometry, was initiated by Finsler from considerations of regular problems in the calculus of variations. Various differential geometric invariants have strong impact on the topology of differentiable manifolds as illustrated by deep results in the Riemannian geometry like the theorems of Hopf-Rinow, Myers, Rauch, Synge. The area of these results has been extended along years to the Finslerian setting. Also, Shen generalized comparison theorems to the Finsler geometry. Afterwards, Wu and Xin proved Laplacian comparison theorems, volume comparison theorems under various flags, and Ricci and *S*-curvature conditions. The validity of the Myers compactness theorem for Finsler manifolds was shown by Shen without any modification. Later, using the weighted Ricci curvature Ric_N , Ohta obtained a compactness theorem and gave an upper bound for the diameter of n-dimensional Finsler manifolds. Wu generalized the Myers theorem to Finsler manifolds in terms of line integral Ricci curvature bound and also gave an upper bound of the diameter. The compactness theorem of Ambrose is a stronger version of the Myers theorem allowing the Ricci scalar to take also negative values in Riemannian manifolds. This theorem is extended to Finsler manifolds by Anastasiei. Yin acquired two Myers type compactness theorems for a Finsler manifold with a positive weighted Ricci curvature bound and an advisable condition on the distortion or the S-curvature. In the present work, we obtain two compactness theorems of Ambrose type for forward complete and connected Finsler manifolds in the context of the weighted Ricci curvature Ric_∞ and Ric_N. The proofs are based on the Bochner-Weitzenböck formula and with suitable sequence choices.

Key Words: Finsler manifold, S-curvature, weighted Ricci curvature, compactness theorem.

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AW(k)-type Salkowski Curves in Euclidean 3-space IE^3

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ABSTRACT

The notion of AW(k)-type submanifolds is introduced by Arslan and West. In particular, many works related to curves of AW(k)-type have been done by several authors. For example, K. Arslan and C. Özgür give curvature conditions and characterizations related to these curves in E^m. B. Kılıç Bayram, K. Arslan consider curves and surfaces of AW(k) (k=1,2 or 3)-type. They also give related examples of curves and surfaces satisfying AW(k)-type conditions. İ. Kişi, G. Öztürk study these types curves according to the parallel transport frame in Euclidean spaces E^3 and E^4.

Salkowski curves, a family of curves with constant curvature but non-constant torsion, and anti-Salkowski curves, a family of curves with constant torsion but non-constant curvature, are introduced by Salkowski in 1909. Thereafter, A.T. Ali studies spacelike and timelike Salkowski curves in Minkowski 3-space.

In this study, we consider AW(k)-type ($1 \leq k \leq 3$) Salkowski (anti-Salkowski) curves with $\kappa \neq 0$ ($\tau \neq 0$) in Euclidean 3-space. We show that there is no AW(1)-type Salkowski curve and AW(1)-type anti-Salkowski curve in E^3. Also, we handle weak AW(2)-type and weak AW(3)-type Salkowski (anti-Salkowski) curves and we show that there is no weak AW(2)-type Salkowski curve in E^3.

Key Words: Salkowski curves, Anti-Salkowski curves, AW(k)-type curves.

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Characteristic Directions of Closed Planar Homothetic Motions

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ABSTRACT

In this study, one-parameter closed planar homothetic motion was considered. The Steiner area formula was calculated for this motion. The area formula was rearranged by using a different parameterization for the integral coefficients. The Steiner point or Steiner normal concepts were described according to whether rotation number is different from zero or equal to zero, respectively. The moving pole point was given with its components. Firstly the relation between the moving pole point and Steiner point when the rotation number is different from zero and then the relation between the moving pole point and the Steiner normal when the rotation number is equal to zero were specified. Furthermore the polar moment of inertia for the closed planar homothetic motion was given. Then the relation between the area enclosed by a path and the polar moment of inertia was expressed. The sagittal motion of a crane was considered as an example. This motion was described by a double hinge consisting of the fixed control panel of crane and the moving arm of crane. The moving arm of the crane can extend or retract during the planar homothetic motion. The results obtained in the theoretical sections of this study were applied for the crane motion.

Key Words: Steiner formula, polar moment of inertia, planar kinematics.

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Darboux Vector and Stress Analysis of Centro-Affine Frame

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ABSTRACT

A set of points that corresponds a vector of vector space constructed on a field is called an affine space associate with vector space. We denote A_3 as affine 3-space associated with IR^3.

The first written sources that can be achieved about affine space curve theory are based on the 1890's when Ernesto Cesàro and Die Schon von Pirondini lived period. From that years to 2000's there are a some affine fames used in curve theory. One of them is centro-affine frame.

The grup of affine motions special linear transformation consist of volume preserving linear transformations denoted by $ASL(3,IR) := SL(3,IR) \times IR^3$ and comprising diffeomorphisms of IR^3 that preserve some important invariants such curvaures that in curve theory as well.

In this study, we separated the matrix representing affine frame as symmetric and antismmetric parts by using matrix demonstration of the centro-affine frame of a curve given in affine 3-space. By making use of antisymmetric part, we obtained the angular velocity vector which is also known as Darboux vector and then we expressed it in the form of linear sum of affine Frenet vectors.

On the other hand, by making use of symmetric part, we obtained the normal stresses and shear stress components of the stress on the frame of the curve in terms of the affine curvature and affine torsion. Thus we had the opportunity to be able to explane the distinctive geometric features of the affine curvature and affine torsion.

Lastly, we made stress analysis of a curve with constant affine curvature and affine torsion in affine 3-space as an example.

Key Words: Centro-affine Frame, Darboux Vector, Shear Stresss.

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Darboux Vector and Stress Analysis of Equi-Affine Frame

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ABSTRACT

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D-Conformal Curvature Tensor on $(LCS)_n -$Manifolds

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ABSTRACT

Recently, in [1], Shaikh introduced and studied Lorentzian concircular structure manifolds (briefly (LCS)-manifold) which generalizes the notion of LP-Sasakian manifolds, introduced by Matsumoto [3]. Generalizing the notion of LP-Sasakian manifold in 2003 [1], Shaikh introduced the notion of $(LCS)_n$-manifolds along with their existence and applications to the general theory of relativity and cosmology. Also, many authors studied various types of $(LCS)_n$- manifolds by imposing the curvature restrictions. After then, the same author studied weakly symmetric $(LCS)_n$-manifolds by several examples and obtain various results in such manifolds.

The concept of D-conformal curvature tensor was defined by Chuman [6]. Again Chuman studied D-conformal vector fields in para-Sasakian manifolds. Adati [7], studied D-conformal para-Killing vector fields in special para-Sasakian manifolds. Shah [8], researched some curvature properties of D-conformal curvature tensor on LP-Sasakian manifolds. Recently, D-conformal curvature tensor were studied by many geometers.

This paper deals with the study of geometry of $(LCS)_n$-manifolds. Motivated by the studies of the above authors, we have studied some curvature properties of D-conformal curvature tensor on $(LCS)_n$-manifolds. In this paper, $(LCS)_n$- manifolds satisfying the conditions, D-conformally flat, R(X,Y).B=0, B(X,Y)S=0 and P(X,Y)S=0 are considered, where and B and P denote the D-conformal curvature tensor and the projective curvature tensor, respectively.

Key Words: D-conformal curvature tensor, projective curvature tensor, Ricci tensor.

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Holditch Type Theorem for Kinetic Energy of Projective Curve under the 1-Parameter Spatial Inverse Motion

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ABSTRACT

H. Holditch have given the following important theorem in 1858:

If the endpoints *A B*, of a line segment *AB* with fixed length $a+b$ are rotated once along an oval (Eilinie) *k* in the Euclidean plane, then a given fixed point *X* $(|AX|=a, |XB|=b)$ on line segment *AB* describes *a* closed, not necessarily convex, curve k_x The area F of the Holditch Ring bounded by the oval k and curve k_x is $F=\pi ab$. W.Blaschke and H. R. Müller have given the following generalization of the classical Holditch Theorem:

Under 1-parameter closed planar motions with the rotation number *v* if the endpoints $A=(0,0)$ and $B(a+b,0)$ on the line segment *AB* trace the closed orbit curves k_A and k_B, respectively, the point $X=(a,0)$ $(|AX|=a, |XB|=b)$ which is collinear with points A and B traces a closed, not necessarily convex, curves k_X. Let F_A F_B and F_X be the orbit areas of the orbit curves k_A, k_B and k_X, respectively. then there is the equation

$$F_X=\frac{aF_B+bF_A}{a+b}=\pi vab$$

In this study, under the homothetic direct and inverse motions. The kinetic energy of the projection curve is calculated. The Holditch theorem are obtained for the kinetic energy of the projection curve. And also some properties are given which fullfill these features.

Key Words: Holditch Theorem, Kinetic Energy, Kinematic

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Lifts of Complex Metallic Structure to the Cotangent Bundle

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ABSTRACT

The lift of geometrical objects, functions, vector fields, *1*-forms etc., defined on any manifold has an important role in differential geometry. Thus, we can generalize to differentiable structures on any manifold to its cotangent bundle. Lifts of structures on any manifold M to its cotangent bundle T^*M were introduced and studied by several authors, see [1, 4, 5].

In 2013, Hretcanu and Crasmareanu [2] introduced a metallic structure which is defined on a differentiable manifold M with a *(1,1)*-tensor field J which satisfies the equation $J^2=pJ+qI$.

In 2016, Yilmaz [3] studied the integrability and parallellism of the metallic structure. The relation between metallic structures and connections are obtained, and some results are given, and the complex metallic structure on manifolds is defined in the cited thesis.

In this study, we studied complete and horizontal lifts of complex metallic structure to the cotangent bundle. Further, we investigated integrability conditions of complex metallic structure in the cotangent bundle.

Key Words: Complex metallic structure, complete lift, horizontal lift, cotangent bundle, integrability.

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New Examples of Timelike Minimal Surfaces in Lorentz-Minkowski Space

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ABSTRACT

A minimal surface has a zero mean curvature in Euclidean space. Björling formula is a way to obtain a minimal surface via complex variables in Euclidean space which is proposed by E. Björling and solved by Schwarz. By using this formula one can obtain the minimal surface from a curve which is called the core curve and a vector field which is orthogonal to tangent of the core curve. This formula is given for maximal surface via complex number and timelike minimal surface via split-complex numbers in Lorentz-Minkowski space. A timelike minimal surface has zero mean curvature at every point in Lorentz-Minkowski space endowed with a Lorentzian metric. In this talk we will give a number of new examples of timelike minimal surface in Lorentz-Minkowski space by using the Björling formula. For obtain the explicit parametrization of the surfaces we use the core curve as a circle. Since a timelike surface include both spacelike and timelike curves, we will take core curve as timelike and spacelike circles. The counterpart of these surfaces in Euclidean space is called the bending helicoids. As particular cases, we will show that these surfaces are invariant by a uniparametric group of rotational motions.

This talk is based on a joint work with R. Lopez [3].

Key Words: Björling problem, timelike minimal surfaces, circle.

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On Contact CR-Submanifolds of a Kenmotsu Manifold

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ABSTRACT

The study of the differential geometry of a contact CR-submanifolds, as a generalization of invariant and anti- invariant submanifolds, of an almost contact metric manifold was initiated by Bejancu and was followed by several geometers. Several authors studied contact CR-submanifolds of different classes of almost contact metric manifolds given in the references of this study. The contact CR-submanifolds are rich and interesting subject. Hence we continue to study in this subject matter.The purpose of this paper is to study the differential geometric theory of submanifolds immersed in Kenmotsu manifold. We obtain the new integrability conditions of the distributions of contact CR-submanifolds and prove some characterizations for the induced structure to be parallel.

In this study, we study the differential geometry of contact CR-submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact CR-submanifold in Kenmotsu manifolds. Finally, the induced structures on submanifolds are investigated, these structures are categorized and we discuss these results. We think that new results are obtained in this study.

We shall define contact CR-submanifolds in a Kenmotsu manifold and research fundamental properties of their from theory of submanifold. Let M be submanifold of an almost contact metric manifold $\bar{M}$, then M is called invariant submanifold if $\varphi(T_xM) \subseteq T_xM, \forall x \in M$. Further, M is said to be anti-invariant submanifold if $\varphi(T_x^{\perp}M) \subseteq T_x^{\perp}M, \forall x \in M$. Similarly, it can be easily seen that a submanifold M of an almost contact metric manifolds $\bar{M}$ is said to be invariant(anti-invariant), if N (or T)

are identically zero in [5]. Now we give definition of contact CR-submanifold which is a generalization of invariant and anti-invariant submanifolds.

A submanifold M of a Kenmotsu manifold. $\bar{M}$ is called contact CR-submanifold if there exists on M a differentiable invariant distribution D whose orthogonal complementary $D^{\perp}$ is anti-invariant, i.e.,

i) $TM = D \oplus D^{\perp}, \xi \in \Gamma(TM)$

ii) $\varphi D_x = D_x$

iii) $\varphi D_x^{\perp} \subseteq T^{\perp}M, \forall x \in M$

A contact CR-submanifold is called anti-invariant(or, totally real) if $D_x = 0$ and invariant(or, holomorphic) if $D_x^{\perp} = 0$, respectively, for any $x \in M$. It is called proper contact CR-submanifold if neither $D_x = 0$ nor $D_x^{\perp} = 0$.

Key Words: Kenmotsu manifold; Contact CR-submanifold.

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On Rectifying Curves in Dual Space D^3

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ABSTRACT

The characterization of a space curve is one of the main subject which has drawn interests of geometers'. Characterizing the curves make it possible to classify them according to their some properties. One example for the characterized curve is a rectifying curve. A twisted curve in the Euclidean 3-space E^3 is called a rectifying curve if its position vector field always lies in its rectifying plane. It is known that rectifying curves have many interesting properties and their centrodes (angular velocity vectors) play some important roles in mechanics and joint kinematics. The examination of the characterization of rectifying curve, provides the ability of commenting about them as kinematically since the position vector field of these curves determines the axis of instantaneous rotation at each point of the curve. In addition to that, the relation between rectifying curves and their centrodes is used to study in not only kinematics but also general mechanics too.

The centrodes of unit speed curves are used to characterize rectifying curves. In this study, we give some characterizations of the rectifying curves in the dual space D^3. We investigate the relations between geodesics and rectifying curves. We study rectifying curves and their centrodes in the dual space via the dilation of unit speed curves. Then we obtain some results about these curves in the dual space D^3. Finally, we give some conditions being a rectifying curve for the centrode of a unit speed curve in E^3.

Key Words: Rectifying curve, centrode, dilated centrode, dual space, dual Darboux vector.

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On Sasakian Manifolds Admitting Concurrent Vector Fields

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ABSTRACT

Let $(\tilde{M}, g)$ be a Riemannian manifold and S be the Ricci tensor of $\tilde{M}$. On a such manifold, if the following equation is satisfied

$$(L_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0, \tag{1}$$

Then this manifold is said to be a Ricci soliton which is as known quasi-Einstein metric in physics literature. Here, $L_V g$ is the Lie-derivative of the metric tensor g in the direction vector field V, which is called the potential vector field of the Ricci soliton, λ is a constant and X, Y are arbitrary vector fields on $\tilde{M}$. A Ricci soliton is denoted by $(\tilde{M}, g, V, \lambda)$. If $L_V g = 0$, then the potential vector field V is called Killing. Also, If $L_V g = \rho g$, then the potential vector field V is called conformal Killing, where ρ is a function. In relation (1), if V is zero or Killing, then the Ricci soliton is called trivial and in this case, the metric is Einstein. So, a Ricci soliton is viewed as a generalization of Einstein metric. Also, if the potential vector field V is the gradient of a potential function $-f$ ($i.e. V = -\nabla f$), then it is called a gradient Ricci soliton. In addition, a Ricci soliton is said to have shrinking, steady or expanding depending on $\lambda < 0$, $\lambda = 0$ or $\lambda > 0$, respectively.

In this paper, we focus on concurrent vector field on Sasakian manifold. Then, it is investigated Ricci solitons on some submanifolds of a Sasakian manifold. Also, it is derived some necessary and sufficient conditions for a vector field on the invariant distribution of M to be concurrent. Finally, it is given a characterization for these submanifolds of Sasakian manifold to be a gradient Ricci soliton.

Key Words: Concurrent vector field, Ricci soliton, Sasakian manifold.

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On The Control Invariants of Planar Bezier Curves For Groups $M(2)$ and $SM(2)$

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ABSTRACT

Let $G = M(2)$ be the group generated by all orthogonal transformations and translations of the 2-dimensional Euclidean space or $G = SM(2)$ be the subgroup of $M(2)$ generated by rotations and translations of R^2.

In classic differential geometry, the following theorem is known for the group $SM(2)$ in [1]:

"Let α and β be two curves in R^2. Then, α and β are equivalent if and only if the curvatures and speeds of α and β are equal."

This theorem shows that a necessary and sufficient condition for two curves in R^2 to be equivalent is that they have the same curvature and speed. Furthermore, the sufficiently proof shows how to find the required isometry explicitly. Here, using differential invariants and Frenet frames of two curves, an isometry transformation which carrying a curve into another curve is calculated.

In [2] ,G-equivalence of two Bezier curves for groups $M(n)$ and $SM(n)$ without using differential invariants of Bezier curves in terms of control invariants of Bezier curves is proved. In this work, starting from the ideas in [2] we address how to compute explicitly an isometry transformation which carrying a Bezier curve into another Bezier curve in terms of control invariants of a Bezier curve for the groups $M(2)$ and $SM(2)$ without using differential invariants of Bezier curves. For solution of this problem, complex numbers are used.

Key Words: Bezier curve, invariant.

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On The Riemannian Metrics Of The Form ${}^{J}G = {}^{S}g_f + {}^{H}g$ On The Tangent Bundle

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ABSTRACT

The tangent bundles of differentiable manifolds are very important in many areas of mathematics and physics. The geometry of tangent bundles goes back to the fundamental paper [5] of Sasaki published in 1958. Although the Sasaki metric is naturally defined, it was shown in many papers that the Sasaki metric presents a kind of rigidity. The three classical constructions of the metrics on tangent bundles are given as follows:

a) The Sasaki metric ${}^{S}g$ is a (positive definite) Riemannian metric on the tangent bundle TM which is derived from the given Riemannian metric on M .

b) The horizontal lift ${}^{H}g$ of g is a pseudo-Riemannian metric on the tangent bundle TM .

c) The vertical lift ${}^{V}g$ of g is a degenerate metric of rank n on the tangent bundle TM .

Another classical construction is the complete lift of tensor field to the tangent bundle. It is well known that the complete lift ${}^{C}g$ of a Riemannian metric g coincides with the horizontal lift ${}^{H}g$ given above. A “nonclassical” example is the Cheeger-Gromoll metric g_{CG} on the tangent bundle TM . Other metrics on the tangent bundle TM can be constructed by using the three classical lifts ${}^{S}g$, ${}^{H}g$ and ${}^{V}g$ of the metric g (for example ,see [6]).

Cotangent bundle is dual of the tangent bundle. Because of this duality, some of the geometric results are similar to each other. The most significant difference between them is construction of lifts [6]. In this study, we present the Riemannian

metrics of the form ${}^{f}\tilde{G} = {}^{S}g_{f} + {}^{H}g$ on the tangent bundle over a Riemannian manifold (M,g) [2], which is completely determined by its action on vector fields of type X^{H} and ω^{V}. Later, we obtain the covarient and Lie derivatives applied to the Riemannian metrics of the form ${}^{f}\tilde{G} = {}^{S}g_{f} + {}^{H}g$ with respect to the horizontal and vertical lifts of vector fields, respectively.

Key Words: Covarient derivative, Lie derivative, Riemannian metrics, Horizontal lift, Vertical lift, Tangent bundle.

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On Vectorial Moment of the Darboux Vector

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ABSTRACT

It is known that any regular curve can be written by means of Frenet vectors and vectorial moment of any regular curve. In a space any given regular curve moves around the Darboux vector field. In this paper we have worked on the curve plotted by the vectorial moment c^* of the unit Darboux vector c.

In the present study we define a new curve c* generated by the vectorial moment vector c* of the unit Darboux vector c whose components are of the Frenet vectors of any regular curve in E^3 . We compute the Frenet apparatus of the curve (c*). It is given that the curve (c*) doesn't form a constant width curve pairs with the main curve. We calculate the Frenet apparatus of the regular curve drawn by the vectorial moment vector of c^* where c is the vectorial moment of the unit Darboux vector of a regular curve α. As a linear combination of Frenet vectors, we can express the curve α in the following way : α(s) = f(s)T(s) + g(s)N(s) + h(s)B(s).

We assert that the curve (N_) generated by the vectorial moment vector N_ of the normal vector N doesn't form a constant width curve pairs with the main curve (α). We also assert that the curve (B^*) generated by the vectorial moment vector B^* of the binormal vector B doesn't form a constant width curve pairs with the main curve (α).When (α) is a regular curve with the unit Darboux vector c.Then we assert that the curve (c^*) generated by the vectorial moment vector c^* of the unit Darboux c doesn't form a constant-width curve pairs with the main curve (α). We finally calculate the Frenet apparatus of the curve (c*) when the main curve is supposed to be an helix and we drew a conclusion too.

Key Words: Darboux vector, vectorial moment.

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Position Vectors of Special Smarandache Curves in Galilean 3-Space

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ABSTRACT

In this work, we have studied general position vectors of special Smarandache curves with Darboux apparatus of an arbitrary curve on a surface in the three-dimensional Galilean space. The main aim of this paper is to determine position vector of Smarandache curves of arbitrary curve on a surface in the three-dimensional Galilean space in terms of geodesic, normal curvature and geodesic torsion with respect to the standard frame. The results of this work include providing Smarandache curves of some special curves such as geodesics, asymptotic curves and line of curvatures on a surface in the three-dimensional Galilean space. Firstly, Smarandache curves of geodesics that are circular helix, generalized helix, Salkowski curve and anti-Salkowski curve have been investigated in Galilean space and their position vectors are obtained. Secondly, the position vectors, have been given for Smarandache curves of asymptotics that are circular helix, generalized helix, Salkowski curve and anti-Salkowski curve have been given in the three-dimensional Galilean space. Then, the position vectors have been given for Smarandache curves of line of curvature that are circular helix, generalized helix, Salkowski curve and anti-Salkowski curve have been given in the three-dimensional Galilean space. Exemplarily; the position vectors of Smarandache curves of any curve on the surface are given. Finally; graphs of these Smarandache curves are given.

Keywords: Special Smarandache curve, Darboux frame, Geodesic curve, Galilean space.

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Recognition Of Plane Paths Under Linear Similarity Transformations

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ABSTRACT

Transformations and invariants of curves, surfaces and graphical objects appear in many areas of computer-aided geometric design, computer graphics, computer vision and pattern recognition. Applications of affine, Euclidean and similarity transformations of curves and graphical objects are considered in many works. For example, in the paper [1], a novel and deterministic algorithm is presented to detect whether two given rational plane curves are related by means of a similarity, which is a central question in PatternRecognition. Moreover, similarity of rational plane curves is determined.

For curves in the similarity geometry, using curvatures of the curve in Euclidean geometry, curvature functions of the curve in the similarity geometry were obtained . This method in the similarity geometry give conditions only for local G-similarity of curves, where G is the group of orientation-preserving similarity transformations. (see [2])

Therefore, the important problem is to find simple but efficient method for the linear similarity check of the paths.

Let E_2 be the 2-dimensional Euclidean space, $LSim(E_2)$ be the group of all linear similarities of E_2 and $LSim^+(E_2)$ be the group of all orientation-preserving linear similarities of E_2.

This presentation concerned with the global invariants of the plane paths under all linear similarity transformations. In this work, the global conditions G-linear similarity of the plane paths for the groups G= $LSim(E_2)$, $LSim^+(E_2)$ are introduced. Clearly, these global conditions can be used to judge whether two paths are G-linear similar.

Key Words: Similarity, path, invariant.

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Rectifying, Normal and Osculating Curves in Minkowski 3-Space

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ABSTRACT

Curves theory has studied in Euclidean $3-$space for a long time. It is well-known that to each unit speed curve $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ with at least four countinuous derivatives, one can associate three mutually orthogonal unit vector fields $T(s), N(s)$ and $B(s)$, called respectively the tangent, the principal normal and the binormal vector fields. The planes which is spanned by $\{T(s), B(s)\}, \{N(s), B(s)\}$ and $\{T(s), N(s)\}$ are known as the rectifying, normal and osculating plane, respectively. The curves $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ for which the position vector $\alpha(s)$ always lie in their rectifying plane, are for simplicity called rectifying curves. Similarly, the curves for which the position vector $\alpha(s)$ always lie in their osculating plane, are for simplicity called osculating curves; and finally, the curves for which the position vector always lie in their normal plane, are for simplicity called normal curves. If all normal or osculating planes of a curve in $\mathbb{E}^3$ pass through a particular point, then the curve is spherical or planar, respectively. It is also known that if all rectifying planes of a non-planar curve in $\mathbb{E}^3$ pass through a particular point, then the ratio of its torsion and curvature is a non-constant linear function. These curves have been studied in many different spaces and many characterizations have been obtained. In this paper, we study rectifying, normal and osculating curves in Minkowski $3-$space. These curves have already been worked on in different spaces, but the significance of this study is to obtain calculations in a different method.

Key Words: Rectifying curve, Normal curve, Oskulating curve

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Ricci Solitons on Ricci Pseudosymetric a Normal Paracontact Metric Manifold

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ABSTRACT

The object of the present paper is to study some types of Ricci pseudosymmetric a normal paracontact metric manifolds whose metric is Ricci soliton such as concircilar Ricci pseudosymmetric, projective Ricci pseudosymmetric. Finally, we constructed an example of concircular Ricci pseudosymmetric a normal paracontact metric manifold whose metric is Ricci soliton. In [2], Hamilton introduced the notion of Ricci .ow to .nd a canonical metric on a smooth manifold. Then Ricci .ow has become a powerful tool for the study of Riemannian manifolds. The Ricci .ow is an evolution equation for metrics on a Riemannian manifold de.ned as follows;

$$\frac{d}{dt} g_{ij}(t) = -2R_{ij}.$$

A Ricci soliton emerges as the limit of the solutions of the Ricci flow. A solution to the Ricci .ow is called Ricci soliton if it moves only by a one parameter group of the diffeomorphizms and scaling. In precisely, a Ricci soliton on a Riemannian manifold (M,g) is atriple (g,V,λ) satisfying

$$l_V g + 2S + 2\lambda_g = 0,$$

where S denotes the Ricci tensor of M, l is the Lie-derivative along the vector .eld V on M and $\lambda \in R$. The Ricci soliton is said to be shrinking, steady and expanding according as $\lambda<0, \lambda=0$ and $\lambda>0,$ respectively.

Key Words: Ricci Solitons; Ricci Pseudosymmetric; Normal Paracontact Metric Manifold.

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Smarandache Curves Obtained from Salkowski Curve

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ABSTRACT

Salkowski curve is a special curve whose first curvature variable, second curvature constant. Similar type Anti-Salkowski curve is the first curvature variable, second curve constant is a special curve. In this paper, tackled Salkowski curves that were gained to science thanks to studies DNA on the spiral of German a chemist Ernst Leopold Salkowski (1844-1923). First, *{T,N,B}* the Frenet frame obtained when received as a position vectors the Frenet vectors of the Salkowski curve, Smarandache curves were obtained using this Frenet frame. Tangent vector *T* with principal normal vector *N* for *TN*-Smarandache, tangent vector *T* with binormal vector *B* for *TB*-Smarandache, binormal vector *B* with principal normal vector *N* for *NB*-Smarandache, tangent vector *T* the binormal vector *B* principal normal vector *N* for *TNB*-Smarandache and this of Smarandache curves calculated second curvature with first curvature for each. Second, *{T*,N*,B*}* the Frenet frame obtained when received as a position vectors the Frenet vectors of the Salkowski curve, Smarandache curves were obtained using this Frenet frame. Tangent vector *T** with principal normal vector *N** for *T*N**-Smarandache, tangent vector *T** with binormal vector *B** for *T*B**-Smarandache, binormal vector *B** with principal normal vector *N** for *N*B**-Smarandache, tangent vector *T** the binormal vector *B** principal normal vector *N** for *T*N*B**-Smarandache and this of Smarandache curves calculated second curvature with first curvature for each.

Key Words: Salkowski curve, smarandache curve.

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Smarandache Curves of Alternative Frame

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ABSTRACT

In differential geometry, special curves have an important role. One of these curves Smarandache curves. Smarandache curves was firstly defined by M. Turgut and S. Yılmaz in 2008. Let α=α(s) be a regular unit speed curve in E^3. This curves Frenet frame and Alternative Frame are $\{T,N,B\}$ and $\{N,C,W\}$, respectively. In there, N is normal vector, W is unit Darboux vector and $C = W \times N$.

In this paper, we created the Smarandache curves according to the alternative frame of the unit speed curve. Firstly, we introduced unit Darboux vector, Smarandache Curves, Frenet frame, Frenet apparatus, Alternative frame and its properties. After that we mentioned the relationship with Alternative frame and Frenet frame. Then we defined four curves. And we calculated curvature, torsion, Frenet frame and Alternative frame of these curves. First curve is $\alpha_{NC} = \frac{1}{\sqrt{2}}(N + C)$. It can be called by α_{NC}_Smarandache curve. This curves curvature, torsion, Frenet frame and Alternative frame are κ_{NC}, τ_{NC}, $\{T_{NC}, N_{NC}, B_{NC}\}$ and $\{N_{NC}, C_{NC}, W_{NC}\}$, respectively. α_{NW} _ Smarandache curve is $\alpha_{NW} = \frac{1}{\sqrt{2}}(N + W)$. This curves curvature, torsion, Frenet frame and Alternative frame are κ_{NW}, τ_{NW}, $\{N_{NW}, C_{NW}, W_{NW}\}$ and $\{N_{NW}, C_{NW}, W_{NW}\}$, respectively. α_{CW} _ Smarandache curve is $\alpha_{cw} = \frac{1}{\sqrt{2}}(C + W)$. This curves curvature, torsion, Frenet frame and Alternative frame are κ_{CW}, τ_{CW}, $\{T_{CW}, N_{CW}, B_{CW}\}$ and $\{N_{CW}, C_{CW}, W_{CW}\}$, respectively. α_{NCW}_ Smarandache curve is $\alpha_{NCW} = \frac{1}{\sqrt{3}}(N + C + W)$. This curves curvature, torsion, Frenet frame and Alternative frame are κ_{NCW}, τ_{NCW}, $\{T_{NCW}, N_{NCW}, B_{NCW}\}$ and $\{N_{NCW}, C_{NCW}, W_{NCW}\}$, respectively.

Finally, we visualized the defined curves and their Frenet and Alternative frame apparatuses with the aid of the maple program.

Key Words: Unit darboux vector, alternative frame, frenet frame, smarandache curves.

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Some Inequalities For Ricci Solitons

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.ABSTRACT

A smooth vector field V on a Riemannian manifold (M,g) is said to be define a Ricci soliton if it satisfies

$$\frac{1}{2}L_V g + Ric + \lambda g = 0,$$

where $L_V g$ denotes the Lie-derivative of the metric tensor g with respect to V, Ric is the Ricci tensor of (M,g) and λ is a constant. Here, we denote a Ricci soliton (M,g,V,λ). The vector field V is called is potential vector field of the Ricci soliton. The Ricci soliton (M,g,V,λ) is called shrinking, steady or expanding if the constant $\lambda<0$, $\lambda=0$ or $\lambda>0$, respectively. A Ricci soliton is said to be trivial, if (M,g) is an Einstein manifold.

In this paper, we start by studying the notion Ricci soliton which is natural generalization of Einstein manifold and its submanifold. Here, we calculate the scalar curvature for the submanifold of Ricci soliton (M,g,V,λ). Considering such a curvature, we establish some basic inequalities and obtain the relations between the intrinsic and extrinsic curvatures for the submanifold of Ricci soliton (M,g,V,λ). As a result, by using these inequalities, we give some characterizations for the submanifold of a Ricci soliton. As a result, we obtain a necessary and sufficient condition such a submanifold of is also to be a Ricci soliton (M,g,V,λ).

Key Words: Riemannian manifold, Ricci soliton, curvature

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Some Notes On The Diagonal Lifts Of Affinor Fields Along A Cross-Section On $T_q^p(M)$

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ABSTRACT

The method of lift has an important role in modern differentiable geometry. With the lift function it is possible to generalize to differentiable structures on any manifold to its extensions. Vertical, complete and horizontal lifts of functions, vector fields, 1-forms and other tensor fields defined on any manifold M to tangent manifold TM has been obtained by Yano and Ishihara [6], Yano and Patterson [7]. E.M. Patterson and K. Yano studied vertical and complete lifts of tensor fields and connections from a manifold M_n to its cotangent bundle $T^*(M_n)$. Afterwards, K. Yano studied the behavior on the cross-section of the lifts of tensor fields and connections on a manifold M_n to $T^*(M_n)$ and proved that when φ defines an integrable almost complex structure on M_n, its complete lift φ^C is a complex structure. The behaviour of the lifts of tensor fields and connections on a manifold to its different bundles along the corresponding cross-sections are studied by several authors. For the case tangent and cotangent bundles, see [6] and also tangent bundles of order 2 and order r, see [3]. In [1], the first author and his collaborator studied the complete lift of an almost complex structure in a manifold on the so-called pure cross-section of its $(p,q)-$tensor bundle by means of Tachibana operator (for diagonal lift to the $(p,q)-$tensor bundle see [6] and for the $(0,q)-$tensor bundle see [5]). Moreover they proved that if a manifold admits an almost complex structure, then so does on the pure cross-section of its $(p,q)-$tensor bundle provided that the almost complex structure is integrable. In [4], the authors give detailed description of geodesics of the $(p,q)-$tensor bundle with respect to the complete lift of an affine connection.

Throughout this study, all manifolds, tensor fields and connections are always assumed to be differentiable of class C^∞. Also we denote by $\Im_q^p(M)$ the set of all tensor fields of type (p,q) on M. In this study firstly, operators were applied to vertical and horizontal lifts with respect to the diagonal lift φ^D of tensor fields of type $(1,1)$ from manifolds to its tensor bundle of type (p,q) along the cross-section, respectively. Secondly, we get the conditions of almost holomorfic vector field with respect to the φ^D on $T_q^p(M)$.

Key Words: Cross-section, Tachibana operators, Vishnevskii operators, Diagonal lift, Horizontal lift, Vertical lift

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Some Properties of 2x2 Complex Semi Quaternion Matrices

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ABSTRACT

Real quaternions were introduced by Irish mathematician Sir William Rowan Hamilton as an extension of complex numbers. The set of real quaternions represented as

$$\mathbb{H} = \{q = q_0e_0 + q_1e_1 + q_2e_2 + q_3e_3 : q_0, q_1, q_2, q_3 \in \mathbb{R}\}$$

where e_0 acts an identity and $e_1^2 = e_2^2 = e_3^2 = -1$, $e_1e_2e_3 = -1$. The real quaternion algebra $\mathbb{H}$ is a four dimensional vector space over the real number field $\mathbb{R}$. In addition, a real quaternion can be represented by 2x2 complex matrix $M_2(\mathbb{C})$, [1]. A real semi quaternion a is a vector of the form $a = a_0e_0 + a_1e_1 + a_2e_2 + a_3e_3$ where a_0, a_1, a_2, a_3 are real numbers and the real semi quaternion basis elements e_0, e_1, e_2, e_3 satisfy the equalities

$$e_1^2 = -1,\ e_2^2 = e_3^2 = 0,$$
$$e_1e_2 = -e_2e_1 = e_3,\ e_2e_3 = e_3e_2 = 0,\ e_3e_1 = -e_1e_3 = e_2.$$

Since $e_1e_2 \neq e_2e_1$, the real semi quaternions are noncommutative, [2]. A complex semi quaternion b is a vector of the form $b = b_0e_0 + b_1e_1 + b_2e_2 + b_3e_3$ where b_0, b_1, b_2, b_3 are complex numbers. The basis elements of a complex semi quaternion e_0, e_1, e_2, e_3 satisfy the same multiplication rules as is real semi quaternions, [3].

In this study, we express complex semi quaternions as a 2x2 complex matrix by using matrix representation of the basis elements of a complex semi quaternion. In this way, we investigate conjugates and determinant of a complex semi quaternion matrix. Moreover, we obtain a method to calculate the determinant of these matrices. Then, we investigate some special matrices for complex semi quaternion matrices.

Key Words: Complex semi quaternion, complex semi quaternion matrix.

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Surface Family With A Common Mannheim B-Asymptotic Curves

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ABSTRACT

Recently, researchers focused on the reverse problem: given a 3D curve, find surfaces interpolating the given curve as a special curve, rather than finding and classifying curves on analytical curved surfaces. The first study related with this problem was proposed by Wang *et al.* [1] in Euclidean 3-space.They constructed parametric surfaces possessing a given curve as a common geodesic. In this construction, they obtained the condition on marching-scale functions, coefficients of the Frenet vectors. Bayram *et al.* [2] studied parametric surfaces which possess a given curve as a common asymptotic.

In this paper, we construct a surface family possessing a Mannheim B pair of a given curve as an asymptotic curve. Using the Bishop frame of the given Mannheim B curves, we present the surface as a linear combination of this frame and analyse the necessary and sufficient condition for a given curve such that its Mannheim B pairs is both isoparametric and asymptotic on a parametric surface. Those that can be developed through these surfaces are also inspected. The extension to ruled surfaces is also outlined**.** In addition, necessary and sufficient conditions have been given for the developing of this ruled surface family. Finally, we present some interesting examples to show the validity of this study.

Key Words: Mannheim B-pair, Asymptotic curve, Bishop Frame, Ruled surface.

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Surface Family With A Common Mannheim B-Geodesic Curves

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ABSTRACT

Recently, researchers focused on the reverse problem: given a 3D curve, find surfaces interpolating the given curve as a special curve, rather than finding and classifying curves on analytical curved surfaces. The first study related with this problem was proposed by Wang *et al.* [1] in Euclidean 3-space.They constructed parametric surfaces possessing a given curve as a common geodesic. In this construction, they obtained the condition on marching-scale functions, coefficients of the Frenet vectors. Kasap *et al.* [2] generalized the marching-scale functions of Wang and obtained a larger family of surfaces.

In this paper, we construct a surface family possessing a Mannheim B pair of a given curve as an geodesic curve. Using the Bishop frame of the given Mannheim B curves, we present the surface as a linear combination of this frame and analyse the necessary and sufficient condition for a given curve such that its Mannheim B pairs is both isoparametric and geodesic on a parametric surface. Those that can be developed through these surfaces are also inspected. The extension to ruled surfaces is also outlined**.** In addition, necessary and sufficient conditions have been given for the developing of this ruled surface family. Finally, we present some interesting examples to show the validity of this study.

Key Words: Mannheim B-pairs, Geodesic curve, Bishop Frame, Ruled surface.

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Surface Family With A Common Natural Asymptotic Lift Of A Spacelike Curve With Spacelike Binormal In Minkowski 3-Space

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ABSTRACT

We encounter curves and surfaces almost in every differential geometry book. Traditionally, authors deals with surface curves to find and characterise them on a given surface. They define surface curves and classify them on surfaces. There exists a lot of papers related with type of problems. However, the more interesting problem is to find a surface passing through a given curve. Recent studies are concentrated on the reverse problem, that is, how to find a surface possessing a given curve as a special curve such as geodesic, asymptotic curve, line of curvature etc. In the present study, we find a surface family possessing the natural lift of a given spacelike curve with spacelike binormal as an asymptotic curve in Minkowski 3-space using the Frenet frame of the given curve. First, we obtain the parametric representation of surfaces passing through the lift of the given curve. Secondly, we express these surfaces according to the Frenet frame of the original curve. We express necessary and sufficient conditions for the given curve such that its natural lift is an asymptotic curve on any member of the surface family. Finally, we illustrate the method with some examples. These examples indicates the validity of the presented method.

Key Words: Surface family, asymptotic curve, natural lift, Minkowski 3-space..

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Surface Family with a Common Natural Geodesic Lift of a Spacelike Curve with Spacelike Binormal in Minkowski 3-space

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ABSTRACT

Surface curves such as geodesics, line of curvatures, asymptotic curves etc. have been a long-term research topic in differential geometry. Regardless of the representation of the surface, most existing work on curves can be viewed as `forward analysis': given a surface, how to find a special curve on the surface in question. In recent years, fundamental research has focused on the reverse problem or backward analysis: given a 3D curve, how can we characterize those surfaces that possess this curve as a special curve, rather than finding and classifying curves on surfaces. Those papers find applications in computer vision and image processing various industrial applications, such as tent manufacturing, cutting and painting path, fiberglass tape windings in pipe manufacturing, textile manufacturing, plate-metal-based manufacturing, astronomy, astrophysics and architectural computer aided design (CAD). In the present study, we find a surface family possessing the natural lift of a given spacelike curve with spacelike binormal as a geodesic curve in Minkowski 3-space. We express necessary and sufficient conditions for the given curve such that its natural lift is a geodesic on any member of the surface family. For simplicity, we assume that the given curve is arc length parameterised. However, the method can be extended for arbitrarily parameterised curves. Finally, we illustrate the method with some examples.

Key Words: Surface family, geodesic, natural lift, Minkowski 3-space.

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The Characteristic Point of Double Hinge Movement under Homothetic Inverse Motion

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ABSTRACT

The principle of least action is a variational principle that, when applied to the action of a mechanical system. The principle remains central in modern physics and mathematics. In physics, the kinetic energy of an object is the energy that it possesses due to its motion. Kinetic energy is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity. Having gained this energy during its acceleration, the body maintains this kinetic energy unless its speed changes. The body when decelerating from its current speed to a state of rest does the same amount of work. In classical mechanics, the kinetic energy of a non-rotating object of mass *m* traveling at a speed *v* is $\frac{1}{2}mv^2$. By using concept of kinetic energy, the principle of least action can be expressed.

In this study the principle of least action of the closed planar motion and the characteristic point of minimal action definitions are given and kinetic energy formula is obtained. By using the formula of kinetic energy, characteristic point of homothetic motion is expressed. These results are investigated for direct and inverse homothetic motion. As an example, characteristic point of double hinge movement under closed homothetic inverse motion is given.

Key Words: Planar Motion, Principle of Least Action, Kinetic energy.

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The Dual Spatial Quaternionic Expression of Ruled Surfaces

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ABSTRACT

In recent years there have been many studies on ruled surfaces. In some studies, the dual expression of the ruled surface has been investigated. However, the ruled surface was not studied as a quaternionic. In [7], Şenyurt and Çalışkan, firstly, the ruled surface is expressed as a spatial quaternionic. Also, the spatial quaternionic definitions of the Striction curve, the distribution parameter, angle of pitch and the pitch are given. Finally, integral invariants of the closed spatial quaternionic ruled surfaces drawn by the motion of the Frenet vectors belonging to the spatial quaternionic curve α are calculated. In this study, the ruled surface which corresponds to a curve on dual unit sphere is rederived with the help of dual spatial quaternions. We extend the term of dual expression of ruled surface using dual spatial quaternionic method. The correspondences in dual space of closed ruled surfaces are quaternionically expressed. As a consequence, dual angle of pitch and distribution parameter, well-known in differential geometry, is rederived with the help of dual spatial quaternions. Then, we compute distribution parameters and dual angles of pitch of closed dual spatial quaternionic ruled surfaces which are formed by representation curves of (T), (N_1) and (N_2) dual curves on dual unit sphere

Key Words: Real quaternion, spatial quaternion, dual spatial quaternion, closed ruled surface, distribution parameter, dual angle of pitch.

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The Integral Invariants and The Gauss Curvatures of Ruled Surfaces Which are Generated by Instantaneous Pfaff Vectors of The Dual Parallel Equidistant Ruled Surfaces

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ABSTRACT

Firstly, in another work, we have defined the correspondence on the dual space of two equidistant ruled surfaces which are obtained assuming generator vectors (tangent vectors) are parallel and the distance between asymptotic planes at corresponding points is constant along their striction curves in Euclidean space. And so, we have called these ruled surfaces as “dual parallel equidistant ruled surfaces”. Further, we have showed the relationships between of these invariants and the integral invariants of these closed ruled surfaces in case of the striction curves of ruled surfaces which are generated by tangent, normal and binormal vectors of these dual parallel equidistant ruled surfaces are close. Besides, we have calculated the Gauss curvatures of these ruled surfaces and we have given the relationships between of these curvatures.

In this work, we have computed the Gauss curvatures of ruled surfaces which are generated by instantaneous Pfaff vectors of the dual parallel equidistant ruled surfaces. Also, the integral invariants belong to the ruled surfaces are calculated in case of striction curves of the ruled surfaces are close. On this basis, we have calculated the integral invariants of the closed ruled surfaces which are generated by instantaneous Pfaff vectors of these dual parallel equidistant ruled surfaces and we are given the relationships between of these integral invariants.

Key Words: Dual parallel equidistant ruled surfaces, integral invariants, Gauss curvatures

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The Ricci Solitons with Concurrent Vector Fields

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ABSTRACT

Ricci solitons exhibit rich geometric properties. They are natural extension of Einstein manifolds and self-similar solutions to the Ricci flow. During the last two decades, the notion of Ricci soliton has been focus of attention of many mathematicians. Especially, it has become more popular after G. Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904.

A Riemannian manifold (M^n, g) is a Ricci soliton if there exists a smooth vector field V on M, such that

$$\frac{1}{2}L_V g + Ric + \lambda g = 0$$

for some constant λ. Here, $L_V g$ is Lie-derivative of the metric tensor g with respect to V, Ric is the Ricci tensor of (M^n, g) and V is called is potential vector field of the Ricci soliton. Hence, the Ricci soliton denotes by (M, g, V, λ). If $\lambda < 0$, $\lambda = 0$ or $\lambda > 0$, the Ricci soliton is called shrinking, steady or expanding, respectively. If (M^n, g) is an Einstein manifold, then the Ricci soliton (M, g, V, λ) is trivial.

In this paper, we survey the concept of Ricci soliton and Lagrangian submanifold of a Kaehler manifold. If such a submanifold admitting a Ricci soliton which has the potential vector field is a concurrent vector field, then we find that the Ricci soliton (M, g, V, λ) is an Einstein manifold.

Key Words: Ricci Soliton, Lagrangian Submanifold, Concurrent Vector Field

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Two Special Linear Connections on a Differentiable Manifold Admitting an Almost Product Structure

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ABSTRACT

The purpose of this present paper is to study two special linear connections, defined by a fixed linear connection on a differentiable manifold admitting an almost product structure. The almost product structure defines two naturally complementary projection operators splitting the tangent bundle into two complementary parts, so there are two globally complementary distributions of the tangent bundle. We investigate the notions of anti parallelism, half parallelism and anti half parallelism for each of the distributions, which are naturally defined by an almost product structure. We also analyze the concept of geodesicity on almost product manifolds with regard to both of the linear connections. First of all, we give the basic definitions, concepts, formulas and results which will be used throughout the paper. We get an equivalent statement related to anti parallelism of both of the distributions with respect to the fixed linear connection. We obtain a necessary and sufficient condition for each of the distributions to be half parallel with respect to the first defined linear connection (respectively, the other defined linear connection). Moreover, we show that both of the distributions are always anti half parallel with respect to both of the linear connections. Finally, we find a necessary and sufficient condition for a curve on an almost product manifold to be a geodesic with respect to the first defined linear connection (respectively, the other defined linear connection).

Key Words: almost product structure, linear connection, anti parallelism, half parallelism, anti half parallelism.

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Ulisse Dini-type Helicoidal Hypersurface 4-Space

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ABSTRACT

We consider Ulisse Dini-type helicoidal hypersurface in Euclidean 4-space E^4. We give some basic notions of the four dimensional Euclidean geometry in section 2. In section 3, we consider Ulisse Dini helicoidal hypersurface. We obtain Ulisse Dini-type helicoidal hypersurface, and calculate its curvatures in the last section.

We calculate the first and second fundamental forms, matrix of the shape operator S, Gaussian curvature K, and the mean curvature H of hypersurface M=M(u,v,w) in Euclidean 4-space E^4.

We define the rotational hypersurface and helicoidal hypersurface in E^4. For an open interval $I\subset R$, let $\gamma:I\rightarrow\Pi$ be a curve in a plane Π in E^4, and let ℓ be a straight line in Π. A rotational hypersurface in E^4 is defined as a hypersurface rotating a curve γ around a line ℓ (these are called the profile curve and the axis, respectively). Suppose that when a profile curve γ rotates around the axis ℓ, it simultaneously displaces parallel lines orthogonal to the axis ℓ, so that the speed of displacement is proportional to the speed of rotation. Then the resulting hypersurface is called the helicoidal hypersurface with axis ℓ and pitches $a,b\in R\backslash\{0\}$. We may suppose that ℓ is the line spanned by the vector $(0,0,0,1)^t$.

Finally, we obtain and calculate its differential geometric properties of the Dini-type helicoidal hypersurface:

$$D(u,v)=\begin{pmatrix}\sin u\cos v\cos w\\ \sin u\sin v\cos w\\ \sin u\sin w\\ \varphi(u)+av+bw\end{pmatrix},$$

where $\varphi(u):I\subset R\rightarrow R$ is a differentiable function for all $u\in I\subset R\backslash\{0\}$, $0\leq v,w\leq 2\pi$ and $a,b\in R\backslash\{0\}$.

Additionally, we find some relations for the curvatures.

Key Words: Dini-type helicoidal hypersurface, Gauss map, Gaussian curvature, mean curvature.

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Ulisse Dini-type Helicoidal Surface in 3-Space

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ABSTRACT

In this talk, we study Ulisse Dini-type helicoidal surface in Euclidean 3-space E^3. We give some basic notions of the three dimensional Euclidean geometry in section 2. In section 3, we consider Ulisse Dini helicoidal surface. We obtain Ulisse Dini-type helicoidal surface, and calculate its curvatures in the last section.

We calculate the first and second fundamental forms, matrix of the shape operator S, Gaussian curvature K, and the mean curvature H of surface M=M(u,v) in Euclidean 3-space E^3.

We define the rotational surface and helicoidal surface in E^3. For an open interval $I\subset \mathbb{R}$, let $\gamma: I\rightarrow \Pi$ be a curve in a plane Π in E^3, and let ℓ be a straight line in Π. A rotational surface in E^3 is defined as a surface rotating a curve γ around a line ℓ (these are called the profile curve and the axis, respectively). Suppose that when a profile curve γ rotates around the axis ℓ, it simultaneously displaces parallel lines orthogonal to the axis ℓ, so that the speed of displacement is proportional to the speed of rotation. Then the resulting surface is called the helicoidal surface with axis ℓ and pitch $a\in \mathbb{R}\backslash\{0\}$. We may suppose that ℓ is the line spanned by the vector $(0,0,1)^t$.

Moreover, we consider Dini-type helicoidal surface:

$$D(u,v) = \begin{pmatrix} \sin u \cos v \\ \sin u \sin v \\ \varphi(u) + av \end{pmatrix},$$

where $\varphi(u): I\subset \mathbb{R}\rightarrow \mathbb{R}$ is a differentiable function for all $u\in I\subset \mathbb{R}\backslash\{0\}$, $0\leq v\leq 2\pi$ and $a\in \mathbb{R}\backslash\{0\}$.

Key Words: Dini-type helicoidal surface, Gauss map, Gaussian curvature, mean curvature.

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LOGICS

A Note On New Ring Structure

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ABSTRACT

Molodtsov[1] introduced the theory of soft sets, which can be seen as an effective mathematical tool to deal with uncertainties, since it is free from the diffuculties that the usual theoretical approaches have troubled. It associates a set with a set of parameters and thus free from the diffuculties effecting existing methods. Soft set theory has rich potential applications most of which have already been demonstrated by Molodtsov. In many fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Sezgin and Atagün [6] discussed the basic properties of operations on soft sets such as intersection, extended intersection, restricted union, restricted diffference defined in [2] and they illustrated their interconnections between each other. Soft set theory has continued to experience tremendous growth in the mean of algebraic structures since Aktaş and Çağman [5] defined and studied soft groups, soft subgroups, normal soft subgroups, soft homomorphisms, adopting the definition of soft sets. In [4], the same authors introduced two new operations on soft sets, called inverse production and characteristic production depending on the relation forms of soft sets and obtained two isomorphic abelian groups called "the inverse group of soft sets" and "the characteristic group of soft sets". In this study, we redefine the operations inverse and characteristic products of soft sets without using relation forms of soft sets. This leads to simplicity and brevity. Also, we construct two ring structure consisting the representations of uncertain objects as elements.

Key Words: Soft sets, group structure, ring structure, inverse product, characteristic product.

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A Research On Symbolic Logic In Last Term Of The Ottoman

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ABSTRACT

Islamic Logic researches mostly described The Logic as " the science which is tool for getting unknowns from known informations" and " the science which protects a mind from error when its rules are obeyed". In this work, a research will be done on Ali Sedad and Salih Zeki with their books within subject of Symbolic Logic's entrance to Ottoman, its development and informations about how Ottoman scholars welcomed it.

The first person who introduced The Symbolic Logic in Ottoman Empire was Ali Sedad. In Ottoman Empire, the person who was accepted as the representative of Symbolic Logic was Salih Zeki.

Ali Sedad's book which was named Mizanu'l Ukul fi'l Mantık ve'l Usul consists of four section and one addition after one preface and entrance. He gave place to Symbolic Logic in his work's add section. In the add section whose name is Application of Math to Logic, Ali Sedad explained Methods of Boole and Stanley Jevons who was a student of Boole with his own comments. Ali Sedad wasn't suppertor to Symbolic Logic but he told it for introduction. He considered it as a difficulty. Logic works which were done in Europan later, provided right him. The most strong example about this was "Deficiency Theory" of Kurt Gödel.

Mizan-ı Tefekkür Book of Salih Zeki was published in 1916. This book consists of one entrance and six sections. In here, Symbolic Logic was worked widely. It was the summary of conferences which Salih Zeki gave in Math Department of Ottoman Daru'l Fünun around 1906. He said in his book that there were a few Symbolic Logic which were different and he explained Boole's Method with Venn's Process. Salih Zeki commented about Symbolic Logic as a necessary and supported it.

Key Words: Logic, Symbolic Logic, Ali Sedad, Salih Zeki.

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Correspondence in the First and Second Order Semantics for Modal Logic

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ABSTRACT

Modal formulae express monadic second-order properties on Kripke frames. In many cases modal formulas correspond to first-order formulas computed by influential algorithms. The first research of this area was “Correspondence and completeness in the first and second order semantics for modal logic” which was written by H. Sahlqvist in 1973. He defined a class of modal formulas which are determined first order conditions on Kripke frames and those conditions can be effectively computed from the modal formulas.

Sometimes a first order equivalent of a modal formula doesn’t exist. In some cases a modal formulae can be correspond to a second-order formulae. Under the circumstances Sahlqvist technique loses its effectiveness. Some algorithms and techniques have been developed for computing a first-order or a second-order equivalent of a modal formula. Several algorithms and techniques are introduced but one of them come forward in these algorithms which is SQEMA, works directly on modal formulae.

Our aim is present differences between Sahlqvist technique and SQEMA algorithm. In this paper consist of four chapter. Chapter one starts with basic definitions and notions of proposition logic and modal logic. In the chapter two, class of Sahlqvist formulae and Sahlqvist theorem are introduced. Then the effectiveness of Sahlqvist technique is explained on modal formulae. In the chapter three, basic definitions and notions of SQEMA algorithm are introduced which are used process of this algorithm. The steps of SQEMA algorithm are examined in detail and are explained with examples. In the end of this chapter, the completeness of SQEMA algorithm is proved with respect to class of Sahlqvist algorithm.

In the chapter four, Sahlqvist technique and SQEMA algorithms are compared with respect to their effectiveness.

Key Words: Modal formuae, Sahlqvist technique, SQEMA algorithm.

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The Historical Course Of Symbolic Logic

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ABSTRACT

Logic is the one of sciences which took place in human life from ancient times. In this work, we aim to explain courses of Logic and Symbolic Logic from first time to present. If we talk on this subject briefly, preparations for setting Logic as a system were done before Aristo and an accumulation occurred. Aristo transformed the reasoning to a discipline and systematized the Logic. He accepted as a base the deduction from kinds of reasoning and founded a basic of Logic species known as Classical Logic today.

Logic studies in Islamic Culture World started with translating Aristo's books to Arabic in Ihvan-ı Safa in Abbasier Period. In IX. And X. Centuries, comments and corrections were done on Greece Logic works. Ebu Bekir er-Razi, Farabi, Ibn Sina and Ibn Rüşd were prominent commenters in this point. The first and famous logic worker who changed this logic according to own culturel goals and added an original sections was Farabi.

Works which were written according to tradition of Ibn Sina and Farabi were studied as Logic Lessons in schools of Ottoman Empire. There were Isa Guci, eş-Şemsiye, Şerfiu'l-Metali' in top of these works. Logic researchers who were foremost were Esirüddin el-Ebheri, Necmeddin Ali b. Ömer el- Katibi, Muslihiddin Efendi, Yanyalı Esad Efendi, İsmail Gelenbevi. Besides Ahmet Cevdet Paşa and his son Ali Sedad were prominent names of Logic History's last period in The Tanzimat Period.

In the New Age, new methods were studied by Bacon and Descartes on account of Naturel Sciences' development. Especially Bacon accepted as a base the induction method opposed to the reduction method. In here, the most important development was works of Symbolic Logic which started in second half of XIX. Century. With this Logic, abstracting The Logic from content and using a mathematichal language were wanted.

Gottlob Frege and after him Bertrand Russel and Alfred North Whitehead with their Principia Mathematica Book which was published in 1916-1917 contributed to being systematized of Symbolic Logic. Kurt Gödel published " Deficiency Theory" in 1933. He demonstrated Logic couldn't be reduced in Math completely with this theory.

There were two values as 1 and 0 in Symbolic Logic. The idea of interval values' being was source for Fuzzy Logic. The development of Fuzzy Logic continues and using in technology is increasing today.

Key Words: Logic, Symbolic Logic, Math.

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MATHEMATICS EDUCATION

5E Learning Model In Mathematics Teaching: A Sample Lesson Plan

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ABSTRACT

The aim of the study is to design a sample lesson plan for the 5E Learning Model based on the constructivist approach in the teaching of multiplication-division processing in natural numbers in the 4th grade elementary school mathematics course. The 5E learning model is important in that it is a constructive model that allows a new concept to be learned or a deep understanding of a known concept (Martin, 2006). While the student is creating a new concept through 5E learning model, he is getting new information from his own knowledge. Each of the 5 E's describes a phase of learning. It is called 5E learning model because each phase begins with the letter "E": Engage, Explore, Explain, Elaborate, and Evaluate.

The study was carried out with 22 students in a primary school in Samsun. In the study, a lesson plan, which was based on the stages of 5E Learning Model, was designed in accordance with the objective “Student will be able to multiply two natural numbers up to five-digit number.”. Intended for students to discover knowledge, the 5E lesson plan included some activities; Musical Chairs, Discovery, Explaining, Multiplication is Easier Now, Time to Think and Operation Skills. As a result of these activities, students were assessed in this process considering the level of curiosity, how they discovered information, whether they used mathematical language, whether they transferred their knowledge to new situations, and whether they used the acquired knowledge and skills.

As a result of the findings, it was observed that the students entertained while learning through the 5E learning model, they discovered the knowledge, they were able to comprehend where the information came from and be aware of the necessity of these information, and they learn maths by doing and experience.

Key words: 5E Learning Model, Maths, Permanence

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6th Grade Students' Construction Processes of Common Multiple Concept

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ABSTRACT

The purpose of the study is to investigate construction process of common multiple concept. In this qualitative study, participants are 6th grade students in a middle school in Samsun. Two classes including 60 students were chosen and these students were grouped heterogeneously in their classes. Teaching process was designed with Realistic Mathematics Education (RME) that allows students do mathematics like mathematicians as a human activity (Freudenthal, 1968). To achieve this, students were asked problems that were constructed according to teaching and learning principles of RME (Gravemeijer and Terwel, 2000; Van den Heuvel-Panhuizen, 2000). According to data which was acquired from observation and group study papers during teaching process, seven students were chosen for clinical interviews. All the process was recorded with camera and voice recorder. APOS theoretical framework (Arnon, Cottrill, Dubinsky, Oktaç, Fuentes, Trigueros and Weller, 2014) was used to investigate these students' construction processes of this concept. After the content analysis of data (Yıldırım and Şimşek, 2006), the findings demonstrate that one student's schema was in action stage. Also, she confused the concepts of common multiple and common divisor. Other students constructed the concept passing through process and object stages. While some of them found common multiple(s) by using repetitive addition or multiplication, some used prime factorization method (Davies, 1980). Finally, it was identified that students who conceptualized common multiple properly could choose least common multiple and could say its meaning.

Key Words: Concept construction, common multiple, realistic mathematics education, APOS

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A Recognition Algorithm of Some Harmonic Chart Patterns in R:Trading Strategies on Financial Markets

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ABSTRACT

There has been always a contradiction between regular and professional investors on the utility of technical analysis to predict the future prices of any asset in any financial markets. The so called professionalist ones having large amount of money are favour of fundamental analysis, while the regular ones seek some chart patterns by studying the past movements of stock prices. Such patterns are not so deterministic to take positions; however, they work so many times since a mass action by people gets place either on buying or selling an asset. This kind of action is non-negligible. For this reason we provided an automated way of recognising such patterns as harmonic gartley or butterfly pattern using the R programing environment. To check the functionality of the code script we considered the price movements of Euro-Usd pair on daily chart between the years 2000 and 2018. All in all, the algorithm provided good results to detect these patterns with some negligible errors of which the future works are dedicated to handling.

Key Words: Economics, Chart Patterns, Programming.

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A Structural Equation Model for the Beliefs of Mathematical Connection and Problem Solving of High School Students

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ABSTRACT

The purpose of this study is to examine the relationship between high school students' mathematical connection (MC), mathematical problem solving (MPS) beliefs and some personal variables with a structural model. The study group consists of 378 high school students studying in grades 9, 10, 11 and 12. Personal information form, mathematical connection self-efficacy scale (Özgen & Bindak, 2018) and problem solving belief scale (Kayan, 2007) were used as data collection tools in the study.

MC and MPS beliefs are implicit variables and are considered as endogenous variables in the model. The MC implicit variable is defined by 5 sub-dimensions and The MPS belief implicit variable is determined by 15 items. Other variables are measurement variables and related information are as follows. Mathematics course score (Mathscore); the mathematics score of a student is an ordinal variable that can be a value between 1-5. Gender (male); dummy variable and the reference group is girls. MKFG; how important are different representations of mathematical concepts (algebraic, verbal, geometric, analytic, vector, etc.) to you? response given to the question, an ordinal variable that can have a value between 1-5. GYMK; Do you take advantage of the lessons learned in mathematics in everyday life? response given to the question and is scored between 1 (no) and 5 (always). FDMK; Do you benefit from concepts learned in mathematics lessons in different disciplines? The response to the question is scored between 1 (no) and 5 (always).

It was decided that the data fit of the tested model is acceptable at X^2/sd = 622.91/273=2.28) and RMSEA =.058 (Bayram, 2013). The path coefficients obtained in the structural equation model can be interpreted as the non-standardized regression coefficient of the related independent variable on the dependent variable

(Bayram et al., 2012). When the path coefficients are examined, it is seen that the coefficients between FDMK-> MC and Mathscore-> MC are significant at 0.05 level and all other coefficients are significant at 0.01 level.

According to the findings, MKFG has a positive effect on both MC and MPS beliefs. This shows that the students who see different representations of mathematical concepts as relatively more important have higher MC and MPS beliefs. It is expected that students with higher MC self-efficacy beliefs are more likely to use in real world which the concepts learned in mathematics lessons (GYMK) according to the model. The hypothesis that GYMK and FDMK levels influence MPS belief wasn't accepted in the model.

The mathematics course score of the student according to the accepted model positively affects the MC self-efficacy belief and the effect on the MPS belief wasn't significant. The gender of the student affects MPS belief negatively, but gender doesn't affect MS self-efficacy belief. Since reference group is girls, it can be said that girl students have higher MPS beliefs at a significantly higher level. Another result is that the MC self-efficacy beliefs of the learners affected the MPS belief positively.

Key Words: Belief, mathematical connection, problem solving, high school students

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A Study on Developing a Scale on Attitude Towards Using Mathematics

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ABSTRACT

In order to keep up with the constant changes in technology and other fields in today's world, there is an increasing need for smart, talented and productive individuals who can take part in mathematical practices, use mathematics efficiently in any situation, develop effective solutions to problems encountered in daily life and who have functional mathematics knowledge and skills. In parallel to this need, the current curriculums of countries are revised and measures are taken in order to make sure that the mathematics learned at school can be used in the real life. In updated curriculums, mathematics-using skills are highlighted (NCTM, 2000; p.4; MoNE, 2018, p.4) and daily-life problems aimed at supporting the development of these skills are frequently given place both while teaching mathematical concepts and practicing them. Besides, it is striking that the number of studies investigating the skills and attitudes of students towards using mathematics in real life is pretty low in the literature. Thus, it has been attempted in the present study to develop a scale aiming to determine the attitude of students towards using mathematics in real life.

The scale is a 5-point likert scale and consists of 31 items. It was administered to 340 secondary-school students. Within the scope of the study, the size of the sample necessary for the factor analysis was investigated and the items having a correlation value less than .40 as a result of the item analyses carried out based on sub/superior group averages and the correlation were eliminated from the scale.

In order to find out the suitability of data for exploratory factor analysis, Kaiser-Meyer-Olkin (KMO) coefficient was calculated and Barlett Sphericity test was administered. As a result, the values obtained were found suitable. In order to find out the construct validity of The Scale on Attitude Towards Using Mathematics, the exploratory factor analysis was carried out and the factor loads were determined to

be minimum .30 (Büyüköztürk, 2006). As a result of the exploratory factor analysis, it was discovered that the items in the scale were gathered under 3 factors having an eigenvalue of greater than 1.

The Cronbach Alpha coefficient was calculated for the sub-dimensions of the scale and the overall reliability. Also, the confirmatory factor analysis was carried out to test the accuracy of the construct introduced with the exploratory factor analysis. In conclusion, it was determined that the scale was applicable in studies to be carried out with students, based on the validity and reliability measures.

Key Words: Using mathematics, attitude, scale development

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Academic Participation View Through Mathematical Literacy Problems

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ABSTRACT

Eliminating the disconnection between school mathematics and real life remains one of the fundamental challenges of mathematics education [1]. Many documents highlight the importance of working with actual data in primary school mathematics courses to eliminate this disconnection [2], [3], [4]. This increases the importance of mathematical literacy (ML), which refers to capacity to use mathematical knowledge acquired in school in real life and makes development of mathematical literacy one of up-to-date goals. Equipping individuals with mathematical literacy is a goal, to organize works for reaching this goal is a need. Assessments for identifying mathematical literacy levels are conducted by allowing students to use their mathematical competencies through addressing them contextual problems. Another need is the lack of academic participation in mathematics classes [5]. Academic participation is an effort to learn the knowledge and skills that are desired to be acquired in the learning process [6]. Academic participation has three dimensions: cognitive, behavioral, and affective [7]. Methods of measuring academic participation are listed in the literature as student self-report, experience sampling, teacher ratings of students, interviews and observations [8].

In this study where the mixed approach was used, 25 sixth grade students were employed. The application was completed in 12 weeks (2 hours per week). The ML problems that are appropriate to the curriculum issues have been studied. Study data were collected through pre-post tests, student participation observation form, interviews and student diary. The student participation observation form was formed by the researchers by examining the scales and student participation frameworks used in the literature. Positive effects on the student participation of ML education were determined with the data collected with the observation form and other data

collection tools. It has been seen that students are beginning to talk to others about new things they learn in school. They seemed to insist on solving the problem when they encountered difficult problems.

Key Words: Mathematical literacy, academic participation, student participation observation form.

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An Analysis of Studies Investigating Mathematical Thinking and Its Stages

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ABSTRACT

An individual will not only learn how to carry out operations in the process of learning mathematics, but also gain the skill of thinking (Tall, 2006). In mathematics, this way of thinking is named "mathematical thinking" (Yıldız, 2016). Mathematical thinking is not peculiar to mathematicians only; it is a way of thinking that needs to be used by individuals from every area today (Alkan & Bukova-Güzel, 2005). Mathematical thinking has a number of components. These components primarily include specializing, generalizing, conjecturing and proving (Arslan & Yıldız, 2010). In this review, the studies investigating mathematical thinking and its stages (specializing, generalizing, conjecturing and proving) were analyzed using the meta-synthesis method. The review included 82 Turkish studies that were published between 2000-2017 and selected via purposeful sampling method. Giresun University Library, National Thesis Center of the Council of Higher Education, TUBITAK ULAKBIM Dergipark and Google Scholar databases were used in the review. Each of the studies was subjected to a content analysis and examined within the context of "the years when the studies were conducted", "gender, title, number of researchers and the universities where they work", "purposes of researchers", "study types and methods", "sample, sampling techniques, sample sizes", "data collection tools and numbers", "data analysis methods and numbers", "statistical package softwares being used", "statistical analysis methods", "validity-reliability measures", "results and recommendations of studies". The findings acquired were indicated using diagrams and tables. As a consequence, it was determined that majority of studies being analyzed "had been conducted by women, assistant professors and researchers of Dokuz Eylul University in 2013" and "were qualitative and case studies with one author". In addition, it was determined that in majority of these studies, "pre-service teachers, criterion sampling, interviews, two data collection tools, SPSS software, frequency or percentage values, a data analysis method, descriptive and content analyses were used"; "the sample size was larger than 100" and "expert opinions were received as validity-reliability measures". Additionally, it was determined that majority of studies being analyzed had been conducted for the purpose of examining the mathematical thinking skills of academicians, teachers, pre-service teachers, secondary education or junior high school students within the context of the stages of specializing, generalizing, conjecturing and proving and it was observed that proving skills of some participants were lower in these studies. Finally, the studies mainly emphasize the necessity of conducting studies regarding raising an awareness by using different proof methods in lessons, as well as the skills and levels of reasoning and proving in every stage of education. It is believed that the present review will contribute to future studies for seeing the deficiencies in the area and generating solutions.

Keywords: Mathematics teaching, mathematical thinking, meta-synthesis method, content analysis.

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An Analysis of the Effect of Critical Thinking Based Applications on Students' Mathematical Achievement and Attitudes

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ABSTRACT

In order to exist in today's information and technology societies, people must have certain skills. Critical thinking is among the most important of them. Thinking and critical thinking skills constitute one of the key building blocks in the lifelong learning process of people and societies.

The purpose of this study is to investigate the effects of critical thinking-based practices on students' mathematical achievements and attitudes. The study was a semi-experimental study with a control group pretest-posttest model. The independent variable that is affected on the experimental group is the critical thinking-based applications. In the control group, traditional applications were made. The study was carried out on 100 (50 experimental and 50 control) students studying in the 6th grade secondary school in the district of Kurtalan in Siirt province during the first semester of 2017-2018 academic year.

To collect the data of the study the 20-item success test developed by the researcher and the mathematics attitude scale developed by İnan (2007) has been rearranged and applied. The obtained data were analysed using the Spss 18.0 packet program. t-test analyses were used for dependent and independent groups to assess the results of the pre- and post-test tests of the experimental and control groups, and findings were made accordingly. Significant difference was obtained in favour of the experimental group when there was no significant difference according to the sex variable. At interviews with school teachers in the practice school, according to teacher's opinion critical thinking has increased confidence and mathematical achievements and attitude of students this idea is supported by the results of this research.

Key words: Critical thinking, success, attitude.

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An Analysis of the Ratio and Proportion Problems in Middle School Textbooks in the Context of Conceptual and Procedural Knowledge

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ABSTRACT

Mathematical knowledge can be explained into two distinct elements: Knowledge about 'how' (skill or knowledge) and knowledge about 'why' (understanding) [1]. Most researchers have defined knowledge about 'how' as procedural knowledge and knowledge about 'why' as conceptual knowledge [2]. Lauritzen [1] revealed that both knowledge are very essential need for students to be competence in mathematics.

Mathematical reasoning is another indispensable component of the mathematics learning and teaching process. Proportional reasoning skills have an important place in mathematical reasoning [3]. Proportional reasoning has an important function in establishing relations between concepts. For this reason, proportional reasoning is considered to be one of the main ideas that constitute the main subject of mathematics curriculum [4]. The development of proportional reasoning is recognized as one of the objectives of the elementary mathematics curriculum in the standards of the NCTM [3].

The updated mathematics curriculum includes the ratio and proportion subjects in our country, and hopes to develop the students' proportional reasoning skills [5]. Furthermore, the mathematics curriculum is student-centered and has a perspective that emphasizes conceptual meaning [5]. Therefore, textbooks are important teaching tools that directly reach teachers and students to teach the subjects, to get the program's focused reasoning skills. In this context, it was aimed to examination of ratio-proportion problems in middle school mathematics textbooks in terms of conceptual and procedural knowledge in the study,

The present study was a qualitative study. The document analysis technique was used to analyse the data. The activities and problems of the ratio-proportion in

the sixth and seventh grade textbooks of the middle school in 2017-2018 academic year were analyzed by using the literatüre [6, 7, 8] in terms of the qualities that characterize conceptual and procedural knowledge. In the study, the descriptive statistics were used by calculating the frequency and percentage ratios according to the criteria that characterize the conceptual and procedural knowledge.

According to the findings of this research, it has been found that the percentages of the ratio problems used in the sixth grade textbook were very close to each other in terms of conceptual and procedural knowledge characteristics. It was found that the problems with the conceptual knowledge character were found to be higher than the problems with the procedural knowledge character in the seventh grade textbook. Furthermore, it was seen that conceptual problems were included in the beginning of each related attainment in terms of ratio-proportion in the same class textbooks. From these results, the activities and problems to improve the students' conceptual understandings and the proportional reasoning skills should be given more in the textbooks.

Key Words: Conceptual knowledge, procedural knowledge, proportional reasoning.

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An Assessment Of Pre-Service Mathematics Teachers' Knowledge And Opinions About Misconceptions

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ABSTRACT

Concept is a general and abstract term, which represents the common features of objects and phenomenon being interpreted in human's mind [1, 2]. When concepts are not being able to be shaped enough in mind, when not associated with existing schemas and used apart from their scientific meanings, misconceptions emerge in individuals [3]. While interview, observations, open-ended and multiple choice tests are used in the studies that are conducted on misconceptions, it is also suggested to use achievement tests, worksheets, concept maps, diagnostic tree, structured grid, and concept cartoons. It is seen that the methods which are mostly used in order to eliminate the misconceptions are conceptual change texts, prediction-observation-explanation (POE) strategy, conceptual analysis, conceptual network, worksheets, drama, analogy, learning journals, and use of technology [4, 5, 6, 7, 8]. This study aims at evaluating the pre-service mathematics teachers' knowledge and opinions on misconceptions in secondary school mathematics. By knowing the pre-service teachers' knowledge and opinions on misconceptions in secondary school mathematics, "Misconceptions in Mathematics" lesson, which is taught in university, will take on an efficient dimension. In this study, 66 pre-service mathematics teachers were administered an achievement test and an assessment survey which include open–ended questions before and after the "Misconceptions in Mathematics" lesson. The results of the achievement test that was administered to pre-service teachers was categorised as "true, partly true, false and unanswered", and the frequency and percentages of the true, partly true, false and unanswered responses that students gave for the questions are presented. The data that were obtained from the assessment survey were analysed with the content analysis method. The responses

that pre-service students gave in the assessment survey were coded according to some certain rules. By using these codes, a systematic way in which some words are identified with some small contextual categories in the data was followed. As a result, it was concluded that pre-service mathematics teachers had knowledge about the misconceptions in secondary school mathematics, they removed misconceptions that were determined on some subjects, and they enriched the lesson with suggestions and solution ways that they developed. Some pre-service mathematics teachers still seem to be misconceptions about symmetry.

Key Words: Misconceptions, mathematics education, pre-service teachers

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An Evaluation Of 7th Grade Mathematics Textbook’s Data Handling Learning Domain: A Document Analysis

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ABSTRACT

The textbook is defined as printed or electronic literature prepared according to curriculum for use formal and informal education **(4)**. In addition to the ease of using textbooks, it is used functionally and continuously by teachers and students as it is accessible by every student **(3)**. Also, the mathematics textbooks play an important role in teaching and learning process **(2)**. It provide interpretation of the curriculum **(6,7)** and it acts as bridge between teachers and students. In other words, it provides an idea of what activities to use according to the level of the students, as well as being a guide on which topics to teach **(2)**. In order these perform these functions, it must be a material that respects the scientific principles and appeals to the student level in terms of language and expression, taking into account design principles and elements **(1)**. The data handling learning domain includes statistical situations related to statistical situations that the student has in their daily life. Therefore, this learning field is of significance in the sense that the student gains knowledge and skills that will be useful both for education and for life **(8)**. This study focused on 7th grade mathematics textbooks’ sections prepared for data handling domain which was taught as a textbook in MoNE schools in the 2017-2018 education period. It was evaluated in terms of preparatory studies, explanations, activities, tasks, assessment and evaluation. Also it was examined in terms of corresponding learning objectives. For this purpose, a document analysis method was adopted and content analysis was used **(5)**. The results of the research showed that the textbooks tasks are inadequate in supporting the statistical thinking of the students from time to time. When the appropriateness of the textbook to the relevant learning objectives is assessed, it is also observed that there are some deficiencies such as not using

technology effectively. Analyses of our research are ongoing. The results will be explained in detail during the presentation.

Key Words: mathematics textbooks, data handling, document analysis

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An Investigation Of Mathematics Contents Of Children Periodicals In Terms Of Problem Specifications

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ABSTRACT

The aim of this study is to analyze the mathematical contents of Bilim Çocuk and TSE Öncü Çocuk periodicals in terms of the physical characteristics of the magazines, the compatibility with the curriculum and the characteristics of the problems. Being qualitative in nature, document analysis is used to analyze the problems in the children periodicals. In the scope of the research, 202 issues of Bilim Çocuk, 74 issues of TSE Öncü Çocuk were inspected and the mathematical content pages were compiled. The data to be encoded are divided into three main groups. At the end of this process, 86 mathematical articles, 314 mathematical games and 674 problems, 1074 contents, in total, were determined.

According to the findings of the research, the mathematical content in the periodicals corresponds to 4% of the total pages. It was determined that more than half of the mathematical content in the magazines (63%) were in problem type, 29% were planned as play activities and only 8% mathematical articles. It is noted that the majority of the images used in the magazines (63%) are suitable for the content. When reviewing the contents of the periodicals, problem-solving skills seem to have come to the forefront of the skills involved. In addition, more than half of the mathematical content is devoted to numbers learning area. Of the mathematical concepts mentioned in the magazines, only 45 (such as palindromic numbers, sierpinski triangle, trepozoite, equilibrium theory, coding) are out of the curriculum of the year they are affiliated. It has been noticed that in children's magazines, the problems of daily life are given less than the problems not related to daily life. The distribution of routine and non-routine problems in children's periodicals is closer to each other, and routine problems are given more in the periodicals. When the cases of open or closed ended problems were examined, it was determined that most of the

problems in the magazines were closed ended questions (89%). It is seen that almost all of the problems (98.3%) in both BÇ and TSEÖÇ journals have adequate knowledge require for solving the problems. In the majority of the problems (67%) were in the form of completely mathematical context whilst an explanatory context has been preferred in about one-third of the problems. When the answer that should be given to the problems is examined, it is determined that more than half of the problems (52%) are problems requiring numerical answer. In the vast majority (84%) of the problems in the periodicals, solution can be achieved with multi-step operations, single-step solutions are required by 16% of the problems. In addition, problem solving abilities are found to be most needed requirement dimension.

Key Words: Children periodicals, problem specifications, mathematics education.

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An Investigation Of Middle School Mathematic Preservice Teachers' Lesson Plans In Terms Of Mathematical Process Skills In Teaching Practice

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ABSTRACT

The rapid changes in the world's developments, science and technology, the changing needs of the individual and the society, have inevitably affected the expectations of the individual. These changes are expected from the individuals within the skills; problem-solving, critical thinking, connection and communication (MEB, 2018). There are many studies on the mathematical process skills of different perspective in the literature. The relationship between middle school preservice mathematics teachers' views on reasoning skills (Işık and Öz, 2017), making connection skill were defined (Özgen, 2013) ant its relationship with problem solving were examined. The opinions of the primary class teachers' views about the skills in mathematics course (1-5) curriculum (Toptaş, 2010) were investigated.

The greatest task in bringing these skills to students falls into teachers who are practitioners of the program. The content of lesson plans that serve as guidelines for teachers this case, handling, application, and the stage of the mathematical skills to be acquired in these lesson plans is of great importance.

In this study, it was aimed to examine preservice teachers' lesson plans in terms of mathematical process skills in teaching practice. For this purpose, 90 lesson plans prepared by the 15 preservice teachers who were senior class students in middle school mathematics teacher education program in 2017-2018 academic year, in a state University in Ankara. In this study, document analysis that kind of the qualitative research methods is used. The data were analyzed using the descriptive analysis approach according to the criteria determined for the purpose of the research. In the research, two probing responses were sought: "Which mathematical

process skills were handled by preservice teachers in their lesson plans? "And ""How did preservice teachers handle the mathematical process skills in the lesson plans?"

As a result of the analysis of the data, it has been determined that some preservice teachers did not mention any skill in the lesson plans. Some of them mentioned the skills in the entrance part in their lesson plans and did not handle them again during the developmental part, and some of them did not mention the skills in the introduction part but also mentioned developmental one.

Problem solving, making communication, reasoning and making connection were the process skills that preservice teachers were handled in their lesson plans. In very few lesson plans these skills were handled all together and in some lesson plans none of them were taken part in. Some preservice teacher were aimed to develop these skills with some activities but they ignored them in process of teaching.

Keywords: Lesson plan, mathematical process skills, preservice teachers

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An Investigation Of The Process That 6th Grade Middle-School Students Construct Knowledge Of Direct Proportion And Inverse Proportion According To Rbc+C Model: A Teaching Experiment

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ABSTRACT

The aim of this research is to analyze the process that 6th grade middle-school students construct knowledge of direct proportion and inverse proportion according to RBC+C abstraction model. In the research, the detailed examination of the process that the students construct knowledge on the concept of direct and inverse proportion and the systematic explanation of the mentality of the individuals in this process are included. In this regard, the teaching experiment method based on the interpretive approach, which is included in qualitative research techniques, has been used in the study. The research has been conducted with two 6th grade students (one of them is girl and the other is boy) studying in a state school where students from the lower socio-economic level studied. The participants have been selected among the volunteer students who are thought to have high mathematical success, but who hasn’t learned the subject of direct and inverse proportion yet and has never studied with these two concepts before. The implementation process has been conducted as a group work lasting almost 33 minutes in order to provide these students to think sophisticated with the help of peer interaction and ensure them to utter their thoughts clearly. Also, all the study process has been recorded by the video recorder. In this study, there are four activities as a data collection tool, included direct and inverse proportion problems prepared by the researcher by analyzing learning outcomes of the Math Lesson Teaching Program (2013) and reviewing literature. The findings have been analyzed by the researcher and an expert qualitatively. In accordance with the findings obtained at the end of the study, it is founded that the participants *build and recognize times* relationship between

proportional quantities and *the table knowledge* by expressing the relationship between directly and inversely proportional quantities as proportional or multiplicative. It is stated that the participants have expressed the relationship between two directly proportional quantities as “linear” both of them increase, both of them decrease” while stating the relationship between two inversely proportional quantities as “contrast” negative” and that the participants constuct the knowledge of direct and inverse proportion. In addition,it has been determined that the participants have built and constructed the knowledge of proportionality constant in the applied activities and that they have reinforced the learned knowledge by using it in the related problem solutions. In the study, it was concluded that participants were quite successful in building prior knowledge about algebraic expressions. Moreover, the students have been observed that in the activity studies, the students are generally confident in themselves and are willing to solve their problems.It is recommended that the abstraction process of student groups with different success levels be examined in other studies.

Key Words: Middle-school students, abstraction, RBC

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An Investigation of Middle School Preservice Mathematics Teachers' Statistical Content Knowledge

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ABSTRACT

Mathematics educators and researchers highlighted that teachers' subject matter knowledge affects their instruction and is associated with students' achievement. That is, teachers must know the matematics they teach [2,4,7,8]. With the growing interest on developing statistically literate citizens, statistics is getting more attention in mathematics curriculum [3,5]. Accordingly, what teachers need to know for teaching statistics becomes an important issue among researchers [6]. It is also important to determine pre-service teachers' subject matter knowledge related to teaching statistics as it helps us improve teacher education programs. Even though statistics is one of the requisite course in the undergraduate programs, research has shown that most pre-service teachers are having difficulty with basic concepts [1,6]. For this reason it is aimed to investigate preservice middle school mathematics teachers' subject matter knowledge related to statistics. Participants are six senior preservice middle school mathematics teachers. Data were collected through 14 tasks and individual semi-structured interviews. Data were analyzed by means of content analysis. Findings showed that preservice teachers were able to determine measures of central tendency when a data set were given, whereas they struggled when the data were presented in a graph. Also it is observed that they had difficulties in representing data by using a variety of representations and formulating research questions. They can read stem and leaf plot but can't interpret. Besides they can interpret scatter plots. The findings of the study suggest that preservice teachers should be provided with opportunuties to engage with each aspects of statistical thinking in dept (i.e., formulating research questions, collecting, organizing, representing, analysing, and interpreting data) through meaningful and real life data.

Key Words: Preservice teachers, Statistical Content Knowledge

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An Investigation of the Mathematical Properties of Spirograph Models with GeoGebra

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ABSTRACT

In mathematics education, different representations of mathematical concepts help students to have deep understanding of essential mathematical concepts and their properties. The dynamic mathematics software GeoGebra allows students to examine different representations of concepts by building dynamic patterns. The patterns produced with GeoGebra may help students' understandings of different representations of a concept. In this regard, the purpose of this study is to develop spirograph models in the dynamic learning environment. These dynamic examples are presented to six mathematics teachers and thirteen university students. The university students construct the dynamic model of a spirograph with dynamic mathematics software GeoGebra in the collaborative learning environment. Their views about building dynamic spirograph models are examined in a descriptive manner. As a result, it can be stated that building dynamic model contributes to examine the mathematical properties of the patterns in the collaborative learning environment, and promotes team study in an enjoyable learning environment.

Key Words: Spirograph, dynamic model, GeoGebra.

An Iplementation of STEM Education in the Context Of Astronomy: Reflections from a Project On Modelling With Robots

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ABSTRACT

Debates about the introduction of engineering education to elementary and secondary students have popularized the STEM (Science, Technology, Engineering, and Mathematics) approach. Since the STEM approach integrates engineering with mathematics, science and technology, it gives children an interdisciplinary point of view and life skills while ensuring that information is brought into action [1]. It is important for students to design and develop materials related to robotic technologies to carry out applications involving technology and engineering disciplines [1,2]. In this study, we introduce the "Modeling with Robots" project that is designed as a practice of STEM education in the context of astronomy. In the project, middle school students were required designing a model that demonstrated the operation of the solar system, the magnitudes of planets, rotation speeds and other properties using the geometry software and robot kits. The study had 32 rising eight grade students who had completed the seventh grade. Activities were carried out at the Science and Art Center in Rize. The diaries of the participants were used for data collection. In addition, semi-structured interviews were conducted with five randomly selected students about their experiences in the project. The treatments took seven days. The theoretical background required for an effective formation of the model was practically given to students in the first five days of the treatments. The participants were instructed about robot construction, software and coding, web 2.0 tools and GeoGebra. At the end of the treatments, each group designed robots that represented one of the planets and students enabled their planet to rotate in its own

orbit on the platform taking into account the planet's volume and velocity that the robot would follow. The robotic solar system model was created by bringing the works of all groups together. Each group designed a platform to activate the solar system model effectively and to allow the planets to move in their orbit in a synchronized manner using the data obtained in the process of training. The results demonstrated that the participants received important learning outcomes in many aspects and this project could be an effective pedagogical model for STEM activities that can be carried out with robotic technology.

Key Words: STEM education, solar system, ronotic, numerical modeling.

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Analysis Of Middle School Mathematics Teaching Programs Implemented Between 1998-2013

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ABSTRACT

The aim of this study is to compare middle school mathematics curriculum implemented between 1998 and 2016 in accordance with general aim of mathematic teaching, teaching-learning process and subject contents.

This research is a qualitative one. The document analysing method was used to analyse middle school mathematics teaching programmes implemented in 1998, 2005 and 2013. In this study, mathematics programmes was examined consideringly with the general aims of mathematic education, the specifications of teachers and students, teaching and learning approaches, measurement and evaluating process, subject contents. At the end of the first coding, researchers calculated reliability coefficient as 86,7 % according to Miles and Huberman (1994) formule. Researchers came together and made meeting about the items that caused incompatibility and had an agreement on each items.

In research findings associated with the distrubitions of general purposes on skill domains, it was seen that, respectively, general purposes were mostly related with cognitive domain skills, then related with domain specific skills and finally related with affective skills. The general purpose expression about psychomotor skills was only mentioned in 1998 teaching programme. When the distrubitions general purposes related with cognitive skills were examined, it was seen that the general purposes related with reasoning were highest in 1998 and 2005 programmes, purposes related with knowledge and practicing skills were highest in 2013 programme. It was seen that the distrubitions of general purposes related with domain specific skills, were mostly related with associating sub-skills in 1998 program, and in 2005 and 2013 programmes they were mostly related with problem solving sub skills. The general purpose statement for the problem posing sub-skills

area was included only in the 1998 program. Teaching methods, techniques and strategies where the students are active and in center and measure and evaluation approaches where the result and process are evaluated with together have been used since 2005 year teaching programme. In the sixth and seventh grade curriculums, the mostly concentrate was on subject contents about “Numbers and Operations” and “Geometry and Measurement”, and in the eight grade curriculum the mostly concentrate was on subject contents about “Geometry and Measurement”. Whereas some contents that were included in 1998 year programme, they were took out in 2005 year programme, but they were again included in 2013 year programme.

Key Words: Middle school mathematics lesson teaching programme, middle school mathematics education, programme review, document analysing.

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Analysis Of Define- And Drawing-Skills Of Secondary School Students: Parallelogram Example

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ABSTRACT

Geometry, which is the subject area of shapes and objects, has an indispensable place in human life. There is geometry in every element created by people such as in science, in art, in architecture, in engineering (Van De Walle, 2001). Geometry, which has important functions such as helping students to better understand the world, is an integral part of mathematics. In addition, geometry contributes to students' critical thinking and problem-solving skills and helps in teaching other subjects of mathematics. However, geometry used intensely in the fields of science and art constitutes an important part of mathematics used in daily life (Baykul, 2002). At the beginning of the geometric concepts that we often use in daily life, are quadrilaterals. Geometry is an important branch of mathematics to teach. The study of geometry contributes to helping students develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof (Jones, 2002). Despite the great importance placed on geometry education included in the mathematics curriculum, much research shows that geometry perception levels of students are not at the expected level (Clements & Battissa, 1992; Carroll, 1998). The topic of quadrilaterals, which holds an important place in primary and secondary school mathematics program, are able to develop some mathematical skills such as defining, classifying geometric shapes, drawing, relational understanding, logical deduction, deductive and inductive thinking (MEB, 2013; 2015). Despite this importance, when the literature is examined, it is seen that the students have some difficulties with the quadrilaterals.

In this study, it is aimed to comparatively examine the students' ability to define and draw parallelogram for each class level. Case study is chosen as the methodology of this study and the working group of the study consists of 120 middle school students from a state middle school in Turkey. Two open ended questions are used to gather data. One of the questions is taken from the study of Fujita (2012) and other question is prepared by researches after examining relevant literature, mathematics curricula and textbooks. The document analysis method is used to analyze data. As a result of the research, it was seen that students at all class levels drawn prototype-parallelogram, and students at all class levels had difficulty in defining parallelograms. It has been determined that students at all grade levels cannot associate with the rhombus, a special form of parallelogram, and do not prefer it in their drawings.

Key Words: Mathematics education, Geometry, Parallelogram

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Analyze of Middle Grade Students' Number Sense

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ABSTRACT

Number sense is a growing concept in mathematics education. According to the standards of National Council of Teachers of Mathematics (NCTM) (2000), pupils gain some skills related to number sense from the preschool to the end of secondary education. With the acquisition of these skills, students are able to use what they learn in a way that is flexible and easy to use in real life conditions, by linking the mathematics they learn at school to the real life. According to Kayhan-Altay (2010), it is important for students with a good sense of numbers to understand the meanings of numbers well, to develop multi-directional relationships between numbers, to understand the magnitudes of numbers, to understand how the effects of their actions are on the numbers, to develop and use it. The study aimed to analyze the sense of number skills of middle school students in terms of gender, grade level, having a private room, age and number of family members. Sample of the study is 84 middle grade students from a public school in Kayseri. Convenient sampling methodology was used to select the school but random sampling methodology was used to select classes. 2 random classes from each grade level were selected. "The Number Sense Test" which was developed by Kayhan- Altay (2010) was administered to middle grade students to collect data. Descriptive statistics, independent sample *t* test, correlation, ANOVA and factor analysis was run to analyze data. The results revealed that middle grade students' number sense level was low. There was no significant gender, having a private room, number of family members difference on the number sense. There was significant grade level difference on the number sense in favor of eight grades. There is significant correlation between number sense test and grade level, age but no correlation between number sense and gender, number of family members. Factor analysis results indicated two factor structure in the number sense test. This study suggests that students need more emphasising skills of understanding and need to develop understanding of number sense.

Key Words: Number sense, middle grades.

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Analysing Mathematical Tasks In Lesson Plans Done By Prospective Mathematics Teachers

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ABSTRACT

Mathematical tasks are defined as a classroom activity which is done in order to get students attention on a conclude that there is a strong relationship between students learning and level of cognitive particular mathematical idea [1]. Mathematical tasks are all the classroom activities which provide students' learning [2]. Stein and Lane [3] examined mathematical task in curriculum materials and used in the classroom. They conducted observation in four different school to understand relation between cognitive level of mathematical task and students' learning. They demand. In 2000, they developed The Task Analysis Guide in order to determine cognitive level of mathematical tasks. The Task Analysis Guide include four categories which are low-level memorization tasks, low-level procedure without connection tasks, high level procedure with connection task and high-level doing mathematics task.

In the forth-year of Elementary Mathematics Education Program, prospective teachers have to take Practice Teaching in Elementary Education course. As a requirement of this course, prospective teachers go to middle schools and lecture. For their lecture, they need to prepare lesson plan. 15 prospective teachers prepare 150 lesson plans which include 5th to 8th grade spring semester mathematical subjects. For this study, these lesson plans were used as data source.

This study is based on document analysis, a method of content analysis [4] were used. Three researchers analysed the task as following task analysis framework developed by Stein and Smith [1]. Memorization tasks are coded as Low-M, procedure without connection tasks are coded as Low-P, procedure with connection tasks are coded as High-P and doing mathematics tasks are coded as high-DM.

Analysis of the data showed that prospective teachers used mostly low level cognitive demanding task in their lesson plans. There were no doing mathematics taking place in the lesson plan of the prospective teachers.

Key Words: Mathematical tasks, level of cognitive demand, lesson plan

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Assessment of mathematical storytelling activities in the context of 7th grade students' abilities to drawing graphs

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ABSTRACT

Graphs are often used in everyday life and other areas as well as in mathematics lessons [1, 2, 3]. It is very important to be able to benefit from graphs in gaining accurate information on many aspects of life. Especially, with data organization and interpretation depending on gradually growing importance of scientific research skills, graph interpretation competencies have become an important factor in terms of understanding today's world and graph interpretation competencies [4]. The power of graphical interpretation and creation helps students conceptually learn mathematics [5]. The contribution of graphics to meaningful learning of mathematics stems from the fact that they offer students the opportunity to switch between different representations [6]. The ability of reading and understanding graphs that is aimed at gaining basic mathematics can be valuable as students can understand what they have learned and adapt it to different situations. In this context; it is aimed to evaluate mathematical storytelling activities in the context of 7th grade students' abilities to drawing graphs in the study. So the study was organised using special case studies from qualitative research methods. The participants of the study are 22 students in the 7th grade who are studying at a public school in Sakarya in the academic year of 2017-2018. The data have obtained from the tasks about students' drawing graphs and mathematical storytelling activities that are prepared by the researchers. Students' responses on drawing and narration activities are examined with details and are analysed with descriptive analysis. According to the findings of the research; it is found that students' drawings are enough substantially, although they have some mistakes. The themes obtained when student drawings are examined; "table and drawing graph is correct, breakpoint is complete", "table and drawing graph is correct, breakpoint is not specified", "table is

correct, drawing graph is missing" and "table is incorrect, no breakpoint is specified." Also, when the mathematical storytelling activity is evaluated, it has been determined that students are not able to write scripts and storytelling difficulties by associating them with graphical drawings and associating the obtained data and adapting them to different situations. When the stories about the graphs of the linear equations which the students made drawings are examined, the themes obtained are; "coherent with the graph", "there is not enough meaning with the graph", "contradictory to the graph", "using one of the graphical data" and "no answer." Students encountered such events for the first time, so they had difficulties in using the information they obtained. This situation may be the cause of the obtained results. Therefore; learning experiences can be provided that enable students to adapt their skills to different situations.

Key Words: Mathematics teaching, geometry, graphs, mathematical narration.

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Assessment Of Preservice Mathematics Teachers' Opinions On Teaching Practice Within The Scope Of Teaching Practices Expressions

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ABSTRACT

Teachers are among the most important factors affecting student achievement [1, 2]. According to Fullan (2007), effective use of classes and schools depends on teacher education [3]. Teacher candidacy, an important emotional and personally difficult experience to be a teacher, is the first step in this process [4]. So, it is very important for the preservice mathematics teachers to improve the efficiency of the teaching practice course in order to prepare the profession for the application of the profession they are going to do and to use the acquired experiences in the first years of the profession and to increase the measures by eliminating the deficiencies. Therefore; in the research, it is aimed to evaluate the opinions of the preservice mathematics teachers about teaching practice within the scope of application lectures. In the study, which was designed as a case study from qualitative research methods, the study group constituted 5 preservice mathematics teacher who attended a college course in the fall semester and continued to the same practice school. In order to determine the opinions of preservice teachers about teaching practice, a questionnaire consisting of 6 open-ended questionnaires developed by the researchers was applied and the observation forms held during the lectures of the preservice teachers were used. In this context, the responses obtained from the opinion forms and the statements contained in the observation forms constitute the data of the research. In the data analysis, content analysis was used, and where necessary, a citation was given. According to the findings of the research; preservice mathematics teachers evaluate the teaching practice course according to the themes of student approach, side by side with students, experience and professional excitement. Also; preservice mathematics teachers' views on teaching practice and

lectures are consistent. In this experience, where teacher candidates find their field of application outside the theoretical courses, the impression that they have in their practice schools and the deficiencies encountered in practice by evaluating their opinions will be determined and necessary proposals will be made for further applications.

Key Words: Mathematics teaching, teaching practice, preservice mathematics teacher.

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Cognitive and Contextual Structure of Prospective Teachers' Mathematical Problems in the Context of Problem Posing

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ABSTRACT

Problem posing activities give many opportunities to prospective teachers such as developing mathematical knowledge, conceptual understanding, critical thinking ability, creativity, and understanding students' mathematical thinking [1, 2, 3]. However, there is limited number of studies deepened on the nature and development of prospective teachers' (PTs) problem posing abilities. For this reason, we aimed to investigate cognitive and contextual structure of prospective middle school mathematics teachers' problems in the problem posing context. Furthermore, we examined what PTs noticed about their own problems in whole-class discussion.

This qualitative study was conducted with twenty five PTs in the scope of school practice course. All participants posed contextual problems by using all data in problem posing task (see Appendix-1) within a week. In their reports, they also wrote the aim and the solutions of the problems, and predictions about how students' responses. Furthermore, we conducted a group discussion with PTs about their problems. For the data analysis, cognitive structure of the problems was analysed based on three cognitive domains of TIMSS 2019 Mathematics Framework (*knowing, applying, reasoning*). Problems grouped in *knowing* domain covers the facts, concepts, and procedures students need to know. Problems grouped in *applying* domain includes the ability to apply knowledge and conceptual understanding to solve questions. Finally, problems grouped in *reasoning* domain includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations [4, p.24]. Contextual structure of the problems was examined from two perspectives. In first, we analysed

how PTs used “the data given in the task” when posing problems. Secondly, we focused on what kinds of contexts PTs used in problem posing activity.

The results indicated that PTs generally used “table as is” or “all data in table verbally” in their problems (see Table 1). From the context types, PTs used similar contexts in their problems such as money saving/spending, diet/sport and the number of test/questions. This showed PTs’ limited contextual knowledge in posing problems. Another important result is that half of PTs prepared their problems by using improper wording.

Table 1. Contextual structure of the problems

Contextual structure of problem		n	%
Use of table in problem	Using table as is	10	40%
	Using all the data in table verbally	8	32%
	Partial use of the data in table	7	28%
Context types of problems	Money saving/spending	12	48%
	Diet/Sport	5	20%
	Solving question/test or reading book	5	20%
	Others	3	12%
Wording of the problems	Proper	13	52%
	Improper	12	48%

Cognitive structure of the problems in Table 2 revealed that 80% of PTs prepared their problems in “*applying”* level and they focused on problems including *multiple-step* solutions. They believed that “multiple-step problems are more difficult than other problems.”

Table 2. Cognitive structure of the problems

Cognitive structure of problem	Problem solving steps		Total
	Single-step	Multiple-step	
Knowing	3 (12%)	0	3 (12%)
Applying	4 (16%)	16 (64%)	20 (80%)
Reasoning	0	2 (8%)	2 (8%)
Total	7 (28%)	18 (72%)	25 (100%)

Key Words: problem posing, contextual problems, prospective middle school mathematics teachers

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APPENDIX-1

Hafta	Ayşe	Rüya	Selim
0	210	158	113
2	202	154	108
4	196	150	105

It is adapted from van de Walle, Karp, & Bay-William (2012, p.353)

Figure 1. Problem Posing Task

.

Determination of Mathematical Competences on Realistic Mathematics Problems

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ABSTRACT

Problem solving is one of the four main purposes of teaching mathematics. As well as mathematical literacy has wide coverage in the literature, the concept of problem has begun to focus on the contextual problem. Mathematical competences is defined as cognition process, which are expected to become active in the process of mathematical intervention to contextual problems [1]. Internationally applied PISA also examines and describes the development of mathematical literacy by focusing on mathematical competencies [2]. The mathematical competencies, which put forward by Niss (2003) [3] and, are the focal points in various researches ([1]; [4]), are listed as follows: Communication, mathematising, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, using mathematical tools. In this study, it is aimed to determine the mathematical comptency(ies) which are difficult to be activated in the problem solving process. In the study conducted as a survey study, 11 contextual problems requiring these competences were applied to 128 eighth grade students. The solutions and the competencies that were lacking in the process were evaluated and analyzed through the rubriks. Accordingly, it has been determined that the students can not demonstrate or weakly showed these competencies in problems that require mathematising, reasoning and argumentation, and representations competencies. The growing awareness of the development of these competencies in the preparation of students for the 21st century has created an environment for the development of basic mathematical literacy concepts [5]. It is thought that the results of the study can be utilized in the organization of training programs and these results can guide teachers in planning the training process. It is believed that the results of

the study can be utilized in the organization of curriculum and, these results can guide teachers in planning the teaching process.

Key Words: Mathematical comptency(ies), problem solving, contextual problems, reasoning and argument.

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Determination of Secondary School Students' Attitudes Towards Mathematics and Their Mathematical Achievements

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ABSTRACT

Today's world expects from mathematics educators to guide student who can produce effective solutions to real problem situations, use mathematics effectively in their lives, be aware of the close relationship of mathematics to the real world and thus enjoy mathematics rather than fear from it [1].

From primary education to university, mathematics lessons are one of the lessons that students are most withdrawn or afraid of. The formation of this fear the positive and negative attitudes that especially occur in the elementary years affect the students. The impact of attitude on achievement in mathematics teaching-learning processes is very important. Attitudes and achievements influence each other [2]. Therefore, it is fundamental for students to be positive towards to mathematics for effective and efficient mathematics education [3].

Attributes such as interest, attitude, value, and achievement that are subject to measurement evaluation may change over time. For this reason, it is essential to use measurements that take into account changes in the process, rather than measuring them at once [4].

In this research, students' attitudes towards mathematics and their mathematical achievements were measured and the relationship between them were investigated. Considering the studies in the literature, having a large sample size is one of the importance of this research.

Relational search model which aimed at determining the presence and / or degree of exchange between two or more variables was used [5]. In this context, 593 middle school students (5., 6., 7. and 8th grade) who study in a public school in Ankara have been applied "Attitude Scale towards Mathematics Scale" developed by Önal (2013) [6]. For students' mathematical achievements, mathematics marks of the

first term of 2017-2018 academic year taken as reference. The data collected have been analyzed by SPSS programme.

As a result of the research, it was determined that middle school students' attitudes towards to mathematics and mathematics achievement differed significantly according to grade level. Also, The Spearman Row Differences Correlation Coefficient calculated to determine the relationship between mathematics achievement and mathematics attitude scores of middle school students showed a positive and significant relationship ($r = 0.53$, $p<0.05$).

Keywords: success, mathematics success, attitude, attitude towards mathematics

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Developing a Checklist in Question Design for Mathematical Literacy and an Application

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ABSTRACT

In the context of measuring mathematical literacy (ML) in PISA studies, we encounter concepts such as content, process, context and level (OECD, 2013). The ML questions used in PISA studies are undergoing various scientific processes, and the examination of various experts in the field and pilot applications are the final result. However, there appears to be a need for the design and implementation of effective ML questions in the classes of teachers responsible for developing and measuring ML of learners. Because it is very difficult for teachers to design a large-scale questionnaire like PISA work. In addition, the compliance of the questions given in the books with the ML and their suitability to the students are seen as other problems. For these reasons, there is a need for a more applicable and effective approach for teachers to have knowledge, skills and experience in question design for ML.

In some studies, it has been determined that questions similar to ML questions applied in PISA have been developed and applied (Demir and Altun, 2018, Gürbüz, 2014, Özgen, 2017). In addition, two creative tools such as ML planning framework and application checklist have been developed (Hill, Friedland & McMillen, 2016) and a 6-factor structure for designing the ML questionnaire (Altun and Bozkurt, 2017). In the studies conducted, it was tried to develop ML questions and activities with methods such as expert opinion taking, pre-application, survey. However, it has been determined that there is no tools of measuring the quality and effectiveness of ML questions developed in these studies. The purpose of this research is to develop and validate a valid and reliable checklist that can be used in question design for the development and measurement of the ML.

In the development phase of the checklist developed for use in ML question design; the process was examined in the form of reviewing relevant literature, getting

expert opinion, getting opinions from prospective teachers, drafting form, getting opinions from teachers and experts, final form and application. It can be said that the checklist which can be used for the ML question design developed and piloted in this study is a valid and reliable measurement tool. Particularly, it is considered that mathematics educators will be a guiding tool in the design process of ML questions and will make the question design process more effective. Furthermore, it is inevitable that the measurement tools developed with this checklist will have more effective and qualified features of ML questions. It is also thought that it may be beneficial to present valid and reliable information to the researcher who studies and investigates ML question design processes.

Key Words: Checklist, mathematics literacy, question design

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Developing Achievement Test to Evaluate the Success of 7th Grade Students at Secondary School for the Topic of 'Rational Numbers'

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ABSTRACT

The aim of the survey is to develop an achievement test to evaluate the success of the 7th grade students at secondary school for the topic of 'Rational Numbers'. While developing the achievement test, the achievement test developing steps of Atılgan (2016) were followed. These steps are respectively; determining the aim of the test point usage, forming the table of specifications, writing the tentative items, revising the tentative items, preparing the tentative test forms, implementing the tentative test, selecting the items by doing test specimen, determining of the ultimate test statistics. For this reason, primarily, the objective which will be used for the test scores has been determined. The main target of the achievement test is to detect what the students have learned and what they haven't reached. Thereafter, the crucial learning outcomes were determined by examining the learning outcomes of the secondary school 7th grade Math curriculum, and in the sequel, 33 tentative questions were written for crucial learning outcomes. With the intent of revising the tentative items and ensuring validity, an expert was consulted and the items were organized. Just after reorganizing the items, on 372 students implementation was done in the secondary schools that were chosen in the province of Samsun. On the data which were obtained out of the implementation, validity, credibility and item analysis were practiced. The data that were obtained from the implementations, were analyzed in FINESSE packaged software. As a result of analyses, the test items which had low distinctiveness and high item difficulty were removed from the test. In the end, the final test which had 20 items was formed. In the ultimate test, 0.45 for average difficulty, 0.81 for reliability were detected. These statistics show that the test is a valid and credible assessment instrument.

Keywords: Achievement test , Test Development, Rational Numbers.

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Development of the STEM Viewpoint Scale: Validity and Reliability Study

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ABSTRACT

STEM education has become widespread in recent years in Turkey. This training, which constitutes the integration of Science Technology Engineering and Mathematics fields is based on students making projects and engineering products using related fields. As STEM's nature blurred the lines between disciplines, it was thought that integration would be more harmonious with nature [1]. In this context, it seems important for students to understand the interdisciplinary relationship. The researches that have been made has shown that the knowledge and skills of teachers influence the learning of students. In this context it is important to determine the views of teachers who are STEM practitioners. This study was carried out in order to develop a valid and reliable measurement tool which will enable the teachers to reveal the perspective of Science Technology Engineering and Mathematics (FETEMM) approach. The data of the study were collected in electronic environment and 366 teachers participated in the study. 209 of the teachers are in mathematics, 157 are in the fields of science. During the scale development process, an item pool was created by taking advantage of the field writing. 49 items were selected from this material pool in the direction of expert opinions and the first version of the measurement tool was created. Validity and reliability analyses were conducted on the data collected in the study. An Attitudinal Factor Analysis (AFA) was performed to determine the factor structure of the scale. It has been determined that the sample size of the scale is sufficient as a result of the Kaiser Meyer Olkin (KMO) test to determine the adequacy of the sample to which the scale is applied. It was also found that the data set also provided the condition of multivariable normality (11709,70, p <.05). As a result of the analysis, 16 items explaining 63.19% of the total variance and a structure consisting of two factors were obtained. Cronbach Alpha internal consistency value was examined in the reliability analysis of the scale. The reliability of the whole scale was calculated as .76. When sub-dimensions are considered in context the reliability value for sub-dimension 1 was found to be .95, and for sub-dimension 2 was found to be .77. It has been determined that the calculated validity and reliability coefficients are within acceptable limits. Based on these results, it can be said that the point of view towards STEM is a measurement tool that produces valid and reliable results and that it can be used by teacher candidates to measure the point of view towards STEM

Key Words: STEM, middle school, scale

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Effect Of Teaching Creative Drama On Teaching Geometry

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ABSTRACT

In developmental process of the mathematical thinking geometry has an important role. In math, especially when it comes to geometry. It is seen as a complex and disliked area. It is not easy to make students like and explain the course while there are such prejudices in their minds. İn this research, it is aimed at using appropriate teaching technics to eliminate these prejudices from students minds and making student feel enthusiastic in learning this course. Whit such purposes, the research focused at teaching area of the triangles to 6th graders with creative drama technics and its effect on students' success.

This research was conducted in 2017-2018 education term with 6th graders of Valiler Middle school in Didim, Aydın and with 6th graders of Gökçek Middle school in Afyon as sample groups.

The samples were divided as control and experimental groups in this study both. The number of students in control and experimental groups in 50.

At the preparation process of the tests, ideas 3experts in the area were taken. The test contains 7 multiple choice and 3 open answered questions. The collected data were analyzed by SPSS programme. İn dependent and independent variable measurement, t- test was used. Analyses show that there is significant difference between control and experimental group. Based on this difference which supports control group positively, drama technic has more effect on increasing students' success while compared to activity in student's book.

Key Words: Creative drama, mathematics education, geometry**.**

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Effect On The Students' Spatial Thinking Of Vocational Areas

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ABSTRACT

To develop students in terms of physical, mental, moral, spiritual, social and cultural qualities, to prepare for the future by equipping them with the knowledge and skills that our time require, respecting democracy and human rights, enable students to gain creative and critical thinking skills is located among general aims of the mathematics curriculum for secondary education institutions[4]. In vocational and technical High Schools' plumbing and air-conditioning technology, electrical-electronic technologies and furniture and interior design are taught technical drawing. Knowledge and skills related to TSE standards and technical drawing rules, perspective views, openings and intermediate sections are given in technical drawing course[3]. Some of the behaviors studied in the literature spatially located in the technical drawing curriculum. If we look at the work done about spatial thinking, the number of studies[1][2] carried out by quantitative method is remarkable. When we look at literature, it is not found any study that examines spatial thinking of vocational high school students by qualitative method. The research examines the influence of vocational high school students on spatial thinking of their practice during vocational courses. The study is carried out with the qualitative research model. One of the approaches of this model, grounded theory, is used. Grounded theory is a method used to develop a theory based on data that is systematically collected and analyzed. The theory can be produced at first. In consequence of comparing of data with themselves and development may also occur later[5] . The data were collected through observation and interviews from the data collection techniques of the qualitative research method. Observations and interviews were taken in video and audio recording. Interviews were made with a student from the furniture department. That his applications of students in technical drawing and vocational lessons positively influences students' spatial thinking is reached the result.

Keywords: Spatial Thinkig, Vocational High School, Technical Drawing

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Examining The Views Of Class Teachers On Mathematics Teaching: A Phenomenological Study

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ABSTRACT

As the ability to gain behaviors related to mathematics forms the basis for applying mathematics, the role of mathematics is gradually increasing. It is known that gaining these behaviors continues from pre-school education to all levels of higher education, thus the increase in the importance of mathematics increases the importance of mathematics teaching [1]. With the foundation of the mathematics laid in elementary school, there are tasks for class teachers to make students truly love mathematics and help them understand it [2], [3]. For this reason, in this study, it is aimed to determine classroom teachers' views on teaching mathematics. This study, in which phenomenological pattern is used, has been realized with the participation of class teachers working in the schools affiliated to Erzurum province center. The data of the study which was carried out with a total of five teachers were collected with semi-structured interviews and analyzed descriptively. Each of the interviewed teachers stated that they felt math anxiety at some stage of their education life and that it was teacher-based. In addition, classroom teachers, who have emphasized that primary school students should learn in a concrete way and mostly by doing and experiencing, have mentioned that these possibilities are inadequate in village schools. They also pointed out that the family support is lacking and that the curriculum is noteworthy for students and families living in the city center, but that this is rarely achieved in village schools. The class teachers, who indicated that the mathematics courses they had attended during their undergraduate education were appropriate in theory but insufficient for implementation, have suggested that mathematics instruction courses should be more productive.

Key Words: Classroom teachers, teaching mathematics, phenomenology

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Examination of 8-Grade Students About Their Performance of Problem Posing: Data Processing Example

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ABSTRACT

Recently, in addition to ability of problem solving of students, ıt is attached importance to posing new problems by considering given situations or constituting new problems by altering the existing problems which states the creation of new ones (Zehir, 2013). Problem posing helps student to understand mathematics easier, it helps to develop their creativity and make possible their learning by their own efforts (Silver, 1997). In this context, the purpose of the research is to examine the eighth grade students about their performance of problem posing related data processing. Participants of this research are 8 eighth grade students that receive education from a state school in center of Elazığ during 2017-2018 education year. Selecting of participants made by using the easily accessible sampling method which is one of the purposeful sampling method. In the research, case study that is based on qualitative approach has been used. It has been used that data collection tool created by researcher called as Problem Posing Performance Determination Test (PPPDT).

In order to classify the problems in this test, Stoyanova and Ellerton's (1996) classification was taken into consideration. The problems were expressed in three categories as *non-structured (free problem posing) semi-structured* and *structured problem posing*. PPPDT consisting of totally 7 problems; 3 free, 1 structured, 3 semi-structured problem posing situations. More than one method has been used to collect data such as observation, interview and written documents of participants. In analyzing the data, qualitative data analysis techniques have been used. In the first step, the participants have been asked to pose problems about different situations on the data processing. For second step, the participants have been interviewed individually about their own answers for about 15-20 minutes. Additionally, it has

been observed the performances about data processing of participants in the classroom.

As a result of research, it has been observed that most of the students can pose mathematics problems that could be solved about data processing. Some other similar observations say that most of the students can pose problems that could be solved about data processing. In the interviews, nearly all students have said that during the posing process, they have had difficulties in order to make the main text of problems clear and understandable. Even a few students have posed problem that contain missing information to make the problem clear and understandable.

Key Words: Mathematics teaching, posing problems, data processing, student.

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Examining Mathematics Teacher's Expectations Regarding to Basic Mathematical Qualifications from Students Who Have Started the Secondary School

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ABSTRACT

The main objective of the Turkish national education is to train individuals with knowledge, skills and attitudes integrated with our values and competencies in a rapidly developing world (MEB, 2018). One of these competencies is mathematical competence. Mathematical competence to bring solution to problems encountered in daily life is the development of mathematical thinking which is built on strong arithmetic skills. Each of the 12-year compulsory education process being implemented in Turkey since the 2012-2013 academic year, the three stages of the period of four years (elementary, secondary and high schools) are available. It is expected that the students who pass the primary school to the secondary school will have the basic competencies determined by Ministry of National Education. So is there a relationship between the expectations of teachers at this level and the objectives of the curriculum? This problem is compounded by mathematics and is more prominent in disciplines that require the study of the basic competencies of the previous learning processes. For this reason, the purpose of this study is to determine the expectation and satisfaction levels of the mathematics from the students who completed the elementary school and have just started the secondary school.

The case study design was used in the study, which is one of the qualitative research approaches. Participants consist of 36 mathematics teacher who teach secondary school mathematics in different cities in Turkey. Within the scope of the research, a scale with 20 questions (five point likert scale) was prepared by using the basic competences expressed in the 4th and 5th mathematics curriculums. The necessary pilot studies were carried out and the participant answered the scale after obtaining the last valid and reliable version. Data were analyzed with descriptive

statistics; interview data obtained from selected participants according to purposeful sampling were analyzed according to content analysis method.

The findings of the study show that participant (mathematics teachers) generally expected from the students to be able to perform addition and subtraction with up to four digits of natural numbers and multiply by up to two digits of natural numbers. Participants also expected from the students to be able to identify time units, convert them to simple problems, and determine the angles (narrow angle, right angle or/and wide angle). Participants agreed that students had difficulty in all four operations, especially that there were inadequate competencies in fractions concept and division process. Relevant results have discussed the relationship between expectations met and the strategies used in the teaching process and the future academic achievements of students.

Key Words: Mathematical qualification, 5th grade, teacher's expectations

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Fourier Method for Inverse Coefficient Euler-Bernoulli Beam Equation

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ABSTRACT

In this study, The inverse coefficient in the Euler-Bernoulli beam equation with over determination conditions is examined. It showed the existence and uniqueness of the weak generalized solution of a mixed problem with periodic boundary condition for a quasi-linear Euler-Bernoulli beam equation are examined, and an estimation of the differences between the exact and approximate solution is obtained.In order to solve the problem, first the test functions are given,then the weak generalized solution of the problem is defined in terms of these functions. The weak solution is expressed as a Fourier series with undetermined variable coefficients, and a system of non-linear infinite integral equations for the coefficients mentioned above is obtained. The existence and uniqueness of the solution of the system are proved by the successive approximation method on the Banach space B . The periodic boundary conditions are used many area.Periodic boundary conditions are used in molecular dynamics simulations to avoid problems with boundary effects caused by finite size, and make the system more like an infinite one, at the cost of possible periodicity effects,heat transfer, life sciences,on lunar theory. Finally, inview of the practical importance of the problem, the norm of the difference between the exact solution and successive approximations of the infinite system is estimated on the space B. A mix problem with periodic boundary condition $u_{tt}(x,t)+\alpha^2 u_{xxxx}(x,t)-g(t)u(x,t)=f(x,t,u)$ is examined for quasilinear quartic partial differantial equations.

Key Words: Euler Bernoulli beam equation; Fourier method; Quasi-linear mixed problem; Periodic boundary condition.

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How Does Pre-service Primary Teachers Explain Area?

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ABSTRACT

Geometry teaching and learning is a familiar concept in literature. Geometry instruction helps learners develop an understanding of “geometric shapes and structures and how to analyse their characteristics and relationships” [2]. Area is the main sub-subject of the elementary grades. In Turkish mathematics curriculum in elementary grades does not cover the area formula. Instead of teaching rea formula, students try to construct area based on unit squares. The purpose of this study is to investigate how pre-service primary teachers estimate the area of unusual shape and how they can explain the area to elementary grades without using formula.

Qualitative methodology was used to collect data. Pre-service primary teachers estimated the area of unusual shape and their solution strategies was examined. The purposive sampling method was used which enables to select the sample in accordance with the specific purpose of the study [1]. The selection criterion in this process was preservice teachers’ lesson experience which was based on activity-based instruction in mathematics education. 72 of 3rd year pre-service primary teachers who are students of state university at the Central Anatolia were participated to study. They wrote their solution strategies on the paper without any restriction.

Pre-service primary teachers stated that they can explain the area of the unusual shape to elementary-level classes (1-4) in mathematics without using area formula. Moreover, they mentioned that it is not an easy task since students sometimes cannot conceptualize the unit squares in a given shape. A few of the pre-service teachers used the area formula, they do not grasp the idea behind using unit squares.

Key Words: pre-service primary teacher, unit square, area, teaching, solution strategy, qualitative study

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Identifying The Spatial Strategies Of Pre-Service Middle School Mathematics Teachers

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ABSTRACT

The spatial ability, defined as imagining, perceiving, manipulating, re-organizing and re-acquiring visual images of objects or forms (Carroll, 1993), has great importance for individuals in both daily and academic life. Literature review shows that there is a positive relationship between spatial ability and geometric success (Ben-Chaim, Lappan & Houang, 1985; Clements & Battista, 1992). The middle school mathematics curriculum (MoNE, 2013) and the NCTM (2003) standards emphasize the importance of spatial ability, and when examining the middle school mathematics curriculum, it is seen that many achievements in the area of geometry learning relate to spatial ability. It is also stated that middle school mathematics teacher should be able to effectively use the spatial ability (NCTM, 2000).

In this context, it is an important obligation of the teachers to obtain the objectives related to spatial ability and to create suitable learning environments for the development of spatial ability. For the reasons mentioned above, it was the aim of studying the strategies pre-service teachers used to solve the problems of spatial ability.

To achieve the purpose, a qualitative methodology, phenomenographic research, was employed. The study was conducted with 12 pre-service teachers in the 4th year of the Elementary Mathematics Teacher Education program. The spatial strategies used by the pre-service mathematics teachers were explored through *task-based interviews* based on the following five branches of spatial ability proposed by Maier (1996); visualization, spatial perception, spatial orientation, spatial relations, and mental rotation.

Based on the results of the study, the spatial strategies used by the pre-service middle school mathematics teachers were categorized into four main groups: *mental*

rotation, mental manipulation, *counting* and *key features*. These strategies were further divided into sub-groups.

The mental rotation strategy, which is defined as moving a given object by turning it as desired, is divided into two sub-strategies as self-moving and object-moving. Also, object-moving sub-strategy is classified as rotating a whole object and rotating part of an object. *The strategy of mental manipulation* is expressed as the process of arranging, folding, opening on the given object (Kayhan, 2012) and is categorized as sub-strategies of imagining the final state of the process, making all steps of the process and making certain steps of the process.

The counting strategy is to perform the necessary operations by counting the parts (row, column, cube) that make up the structure instead of thinking as a whole in the problems such as isometric or orthographic drawing (Kayhan, 2012). These counting operations can be classified as layer counting and unit counting. The sub-strategies of *the key feature strategy*, which is defined as defining important features on the object and solving the problem using these properties, are classified as follows: comparing associated positions, comparing associated distances, symmetry, painting, pattern formation and determining the structural features of an object.

It is suggest creating courses that will enable the development of spatial ability at the undergraduate level so that pre-service teachers have the necessary competencies. In this way, pre-service teachers can develop strategy repertoires and thinking styles with these courses.

Key Words: spatial ability, geometry, pre-service teacher.

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Instructional Design To Enhance Students' Learning Of Mathematics

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ABSTRACT

Mathematics teaching processes should be designed within the contexts including the skills, values and concepts underlined in the Mathematics Program (MEB, 2017) for students to acquire problem solving, communication, production and creativity, analytic thinking, entrepreneurship, reflective thinking, critical thinking, basic mathematical literacy and daily life skills. The aim of this study is to provide example for an instruction which was designed in the light of principles underlined in Understanding By Design Model (Wiggins and McTighe, 2005) which aims to enhance students' learning of mathematics. The study provides a sample for mathematics unit preparation, implementation and evaluation processes. In this mathematics unit, the core elements namely, the knowledge, skills and attitudes stated in the Mathematics Program (MEB, 2017) are included by adopting an interdisciplinary and real life-based point of view. The main steps in instructional design are as follows; (1)Identification of students' characteristics, learning profiles, readiness and interests (Tomlinson, 2010), (2)Identification of the main items of the unit by using Understanding By Design Model (UBD) (Wiggins and McTighe, 2005); (3)Deciding on primary and secondary aims of the lesson, (4)Defining the descriptors of achievement at the end of the unit, (5) Selecting and designing the activities, (6) Implementing the activities and assessing the process formatively, (7) Conducting summative assessment, (8) Reflection on the process, (9) Recording and revising for the future implementations. By means of such an instructional design, it is proposed that a convincing answer to students pondering about the question "What will be the use of learning this (mathematical concept) to me?" can be given in a meaningful way. The study was performed with 70 6th grade students in one of the Foundational Middle Schools in Ankara. The data collection tools included written documents, students' reflection papers, and field notes kept by the researcher. After the first

implementation, changes in the instructional unit are done in the light of data analysis process.

Key Words: instructional design, mathematics, students’ learning

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Investigation The Possible Causes And The Prevention Suggestions Of Misconception In The Exponential Expressions Based On The Views Of The Teacher Candidates

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ABSTRACT

The exponential expressions is one of the most difficult topics in mathematics. In previous studies on exponential expressions, this issue has been dealt with in different dimensions. Some studies have examined the misconceptions that have been experienced while others have examined them in terms of number sense components. When the results of the studies were examined, it was stated that the students had serious misconceptions about the rooted and exponential expressions and that they complained about the rules (Cengiz, 2006) and that they were challenged in this subject (Kutluca, 2009).

This study aims to examine the question of what can be done to minimize these troubles based on the opinions of the candidates of middle school mathematics teachers.

This purpose was carried out with 29 senior middle school mathematics teacher candidates studying at a state university. Qualitative research method was used in the study. In the research, two probing responses were sought: “What are the opinions of the candidates of middle school mathematics teachers for possible causes of misconceptions in exponential expressions?” And and “What are the opinions of middle school mathematics teacher candidates in preventing misconceptions in exponential expressions?

In the data collection process, first the participant applied the question form prepared by using the misconceptions about the exponential expressions that are appropriate to the level of middle class education encountered in the field writing, then followed by detailed interviews with the volunteers. The "content analysis"

technique was used in the qualitative analysis approaches to identify repetitive codes and themes in the forms in which teacher candidates identified possible reasons and opinions about conceptual misconceptions about the exponential expressions.

As a result of the analysis of the data, the concept indicated that the possible causes of misconceptions were caused by the lack of conceptual information in the exponential expressions, which led to the students' misjudgment of many rules in the exponential expressions. In order to prevent misconceptions, teacher candidates stated that more examples of exponential expressions should be resolved, at the beginning of the subject concepts such as base and exponent should be emphasized and rules should be taught by pattern and numerical and verbal expansions should be taught while exponential expressions are explained.

From results of the study, to avoid such troubles in this subject, introducing this new number form carefully at the beginning of the subject, focusing on how to express these numbers in other ways and the place of these numbers in number line, are given as recommendations of the study.

It should also be emphasized that the exponential number corresponds to a rational number, and in cases where the value is too large or too small, it should be thought of as an estimation.

Key Words: Exponential numbers, misconceptions, middle school mathematics teacher candidate

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Investigate the Development of Preservice Teachers' Knowledge of Teaching Strategies by the Lesson Study Method

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ABSTRACT

The objective of this study is to investigate the development of preservice teachers' knowledge of teaching strategies, which play a key role in their pedagogical content knowledge, by the lesson study method. The lesson study method is a method that enables the mutual professional development culture and develops the professional knowledge by providing teachers with the opportunity to observe their colleagues. In this context, the qualitative research approach was adopted, and the case study method was used in the study. Participants of the study consist of 5 preservice teachers enrolled for the program of elementary mathematics teaching, studying at the 4th grade, and already taking the teaching application class. The study data were collected within the scope of the teaching application class. In this process, first, each preservice teacher was asked to prepare a lesson teaching plan and teach secondary school 7th-grade students according to the plan they prepared. Lesson teaching performed by the preservice teachers was observed by a specialist, a teacher, and other preservice teachers (4 preservice teachers, except for the one performing teaching). In the next stage, the stage of preparing a lesson plan according to the lesson study method, one specialist, one teacher, and 5 preservice teachers gathered and prepared a joint plan. In the implementation stage of the plan, one specialist, one teacher, and 4 preservice teachers, except for the one teaching the lesson, observed the lesson, took observation notes, and contributed to the next lesson plan. The plan revised with observation notes was implemented again to another branch by another preservice teacher and was observed by the same observers. In this process, the observation form developed by Gökkurt (2014) was used. Moreover, semi-structured interviews were held with preservice teachers and the data obtained were analyzed by the descriptive analysis technique and evaluated. In the light of the findings of the study, it was concluded that the knowledge of preservice teachers about teaching strategies was enriched and improved by the lesson study method, and it can be suggested that these kinds of methods should be used for the improvement of the knowledge of preservice teachers and the teaching application class should be carried out by methods such as the lesson study method.

Key Words: Preservice Teacher, lesson study, knowledge of teaching strategies.

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Investigate the Levels of Meaningfully Remembering Geometric Formulae of Preservice Mathematics Teachers'

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ABSTRACT

The ability to learn geometry that includes many formulae and rules requires students to remember formulae and the relationship between formulae and geometric shapes [1]. Therefore, when teaching mathematical rules and formulas the importance must be given on understanding and reasoning instead of trusting the memorization capabilities of the students [2]. Because, it is important for preservice teachers to learn and remember formulae meaningfully. In this context, this study aims investigate the levels of meaningfully remembering geometric formulae of preservice mathematics teachers'. In this study that adopted the qualitative research approach, the case study method was used. Participants of the study consist of 74 undergraduate students studying at a state university in Turkey. The study data were collected by the knowledge test consisting of geometry questions and were analyzed by the descriptive analysis technique. The findings of the study were explained by directly quoting the answers of students, in addition to frequency and percentage tables. In the light of the findings obtained, it was observed that students could not remember particularly the area and volume formulae in a meaningful way. On the other hand, it was also observed that most students did not need to use formulae in questions related to angles.

Keywords: Preservice mathematics teachers'; geometric formulae; meaningfully remembering

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Investigating Preservice Mathematics Teachers' Proving Skills[1]

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ABSTRACT

Proof is a serial of formal steps based what makes a proposition is true in case of being true or what makes it false in case of being false on some persuading evidences (Hersh, 1993; Hanna & de Villiers, 2012; Baki, 2015). Proving could be simply defined as the process of doing a proof. While proving is a process, proof is a cognitive product appearing at the end of this process (Harel, 2008). On the other hand, proof schemes are cognitive characteristics of the proving process (Harel & Sowder, 1998). Sowder and Harel (1998) categorize a proving process into two sub-processes called "*ascertaining*" and "*persuading*", while they categorize proof schemes into three main classes called *external proof schemes*, *empirical proof schemes* and *analytic proof schemes*. When providing justification for correctness or incorrectness of a proposition, students having *external proof schemes* apply for an authority such as a teacher or a book, or refer to pre-acquired habits or meaningless manipulations of symbols (Harel & Sowder, 1998). However, students having *empirical proof schemes* apply for one or a few examples, use their intuitions and one or a few simple drawings (Sowder & Harel, 1998). Finally, students having *analytic proof schemes* use generalisation, logical deduction and operational thinking together (Harel & Sowder, 1998). A number of students in this main scheme can regulate these within an axiomatic structure (Sowder & Harel, 1998).

Aim of this study is to investigate the proving skills of upper secondary preservice mathematics teachers. The proving skills were determined based on whether they chose appropriate proof methods or not, they did an acceptable proving or not, and also proof schemes that they used during proving.

[1] This study has been produced from the dissertation which is being conducted by the first author under the supervision of the second author. The study is a part of a research project sponsored by Marmara University Scientific Research Projects Committee with project number EGT-C-DRP-120418-0202.

This study used case survey method. Research sample was 22 students attending a program in secondary mathematics teaching department at a state university in Istanbul. Data collection tool is a questionnaire including 7 questions related to basic mathematics subjects. Data analysis was based on whether or not preservice teachers chose an appropriate proof method (as appropriate, inappropriate and no response), and whether or not they did proofs correctly (as correct, partially correct, incorrect and no response). Finally proof schemes were evaluated according to the classification of Sowder and Harel (1998). 154 answers given by 22 preservice teachers were analyzed.

The findings revealed that preservice teachers had difficulties in choosing the appropriate methods of proving. Of 154 proofs made by preservice teachers, 23 of them were evaluated as correct, 6 of them partially correct, 101 of them incorrect and 24 of them as no response. Another finding was that participants mostly used external proof schemes in their proofs. They were followed by empirical proof schemes and analytic proof schemes respectively. More than one proof scheme was used in 4 proofs. 70 external proof schemes, 32 empirical proof schemes, 29 analytic proof schemes and 27 no response were determined. Considering these findings, it has been suggested that intervention studies should be conducted to improve proving skills preservice mathematics teachers.

Key Words: Proof, proving, proof schemes, proof methods, upper secondary preservice mathematics teachers.

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Investigation of Middle School Mathematics Teachers' Skills of Using Mathematical Language in Problem Posing Process

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ABSTRACT

Mathematics is a language that has its own concepts, terms, and symbols. The mathematical language is actively used in the learning and teaching process. The use of mathematical language with a correct and logical ordering leads to a more permanent learning of the information (Toptaş, 2015). Correct and understandable creation of mathematical language in the mind is a factor that influences the learning and success in the positive direction. In the process of learning mathematics, students need to use mathematical terminology and language correctly to express their mathematical thoughts. Teachers have great tasks in this frame of effective and correct use of mathematical language. The purpose of this research is to examine the skills of using mathematical language in the problem posing process of middle school mathematics teachers. The participants of the study consist of 8 mathematics teachers who are working in different state middle schools. A purposeful sampling method was used in the selection of the participants. Case study method based on qualitative approach was used in the study. As a data collection tool, 6 problems were prepared by the researchers. These problems were prepared considering Stoyanova and Ellerton (1996) 's semi - structured problem posing category. Geometry, one of the sub-learning areas of mathematics, is chosen from geometry learning if the frequent use of mathematical language is taken into consideration. In the problems, geometrical concepts and rules (point, line, line segment, ray, angle, pisagor relation, similarity) are given visual elements for expressing mathematical language. At the first stage, teachers are asked to use mathematical terminology and language when constructing problems. At the second stage, participants made 10-15 minutes of interviews on the problems they had and the mathematical

language they used. In the analysis of the data, qualitative data analysis techniques were used.

As a result of the research, it was determined that most teachers can establish a solvable mathematical problem from the forms and concepts for geometry learning. When the content of the problems is examined in terms of mathematical language, it is seen that some of the teachers use mathematical language with their verbal expressions and some of them use with the help of symbols. As a result of the study, it has been found that some teachers mistakenly use (misuse of the closed half line, not using the degree symbol in angle measurement etc.) the mathematical language. An example of this mistake is the writing of "[AB] line segment" instead of the " AB line segment " and writing of ‘’How much degree is the angle of m(Â) ? ’’ instead of ‘’How much degree is m(Â) ? ’’. Based on the obtained data, it is concluded that these mistakes what teachers did are the result of lack of attention, not lack of knowledge.

Key Words: Mathematical language, middle school mathematics teacher, problem posing

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Investigation of PISA Problem Posing Skills of Mathematics Teacher Candidates

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ABSTRACT

PISA is an international exam held by OECD once in every 3 years to evaluate fifteen aged children's mathematics literacy skills [OECD, 2018]. The most important component of mathematics literacy skill is problem solving (Altun, 2013; Salz, 2009). The mathematics problems placed in PISA are equally distributed to the different contexts and different contents. Also in this exam which requires solving problems includes daily living situations Turkey could not achieve the success intended [OECD, 2018]. It is an acknowledgement that one of the most important factors affecting the students' achievement is teachers (Ball, Thames and Phelps, 2008). Therefore teacher candidates who are teacher of future must be more conscious about PISA for our country to be more succesful in PISA. In this context mathematics teacher candidates' problem posing skills appropriate to the nature of PISA were investigated in this study. The case study method, one of the qualitative research methods, was used in the study. Participants of the study are constituted of 55 mathematics teacher candidates studying in education faculty of a university. In the study, initially, teacher candidates were educated about PISA exam. During the education, knowledge about the purpose of PISA and held by whom, which skills are assessed, how the questions are prepared and evaluated, which features the questions have and information about the performances of the other countries were given. Later the questions which were placed in PISA were analyzed under categories of context, content, process skills and question types. Teacher candidates were also given detailed information on how these questions were evaluated. After this training, teacher candidates were given a research project to prepare three questions appropriate to the nature of PISA. The problems that teacher candidates have prepared are analyzed by two researchers with the data analysis framework developed by the researchers. As a result of the study, it has been found that many of the problems that teacher candidates have posed are appropriate to the nature of PISA. In addition, it has been seen that the teacher candidates usually posed open-ended problems, but they less prefer to pose multiple-choice and short-answer problems. When teacher candidates' problems are examined in terms of content, context and process skills, it has been determined that they most posed problems in the content of quantity (Numbers and Operations), in the personal and occupational context and generative skill oriented. In addition, most of the teacher candidates made an explanation for the exactly correct category and the zero categories for evaluating the problems they posed, but not for the partially correct category.

Key Words: PISA, problem posing, teacher candidate.

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Investigation of Pre-service Elementary Mathematics Teachers' Beliefs about Mathematics Teaching Self-Efficacy and Mathematics, Teaching and Learning

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ABSTRACT

The present study aims to find out pre-service elementary mathematics teachers' beliefs about mathematics teaching self-efficacy and mathematics, teaching and learning and to investigate their beliefs in terms of gender and years of study. In this regard, relational screening model from general screening models was employed in the study. The study group consisted of 198 pre-service mathematics teachers studying at 2nd, 3rd and 4th year in Primary Mathematics Teacher Education Program at a public university in 2017-2018 academic year as well as the pre-service teachers who graduated the previous year from the same program. For data collection, "The Mathematics Teaching Self-Efficacy Scale" developed by Enochs, Smith and Huinker (2000) and adapted to Turkish by Hacıömeroğlu and Şahin-Taşkın (2010) and the "Beliefs About Mathematics Teaching and Learning Scale" developed by Kayan, Haser and Işıksal-Bostan (2013) were used. The pre-service mathematics teachers' beliefs about mathematics teaching self-efficacy were investigated under the sub-dimensions of 'Self-Efficacy', 'the Role of Teachers in Effective Learning' and 'Teaching Performance' whereas the beliefs about Mathematics Teaching and Learning were handled separately under the dimensions of 'constructivist beliefs' and 'conventional beliefs'. Independent Samples t-test was administered in order to find out how the beliefs of pre-service teachers changed according to gender while one-way Anova test was administered to find out how their beliefs changed according to their years of study. Furthermore, the Pearson correlation coefficient was calculated in order to discover the relationship between mathematics teaching self-efficacy beliefs and the beliefs about mathematics, teaching and learning. The data were analyzed at .05 significance level. Ultimately, some recommendations were presented in line with the results obtained.

Key Words: Self-efficacy beliefs, mathematics related beliefs,

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Investigation of Pre-service Elementary Mathematics Teachers' Knowledge of Interpreting Line Graph

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ABSTRACT

Using mathematical representations such as tables, graphs and algebraic expressions effectively plays a significant role both in problem solving and conceptual understanding. It is important to use all of these representations effectively, but the graphics have a separate importance because they can present a complex relationship or information that is difficult to describe with words in a more simple and visual manner to the reader (Özgun-Koca, 2008). The present study aims to find out 3rd and 4th grade pre-service elementary mathematics teachers' knowledge of interpreting a contex-based line graph with relation to Curcio's graph interpration levels to and also investigate their graph interpretation performance in terms of gender and grade. As a data collection tool, a task which developed by Patahuddin and Lowrie (2018) including 23 true-false questions concerning line graph, was adapted to Turkish. In this task the graph shows how the speed of two cars changes to each other and over the time. The data were analyzed by using descriptive analysis and t test. Independent Samples t-test was used in order to find out how the performance of pre-service elementary mathematics teachers changed according to gender and grade. At the end of the study, some recommendations were presented depending on the results obtained.

Key Words: line graph, graph interpretation

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Investigation of Secondary Pre-service Mathematics Teachers' Pedagogical Content Knowledge of Proof[2]

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ABSTRACT

Teaching proof is a highly difficult issue for teachers to overcome (Heinze & Reiss, 2004; Knipping & Reid, 2015). For effective teaching of proof, teachers should first realise students' difficulties with proof and their reasons, and then develop teaching strategies to overcome these difficulties and their reasons, and implement them in their lessons. Furthermore, teachers should be able to determine types of justification used by students for the trueness or wrongness of a proposition and help them enhance their justification types to be able to reach axiomatic level.

The aim of this study is to investigate secondary pre-service mathematics teachers' pedagogical content knowledge of proof with its four components: (a) knowledge of students' difficulties and reasons behind them, (b) knowledge of teaching strategies that can overcome these difficulties, and (c) knowledge of types of justification used by students.

This study uses case survey method which aims to make a description of an existing case related to the topic of research (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2016). Participants are 22 secondary pre-service mathematics teachers studying at a state university in Istanbul. Data collection tool is a questionnaire consisting of two parts. The first part of the questionnaire has five questions aiming to gain information about participants' knowledge of student difficulties regarding proof, the reasons for those difficulties and teaching strategies. While preparing the questionnaire entries, a meticulous attention has been paid on the entries to contain various difficulties, various reasons for these difficulties and various strategies. The second part of the questionnaire consists of vignettes

prepared with the aim of finding out whether pre-service teachers can determine students' justification types.

With regard to the first part of the questionnaire, difficulties, reasons and strategies have been analyzed separately. For the analysis of student difficulties with proof, a literature based classification has been made. In this classification, student difficulties consist of five sub-headings. These five sub-headings are: (a) difficulties in understanding the nature of proof, (b) difficulties with the content area involved in the proof, (c) difficulties with proof methods, (d) difficulties based on problem solving skills and mathematical reasoning, and (e) difficulties in using mathematical language. To analyze participants' knowledge of the reasons behind these difficulties, Cornu's (1991) classification was used: *genetic*, *didactical* and *epistemological difficulties*. To analyze participants' knowledge of teaching strategies, strategies suggested by the related literature were used. With regard to the second part of the questionnaire, the justification types determined by pre-service teachers were coded as *external*, *empirical* and *analytic proof schemes* of Sowder and Harel (1998).

The findings of this study has indicated that pre-service teachers have problems with determining students' difficulties regarding proof and reasons behind these difficulties, and thus are not able to design teaching strategies to overcome these difficulties. Another finding is that pre-service teachers are not in a desired position about determining the justification types used by students. Considering the findings, it is recommended that some intervention studies should be conducted to develop secondary pre-service mathematics teachers' pedagogical content knowledge of proof.

Key Words: Proof, proving, proof schemes, pedagogical content knowledge, upper secondary pre-service mathematics teachers, student difficulties, teaching strategies.

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Investigation of Secondary Pre-service Mathematics Teachers' Difficulties with Proving[3]

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ABSTRACT

"Learners and their characteristics" have an important place in teaching (Shulman, 1987, p.8). The difficulties that learners experience are also one of them. Students at all levels of education have difficulties towards proving (National Council of Teachers of Mathematics [NCTM], 2000). This case is also valid for pre-service mathematics teachers (Almeida, 2003). At this point, research should focus on determining pre-service mathematics teachers' difficulties towards proving.

Aim of this study is to examine upper secondary pre-service mathematics teachers' difficulties towards proving. The pre-service teachers' difficulties towards proving were determined based on the first and most distinct difficulty that prevented them from a valid proof.

In accordance with this aim, case survey model was used in this research. Case survey model, which is more comprehensive than general survey models, gives more realistic results (Karasar, 2014). Participants are 22 pre-service mathematics teachers who were at the second grade in a teacher preparation program in Secondary Mathematics Teaching Department at a state university in Marmara Region. The data collection tool consists of 7 questions which require proofs on general mathematics topics. Data analysis was based on five categories of difficulties that the pre-service teachers might experience in the proving process as reported in the literature. These five categories are: (a) difficulties in understanding the nature of proof, (b) difficulties with the content area involved in the proof, (c) difficulties with proof methods, (d) difficulties based on problem solving skills and mathematical

[3] This study has been produced from the dissertation which is being conducted by the first author under the supervision of the second author. The study is a part of a research project sponsored by Marmara University Scientific Research Projects Committee with project number EGT-C-DRP-120418-0202.

reasoning, and (e) difficulties in using mathematical language. 154 answers given by 22 pre-service teachers for 7 questions were analysed according to the categories above.

According to the findings of the research, no difficulties were encountered in 23 answers. Difficulties were identified in the remaining 131 responses. Of these 131 difficulties, 37 of them were difficulties in understanding the nature of proof, 21 of them were difficulties with the content area involved in the proof, 57 of them were difficulties with the proof methods, 12 of them were difficulties based on problem solving skills and mathematical reasoning, and 4 of them were difficulties with the use of mathematical language. Based on the findings of the research, it has been recommended that courses in mathematics and mathematics education departments should deal with students' difficulties with proof.

Key Words: Proof, proving, difficulties towards proving, upper secondary pre-service mathematics teachers.

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Investigation of Teachers' Awareness of Interdisciplinary Mathematical Modeling Problem

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ABSTRACT

In order to meet the 21st-century learning competencies, the STEM approach is at the forefront, and its importance gains more attention [1]. Mathematical modeling (MM) can be seen as an important tool in STEM education because it can provide opportunities for students to engage in different disciplines along with mathematics. We call this kind of activities Interdisciplinary Mathematical Modeling (IMM). The role of the teacher is very important in the transfer of IMM to the classroom environment [2]. Thus, teachers need to understand IMM approach, to feel the necessity and to gain practical experience about IMM. The purpose of this study is to explore Mathematics and Science teachers' competencies of determining the subjects/concepts and identifying the problem situation in the IMM activity.

We conducted individual interviews with twenty teachers (10 Science-10 Mathematics). They were asked to evaluate an IMM activity which is called "Organ Transplant Center" problem that prepared by the researchers. The problem situation asked to determine a place between Istanbul, Ankara, and Izmir to build an organ transplant center. The activity is basically suitable for teaching subjects and concepts of Science and Mathematics courses and also has characteristics related to different disciplines.

The results of the research show that the teachers had difficulties in determining the problem situation and presenting the appropriate solution proposal. Their approach to the interdisciplinary relationship was only at surface level. In particular, the science teachers had difficulties to express the problem situation. They claim that problem situation was about not having enough organ donation or about the lack of

the number of the organ transplantation centers in Turkey. However, those pieces of information were given in the problem to highlight the importance of organ donation. Most of the teachers said that the problem first belonged to their own branches but then were able to see the link with other disciplines: Science and Technology (Organ Donation and Organ Transplant), Mathematics (Proportion, Data Collection, Center of Gravity, Percentage Calculations), Geography (Mapping Information), and Moral Education (The Importance of Organ Donation). One of the interesting results was that most of teachers did not see the mathematics in the solution of the problem. Some of the teachers have associated mathematics with only taking into account the numerical information given in the text of the problem; they did not realize that mathematics was needed to solve the problem. The mathematics teachers' assumptions about the location of the Organ Transplantation Center were based on factors such as distance from each other, patient or population density, and transportation, while assumptions of science teachers were limited and mathematical based.

Key Words: Mathematical modeling, interdisciplinary mathematical modeling, STEM

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Investigation of the Opinions of Students Studying in the Rural Area on the Use of Mathematics in Agriculture

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ABSTRACT

Secondary school mathematics program allows students to establish a connection between primary school mathematics and high school mathematics while at the same time preparing students for everyday life [1]. This being the case, many students still do not realize that mathematics can be used in everyday life. Students may consider mathematics as a lesson that consists of abstract structures and a useless lesson. In addition to this, the fact that some students consider mathematics all about arithmetic and realize its relationship with other disciplines may be a source of inner motivation to learn mathematics [2]. Accordingly, the study was conducted in order to examine the opinions of students studying in the rural area on the use of mathematics in agriculture. The study was conducted by the case study method among qualitative research patterns. Four students from each grade level at the secondary school level (5th, 6th, 7th and 8th), sixteen students in total, participated in the study. All the students participate on a voluntary basis, and they study at a school in a rural area in a Northern province of Turkey in the 2017-2018 academic year. The data of the study were collected through a semi-structured interview form. In the interview form, the students were asked questions such as “Is the knowledge of mathematics necessary for working in the field of agriculture?”, “In which field do you think mathematics can be used in agriculture?”, “Do you think that mathematics lessons can be taught with agriculture?” The data collected in the study were subjected to the content analysis. The results obtained in the study showed that students studying in the rural area think that mathematics is necessary to work in the field of agriculture. Students think that mathematics can be used mainly to divide the agricultural land. Nevertheless, opinions that the knowledge of mathematics is

necessary in fields such as measuring agricultural lands and making economic calculations emerged. Furthermore, it was observed that students stated that mathematics lessons can be taught with agriculture, whereby the lesson will be more enjoyable and they will be able to solve questions in a practical way. Students also noted that mathematics can be used mainly to teach subjects such as area-circumference, fractions, gcd-lcm, and area measurement units to work in the field of agriculture. When the findings of the study were examined, it was concluded that students studying in rural areas realized that mathematics is used in agriculture and mathematics lesson is related to daily life in this way.

Keywords: Rural area, agriculture, mathematics, secondary school students.

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Investigation of the problem solving processes of "definite integral" of science student teachers

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ABSTRACT

Integral, one of the basic concepts of mathematics, is one of the concepts that students are forced to understand (Orton, 1983; Tall, 1993; Rasslan and Tall, 2002, Skemp, 1976, Tall and Vinner, 1981;). Like other concepts of integral mathematics, require a strong pre-cognitive knowledge. In order to solve the integral problems, it is expected to have conceptual knowledge level in many contexts such as knowing the properties of mathematical functions and drawing graphs, derivation, changing variables. The purpose of this study is to examine the problem solving processes of "definite integral" of prospective science teachers. To this end, the working group constitutes 44 prospective teachers registered to the first degree program of science teacher education of a state university in the province of Istanbul. The study was defined as an exceptional case with a qualitative interpretive paradigm. The " definite integral problem test" was used as a data collection tool. This test consists of ten open-ended definite integral questions. Within the scope of this study, two questions about the test were examined. Findings have generally preferred geometric solution in the process of solving definite integral problems of prospective science teachers. It has also been found that prospective science teachers draw graphs of functions and are unable to write equivalents of trigonometric functions. With regard to the research findings, suggestions were offered for the future studies by emphasizing the necessity to take some precautions to increase pre-service teachers' knowledge on mathematical language as well as its use.

Key Words: Preservice science teachers, definite integral, problem solving.

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Investigation of Written Arguments of Pre-service Teachers about Fractions

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ABSTRACT

Fractions and operations with fractions are difficult topics for students to understand. The studies revealed that both students and pre-service teachers struggle wit to understand fractions (Van Steenbrugge, Lesage, Valcke, & Desoete, 2014). If pre-service teachers struggle with their understanding of fractions, they are likely to struggle in teaching fractions to their future students (Van Steenbrugge et al., 2014). If teachers struggle in teaching fractions to students, their students, in turn, are likely to struggle with understanding fractions and have difficulty contextualizing and decontextualizing word problems that contain fractions. Based on the literature, teachers and pre-service teachers have conceptual misconceptions about concept of fraction and multiplication and division with fractions (Işıksal, 2006; Zambat, 2007). Although pre-service teachers solve problems related to multiplication and division with fractions, their reasoning skills to understand the meaning of these processes are at low level (Işıksal, 2006). Pre-service teachers can divide fractions easily. However, they have a difficulty to explain why the second fraction is inverted and multiplied with first fraction (Zambat, 2007). For this reason, the purpose of this study is to investigate pre-service mathematics teachers' written arguments about fractions in terms of Toulmin's Argumentation Model and argumentation levels. Argumentation levels were categorized based on Toulmin and Osborne, Erduran and Simon's work.

There are six basic elements in Toulmin's argumentation pattern: data to support claim (D), claim which is a value or opinion about existing situaton (C), reason which explains the relation between data and claim (R), support which is a basic assumption validating a specific reason (S). Except from these elements, there are exceptions, in others words restrictives, where reasons are not valid (R) and rebuttals through which claims are rebutted (R) (Driver, Newton, & Osborne, 2000).

As Toulmin's model is intended to be applicable to arguments in any field, it has provided researchers in mathematics education with a useful tool for research, including formal and informal arguments in classrooms (Knipping, 2008). Among the qualitative research designs, case study was the methodology of the current study. A total of 86 pre-service mathematics teachers from Giresun University were participated in the study. The findings of the study show that 62.8% of the arguments constructed by pre-service teachers related to division of fractions were found to be correct in terms of the selection of the claim, 36 % of them have an acceptable warrant, and 9.3 % of the warrants completely support the claim. When they explain the claims about division of fractions, they stated that the results are true for other examples. However, they have a difficulty to explain the reason of the argument as stated in the literature (Işıksal, 2006)

Key Words: argumentation, fractions, pre-service teachers

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Investigation of Concept Images of Elementary Preservice Mathematics Teachers in The Context of "Convergence of A Sequence.

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ABSTRACT

The purpose of this study is to determine concept images in the context of the "concept of convergence of a sequence" of preservice teachers who read in 3th grade of elementary mathematics teacher education. Preservice teachers are able to respond to questions about the convergence of a sequence, which is one of the fundamental concepts of mathematics, in an instrumental sense. It has been necessary to study in order to determine whether preservice teachers who are able to answer transactional questions construct and construct the concept of convergence in a conceptual sense. Concept images of preservice teachers have been identified using Vinner's "concept definition" and "conceptual image" methodological framework [1,2]. The study was conducted with the participation of 47 preservice teachers who attended the third grade of Elementary Mathematics Teacher Education Department of a State University located in the province of Istanbul. The "Sequence Concept Test" consisting of five questions was used as a measurement tool. Preservice teachers' answers are conceptually coded as insufficient, inadequate, and acceptable. Findings were analyzed with qualitative and quantitative methods. At quantitative data were used to support qualitative data. Quantitative data were obtained from frequency and percentage, and qualitative data were obtained from students' written answers to questions. As a result of the research, it was determined that they could not construct the concept images of elementary preservice mathematics teachers in the context of "convergence of a sequence [3]. It was also found that prospective teachers were constrained to conceptual convergence and failed to protect conceptual information at the point where they were forced. With

regard to the research findings, suggestions were offered for the future studies by emphasizing the necessity to take some precautions to increase pre-service teachers' knowledge on mathematical language as well as its use.

Key Words: Preservice mathematic teachers, conceptual understanding, concept image, convergence a sequence.

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Mathematics Teachers' Views On The Web Site 'Education Information Network' Known As 'Eba' In Turkey

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ABSTRACT

In the 21st century's information age, the meaning of the concept of information and its ways of reaching this information are changing and even new ways are emerging. In this evolutionary process of knowledge, the ability to reach the knowledge expected from today's schools and to use it effectively has also changed. In order to develop these skills in individuals, schools should be benefiting from information and communication technology. In the scope of this main objective, the Ministry of Education has aimed to offer a student-centered learning by combining the education system with advanced technologies1 and has provided "Education Information Network" (EBA) for education and training. The purpose of the study is to determine the opinions of mathematics teachers regarding the web site EBA which is very important in the concretization of abstract concepts particularly in mathematics courses. In this research, case study, which is one of the qualitative research design was used. Therefore, it can be said that the research is a qualitative study based on the case study. In the case study, the fact is handled within its own life frame. The study group is composed of 30 Math teachers in secondary schools in Sivas city center. In terms of socioeconomic level, the schools included in the sample are divided into three categories as good, middle and lower level. The sampling includes 15 secondary schools. Two Math teachers have been selected from each secondary school. Determining the socioeconomic characteristics of the schools were based on the place of the school, the views of the Directorate of National Education, the views of school principals and teachers. In this research, a semi-structured interview form consisting of eight questions was used as a data collection tool. The questions in the interview form directed by face to face interviews with teachers and their responses recorded. The content analysis method which is in accordance with the qualitative research design was used in the evaluation of the data obtained from teachers. Study results revealed that most of the teachers have been using EBA website both during and after class. It has been determined that the EBA website has contributed mostly to visualize and concretize the lesson and it also draws attention and excites curiosity. It is determined that the most common problems are the inadequacy of the content (lecturing) and the internet connection problems. The other most gripping part of EBA for students is games and videos on the website. Consequently, the majority of teachers emphasized the usefulness of the EBA and the rest requested for more up-to-date, diverse and much more questions on the website.

Key Words: Education Information Network (EBA), Mathematics Teaching

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Mathematical Literacy Levels of 11 Years Old Students

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ABSTRACT

Mathematical literacy in literature is considered a good opportunity to fill the gap between school mathematics and real life problems. Many studies have been realised on teacher candidates, teachers [1], [2], [3] and on students [4], [5], [6] in order to improve the level of mathematical literacy achievement. However, the studies carried out in order to determine the mathematical literacy of the students are mostly limited to the group of 15 years old. The aim of this study is to determine the mathematics literacy levels of elementary school fifth-grade students who are 11 years old. In the scope of the study, mathematics literacy test consisting of 12 questions in total was applied to 20 elementary fifth-grade students and the mathematics literacy levels of the students have been determined with the help of the scores. In this study, qualitative research method was used and the data was analysed by content analysis. As a result of the content analysis, the questions in the mathematics literacy test were grouped and divided into three categories. These categories include “space and shape”, “uncertainty” and “quantitative” questions. According to the findings of the study, it has been found that students mostly have difficulty with quantitative questions. The students have experienced the quantitative questions only in the form of questions expressed in numbers; however the quantitative questions in the mathematics literacy test were in the form of the real life problems. In this case, it has been concluded that when the students deal with a question in the form of verbal expression, which they were not accustomed, they either do not answer the question or solve it incorrectly. It has been determined that the category that the students have been successful the most is the uncertainty category. The reason for this success can be explained by the fact that the uncertainty category is a topic frequently encountered by students in their everyday life and learning environments. When the students have been evaluated one by one in the direction of the findings of these three determined categories, 95% of the participants remained at the middle or lower level of mathematics literacy.

Key Words: Mathematical literacy, mathematics literacy achievement level, elementary education

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Mathematical Modeling of Ablation of Carbon Fiber Reinforced Plastics by Laser

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ABSTRACT

Major developments, especially in the aircraft, automobile and energy sectors, have led to the need to use highly featured new materials used in these fields [1]. Carbon fiber reinforced plastics (CFRP) have been used frequently in the transportation and energy sectors because of their high strength-to-weight ratio [2]. Lasers are often used in cutting, drilling and surface treatments since they have the ability to perform high energy transfer in various forms [3]. Because of the quality degradation in the laser process, especially in the efficient processing of CFRPs, high energy and other laser parameters compatible with these energy levels are required. The main quality reducing effect during laser processing of CFRP is the thermal damage which is influenced by the laser intensity on the work piece and the heat accumulation effect [4].

Mathematical Modeling of laser beam ablation of carbon fiber reinforced plastic (CFRP) is performed. It is important to understand the mechanism about generation of heat-affected zone (HAZ) in order to improve quality of CFRP ablation by laser. Finite difference method is applied to calculate heat transfer inside CFRP and succeeded in simulating HAZ formation. Heat transfer equation based on Fourier's law of conduction is used for calculation of the temperature distribution in the target material. The transient temperature field is obtained by solving one-dimensional heat conduction equation. The governing equation for the one dimensional heat transfer can be written as,

$$\frac{\partial T(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(x,t)}{\partial x^2}$$

where

$$\alpha = \frac{k}{c.\rho}$$

k denotes the heat conduction coefficient, ρ denotes density and c specific heat. Under some natural regularity and consistency conditions on the input data the existence, uniqueness of solution are shown by using the generalized Fourier method.

Finally, the experimental data are compared to the results calculated by heat conduction models.

Key Words: Laser ablation, mathematical modeling, Fourier method.

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Mathematics Through Faculty Of Education Students' Eyes

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ABSTRACT

Life is intertwined with mathematics and knowing mathematics is a force. As in the past, today and tomorrow; it is obvious that individuals will have a close relationship with mathematics. Again, it is accepted that mathematics is effective in all fields of human life, and that the path leading to a distinguished life passes through mathematics [1]. Because a normal person can encounter and need at any moment in his daily life; counting, reading time, paying for shopping, weighing and measuring, understanding simple graphics and diagrams, doing arithmetic operations are all included in the basic concepts of mathematics. For this reason, students who will be tomorrow's teachers; understanding what mathematics is, what mathematics are, how to do it, and what the teacher means are important to understand tomorrow's students' learning. For this reason, the aim of the study is to determine the opinions of learners in education faculty of on the usefulness of mathematics and mathematics and to examine them in terms of different variables.

The research has a quantitative research design and the survey method is used from the descriptive research techniques in the research. Descriptive research is often carried out to explain a given situation, make assessments, and identify possible relationships between events [2]. The survey technique is a non-experimental technique using a wide field of view. It is defined as a method of gathering information from a community through sampling of the sample representing it (usually by questionnaire) [3]. The sample of the study is the students who are educated in the education faculty of a university located in North Eastern Anatolia Region. Collect the data of the study with the help of a questionnaire developed with

literature support. This questionnaire consists of 5-Likert types, open-ended questionnaires, ranking items and selection items. In the analysis of the obtained data, descriptive and predictive statistical methods were analysed with the help of SPSS program. However, in the evaluation of the answers obtained from the open-ended queries, both frequency and percentage values analysis and content analysis method were used and codes were created and interpreted.

As a result, the majority of students see mathematics as a useful area and problem-solving techniques and a mental task for the development of mental abilities. Most of the students have argued that mathematics was created or created by humans in order to solve the problems that people face in their practical or daily needs. However, the number of students who argue that mathematics is discovered by humans while trying to understand the world in which they live is less. Again, many students see mathematics as a tool. So, some of the students are concerned with "mathematics is a complex logical system and a way of thinking", while some students are concerned with "mathematical numbers, rules, formulas and geometric forms". In that case, while some students have a holistic understanding of mathematics, some have a discrete understanding.

Key Words: Mathematics, mathematics in everyday life, usefulness of mathematics, education faculty students.

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Middle School Students' Transition from Arithmetic to Algebra in The Context of Equation Problems

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ABSTRACT

Algebra is one of the basic elements of mathematics; however, students experience vital difficulties while shifting from arithmetic to algebra. Transition from arithmetic to algebra is a problematic process for most of the middle school students. On the other hand, this transition to algebra is required precondition for equations topic, since most of the middle school mathematics curriculum depends on algebraic equations. The aim of this study was to investigate middle school students' transition levels from arithmetic to algebra and in which type of problems they could interpret their learnings easily in equations topic.

This research was designed as a survey type study to describe the transition levels of students from arithmetic to algebra. Study was conducted with 109 sixth, seventh and eighth grade students. While collecting data, students were asked to work on a written exam paper that includes six problems related to equations and a mini-questionnaire that contains four items related to the students' views about these questions. Some of the problems were start-unknown and some were result-unknown. Because result-unknown problems were considered to be arithmetic, the start-unknown problems were considered to be algebraic. Additionally, half of the problems were story-type daily life problems and other half of the problems were only algebraic-symbolic form of first three problems. Analyses of students' work were assessed with four-point rubric.

Results showed that most of the students did not satisfy the expected algebraic levels for their grade. Moreover, it did not matter that which type of question was

used to teach equations topic and in measurement and assessment part to evaluate students work.

Key Words: Transition, Arithmetic, Algebra, Story-type problems, Algebraic-symbolic problems.

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Misconceptions of Sixth Grade Secondary School Students on Fractions

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ABSTRACT

Fraction is a fundamental subject which has critical importance for students and essential for algebra and many advanced mathematics topics (Van De Walle, Karp, & Bay-Williams, 2012). It is one of the main topics in the numbers and operations learning area in the secondary school mathematics curriculum at all grade levels (MoNE, 2013). Fraction concept is more difficult for students than many of the topics in the curriculum and it is stated that it is very difficult for students to understand fractions as numbers and how to do operations with these numbers (Olkun & Toluk-Uçar, 2007). When the literature is examined, it is seen that the students at all levels of education have many misconceptions about fractions.

The purpose of this study was to determine the misconceptions of 6th-grade secondary school students in terms of part-whole relation in fractions, representation of fractions on the number line, comparison of fractions and operations in fractions. The sample of this study was composed of 104 students who are being educated in 6th-grade of five middle schools in 2017-2018 teaching year in a province which takes place in the north part of Turkey. Fractional information test, which consists of 5 open ended questions developed by researchers, was used as data collection tool. The obtained data were analyzed using coding and frequency/percentage tables according to the content analysis method. By examining the explanations that the students have made, the incorrect answers were divided into categories, and the misconceptions in each question were presented separately. As a result of the study, it was seen that students have misconceptions in terms of parts-whole relation in fractions, representation of fractions on the number line, comparison of fractions and operations in fractions.

Key Words: Mathematics education, misconceptions, fractions

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Model and (Mathematical) Modelling: Academicians' Knowledge

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ABSTRACT

Model and modelling are concepts that are used in different meanings in different disciplines. While the model is a collection of systems representing an event, object, and thought in a general situation [1], modelling is the whole of the scientific processes used to build the model [2]. In this sense, models should be seen as necessary and useful tools in the realization of the modelling process and as products that must be achieved during the modelling process [3], [4]. However, mathematical modelling expressed as a cyclical process in which real life problems are expressed and solved by use of the mathematical language, and solutions are tested through interpretations based on reality [5], is defined as the explanation of real life situations and the relationships within these situations by use of mathematics [6]. More recently, the view that mathematical modelling should constitute a significant part of the content of mathematics courses at different levels of instruction is emphasized and the assumption that the mathematics of the students will be more meaningful and helps their learning related to the real life model is required to be used in mathematics education. In particular, creating a mathematical model for academics trying to find suitable solutions to a problem using knowledge of science and mathematics allows the problem to be understood and tested in the default solutions. Already, mathematics is a science called the language of different disciplines, and mathematical modelling has important individuals in university education. Despite recent models, modelling and mathematical modelling, which are the most debated and popular topics in university education, it has been found that the quantitative studies after the domestic literature search are inadequate. The aim

of the study is to analyse statistically the models, mathematical models and mathematical modelling knowledge of the academicians in terms of different variables.

The research has a quantitative research design and the survey method is used from the descriptive research techniques in the research. Descriptive research is often carried out to explain a given situation, make assessments, and identify possible relationships between events [7]. The survey technique is a non-experimental technique using a wide field of view. It is defined as a method of gathering information from a community through sampling of the sample representing it (usually by questionnaire) [8]. The study was conducted with academicians who work at a university in the Eastern Black Sea Region in Turkey. Two different questionnaires and questionnaires developed by the researcher were used in the study and a suitable questionnaire was created for the purpose of the study. Descriptive and predictive statistical methods will be used in the analysis of the obtained data. However, in the evaluation of the data obtained from open-ended questions, both frequency and percentage values analysis and content analysis method will be used. Content analysis will generate and interpret codes.

The data collection and analysis process is ongoing and the results will be presented later.

Key Words: Models, mathematical models and modelling, academicians

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Negative Automatic Thoughts of Secondary School Students When They Encounter Verbal Story Problems

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ABSTRACT

Negative automatic thoughts can be explained as situations that come to mind when a person encounters a specific situation. For example, when a teacher asks a question in the class, negative thoughts that emerge in a student's mind such as "The teacher may get angry at me if I answer wrongly", "My friends may laugh at me if I have difficulty in answering" are negative automatic thoughts. The situation that is formed in the minds of students before they begin to solve a problem when they encounter it, in other words, negative automatic thoughts, will affect their willingness and determination to solve the problem [1]. Accordingly, it is important for secondary school students to determine negative automatic thoughts in the problem-solving process in order to improve the problem-solving process of students. For the reasons explained above, this study was conducted to examine the automatic thoughts that secondary school students had when they encountered verbal story problems. The study was conducted with the case study method among qualitative research designs. Factors related to one or more situations are investigated in depth in the case study with a holistic approach and an in-depth investigation is conducted on how they affect the relevant situation and how they are affected by the situation in question [2]. Eight students in total participated in the study at each grade level (5^{th}, 6^{th}, 7^{th} and 8^{th} grade), two students from each grade level, at the secondary school level. All the students participate on a voluntary basis, and they study at the same school in a Northern province of Turkey in the 2017-2018 academic year. The data of the study were collected through activity cards and a thinking aloud protocol. On the activity cards, two different verbal story problems were presented to the students, and they were asked to solve them by thinking aloud. The data collected in the study were subjected to the content analysis. The results of the study showed that students often have negative automatic thoughts such as “I cannot solve this question”, “What kind of a question is this?”, “I do not understand anything”, “You will not be angry with me if I cannot do it, will you?”, and “It’s a very complex question” when they encounter a problem. In addition to these, some of the automatic thoughts determined in the study are “Will such a question be asked in the exam?”, “I don’t like problem questions”, “I cannot do it on my own”, “Will this affect my grade if I cannot do it?”, “I will definitely not be able to do it”, “I panic when I solve problems”. As a result of automatic negative thoughts formed in the mind of students, it was determined that the students had difficulty in adapting to the question, and they could not fully understand the question.

Keywords: Verbal story problems, negative automatic thoughts, secondary school students.

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Obtaining Acquisitions on the Values in the Secondary Education Mathematics Curriculum through Creative Drama Activities

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ABSTRACT

Values education, which was added to the curriculum of mathematics in the academic year of 2017-2018, has been revised and renamed as Our Values in January, 2018. Out values are significant in enabling communities to continue their existence, preserve their unity, and allowing individuals to peacefully and safely live within a society. Moreover, as in every other aspect in life, it is necessary for individuals to have a well-developed values systems in order to obtain the true success in their educational lives. Enabling individuals to obtain knowledge and a profession, which have been the primary goal in the process of education and training throughout the history, have played a significant role in shaping individuals' behaviors, characteristics, and personalities. In our rapidly-growing century, preserving the traditional values that have been refined with experiences of thousands of years, has become one of the most important study fields. In the process of mathematics education, affective acquisitions are as important as cognitive acquisitions. The value of helpfulness under the heading of Our Values in the curriculum of the studies examined in this research, has been studied and a model implementation has been presented. In line with that purpose, 100 eighth grade students studying at four secondary schools affiliated to the Ministry of National Education, has participated in this study. The research aimed at enabling students to obtain the acquisitions of ''recognizing right prisms, detecting and building fundamental elements, and drawing their development'' (MoNE, 2018).

The target audience is the secondary school students studying at Banaz- UŞAK, Serik- ANTALYA, Didim- AYDIN and Gökçek- AFYON, where the researchers work as teachers. 50 students have been selected for the experimental group and 50 other students have been detected for the control group. The lessons were instructed with the traditional method in the control group, whereas; in the experimental group, the

teacher instructed the subject of right prisms with the method of creative drama. The creative drama activity has not only contributed to teaching the subject of prisms, but also to enabling students to acquire the value of helpfulness. By using these means, it was aimed to help secondary school students to gain the value of helpfulness.

Key Words: Mathematics teaching, values education, creative drama, helpfulness.

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Opinions of High School Students About the Writing Technique Applied in the Mathematics

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ABSTRACT

The purpose of this study is to reveal the views of high school students on writing exercises used in mathematics lessons to develop metacognitive behaviors. The research was designed as a qualitative study and the findings were interpreted with content analysis and descriptive analysis. The study group consists of 64 students who are studying in 9th and 10th class in Akdağmadeni Anatolian High School in Akdağmadeni district of Yozgat province in second term of 2015-2016 education year. When students are given the opportunity to reflect on their work through writing, metacognitive behaviors emerge allowing true understanding to occur **[1]**. So that during the 6 weeks, students were given different writing activities related to the topic, which is usually held every week in the last math classes of the week. These activities are based on writing activities that develop metacognitive behaviors unlike traditional writing in mathematics lessons. In the first weeks there are activities where there are more orientations to get students to practice. Over the following weeks, however, activities have been prepared which allow students to write more freely by reducing their orientation. Every week, the activity papers of all the students were collected by the teacher who was also the researcher, reviewed and sent back to the students the following week. Students are also asked to obtain a math journal. When students go home after each mathematics lesson, they are asked to write about the subjects that are being discussed that day, understanding what they are about, what parts they are interested in, where they are challenged, what they feel and how they feel. At the end of 6 weeks, students were told to write an essay which was evaluated the process. Written essays were collected and their views on the writing technique used were analyzed. As a result, students were generally satisfied with the applied writing technique. Students have touched upon the affective, cognitive and metacognitive contributions of these activities. However, it is understood that some students are reluctant to write journal.

Key Words: writing technique, writing in the mathematics, high school students.

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Orthographic and Isometric Drawing Ability of 5th Grade Students

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ABSTRACT

The spatial ability is used to better understand the living environment and to locate objects in space (National Research Council, 2005). This ability comes to the forefront in almost every subject in the field of geometry, especially in isometric and orthographic drawings and in the study of 3D objects. Studies show that having better spatial ability have positive effect on mathematical problem solving and mathematics achievement (Battista, 1990).

Spatial ability is defined as the ability to move objects in the mind in 2 or 3 dimensions (Olkun, 2003). Studies that examine the spatial competence of primary school students reveals that this ability is often weak in students (e.g. Kurtuluş and Yolcu, 2013). Moreover, students have difficulties finding the number of unit cubes in buildings made from small cubes (Ben-Chaim, Lappan and Houang, 1985; Kurtuluş and Yolcu, 2013; Olkun, 2003).

The aim of this study is to investigate 5th grade students' ability of orthographic and isometric drawing through an exploratory case study method. The students had five class hours practice including determining the number of unit cubes of the structures and to make isometric and orthographic drawings of the structures.

According to the observation notes some students had difficulties in isometric drawings. They didn't have any problems in orthographic drawings when they construct structures with unit cubes. Students only considered the cubes they could see while finding the number of cubes. According to the content analysis of students' answers to the Isometric and Orthographic Drawing Test developed by the

researchers, they had difficulties in isometric drawings of 3-D structures given with orthographic drawing. Content analysis results will be discussed in detail by giving examples of students' answers.

Key Words: Orthographic drawing, isometric drawing, spatial visualization, 5th grade

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Preschool Teacher Candidates' Content Knowledge on Two Dimensional Shapes

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ABSTRACT

Preschool children are expected to comprehend geometrical concepts such as; "circle", "round", "triangle", "square", "rectangle", "oval", "edge" and "angle" (MONE [Ministry of National Education], 2013). Therefore, preschool teachers should have sufficient knowledge on these concepts. The purpose of this study was to investigate prospective pre-school teachers' content knowledge on two dimensioned geometrical shapes. *Content Knowledge* refers to educators' knowledge and their manipulating the knowledge in their mind (Shulman, 1986). For this purpose, 46 second grade and 51 third grade prospective teachers, who were decided according to *Convenience Sampling* participated this study. This sampling method ensures us to study with accessible and practical samples (Creswell, 2012). Third graders have attended a course on mathematics education regarding pre-school terms, but second graders not. A written interview form developed by researchers was offered to participants. They were expected to define, to give examples of daily life objects and to draw examples for each shape (circle, round, square, triangle, rectangle and oval). Data were *descriptively* analyzed. Definitions and drawings were analyzed according to correctness. Correctness category includes correct, partially correct or inadequate sub-categories. Examples of daily life objects were categorized into scientific, synthetic or naïve model sub-categories, as Saçkes & Korkmaz (2015) used in their study. Findings were represented by percentages and frequencies tables. The results of the study show that majority of both 2nd and 3rd grade prospective teachers defined each shape partially correct, but they defined oval concept, inadequately. Examples of daily life objects for each shape were mostly synthetic and naive models. Majority of prospective teachers drew mathematically correct examples for each shape. And also, majority of them drew examples as prototypical representations for each shape.

Most of them defined each shape partially correct but drew mathematically correct examples for each shape. More practical courses, manipulatives related to the mathematical concepts and enriched training programs will help prospective teachers to develop their content knowledge regarding geometrical shapes.

Key Words: Pre-school, mathematics, prospective teacher, content knowledge, shape.

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Preschool Teachers' Concept Definitions: The Case of 2d Shapes

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ABSTRACT

Teaching in pre-schools should be based on a communication between pre-school teachers, children and the content of the curriculum. Pre-school teacher should be able to plan the content of teaching. For this reason, teachers' Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), which influences what and how to teach, are important factors for a quality instruction (Bäckman & Attorps, 2012).

Geometry teaching in preschool period is a challenging task for teachers. The preschool teacher has an important role in fostering children's geometrical knowledge (Tsamir, Tirosh, Levenson, Barkai & Tabach, 2015). Preschool teachers must go beyond providing children with just shape-naming activities. Teachers are also required to help children distinguish complementary features like sides and corners of shapes based on various characteristics such as dimension, transformation, orientation, rotation, kurtosis and skewness. In addition to these, teachers must use mathematically correct terms when they introduce notions of geometry to children (Clements & Battista, 1992). Research shows that preschool teachers have difficulties in teaching geometry and have insufficient knowledge and skills to do so (İnan & Temur-Dogan, 2010; Zembat, Sezer, Koçyiğit & Balcı, 2014).

We think that preschool teachers' subject matter and pedagogical content knowledge can influence their geometry teaching. Thus, it is important to investigate how geometric shapes, especially two-dimensional ones are definition, recognized and taught by preschool teachers. Therefore, the purpose of this study is to examine the concept definitions of two-dimensional geometric shapes (triangle, rectangle, circle and square) of preschool teachers. It is also aimed to investigate how preschool teachers use correct and precise mathematical language when defining two-dimensional geometric shapes. This a case study and the participiants were 15 preschool teachers from various pre-school education institutions in Samsun who participated voluntarily. The data of the study were obtained using semi-structured interviews and analysed and interpreted through content analysis technique. The results of the study showed that pre-school teachers had some problems when describing geometric shapes, and most of them could not use the mathematical language effectively.

Key Words: 2d shapes, geometry, preschool teachers

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Preservice Elementary Mathematics Teachers' Concept Images Related to the Ratio and Proportion Concepts

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ABSTRACT

Mathematics is a science based on concepts and processes with a certain and logical order [1]. We can see mathematical concepts; mathematical theorems, and mathematical ideas as the basis upon which to base their definition. Therefore, in a study conducted in the name of mathematics, concepts have an important place. The importance of mathematical concepts is also emphasized in secondary school curricula and it is shown among the aims of the curriculum that the students can understand the mathematical concepts and use these concepts in daily life [2]. There are many models about the process of formation in the minds of the concepts that have important roles in teaching. One of these is the concept image-concept definition model that Vinner put forward in his 1983 work [3]. In his work Vinner described the concept image as 'consists of all the cognitive structure in the individual's mind that is associated with a given concept.'. The concept definition is, to be a form of words used to specify concept [4]. Ratio and proportion is commonly used in daily life [5] which is related with other learning field as well [1]. The preservice teachers who are teachers of the future should be brought out the concept of ratio and proportion can be useful in terms of determining key point of teaching this concept and related to this concept misunderstandings can be prevented. The aim of this study is therefore to identify the images of preservice elementary mathematics teacher at different grade levels on the concepts of ratio and proportion. To this end, a ratio-proportion test consisting of 10 questions was developed by the researchers. While the test questions were being developed, two experts were consulted to see if the data collection tool was suitable for the purpose to be used. Qualitative research

method will be used in the research. The obtained data will be analyzed by content analysis technique. The data obtained in the research will be analyzed by content analysis technique. Content analysis will be able to explain concepts and relations the data that is collected for the purpose of reach is a method of data analysis [7]. It is among the expected results of the research; preservice teachers have a limited sense of proportion and proportion concepts, even if they define that the ratio is a multiplicative relationship between two multiplicities, they do not consider it in other questions and as Vinner stated in his 1983 study [3], students responded with images in their minds without resorting to definitions when solving questions.

Key Words: ratio-proportion, concept image, concept definition

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Pre-service Mathematics Teachers' Construction Process of Nine Point Circle with Geometers' Sketchpad: The Case of Emre

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ABSTRACT

The purpose of the study is to investigate pre-service teachers' construction processes of nine point circle in the context of Geometers' Sketchpad (GSP). Its six points were discovered by Euler in 18th century and other three points were added by Brianchon and Poncelet in 19th century (Villiers, 2005). This qualitative research is a part of a study that was implied in the course of Computer Assisted Mathematics Education. Participant of the study is a pre-service teacher in the 4th year of Undergraduate Program in Mathematics Education at Ondokuz Mayıs University and he had experience about GSP during the course. While he tried to construct nine point circle by individually with GSP, researchers guided him. During his construction process, he has been wanted to think aloud and all the process was recorded with camera. APOS theoretical framework (Arnon, Cottrill, Dubinsky, Oktaç, Fuentes, Trigueros and Weller, 2014) is used for content analysis (Yıldırım and Şimşek, 2006). To construct nine point circle properly, it was accepted that he had information about the concepts like median, angle bisector, height, inscribed circle, and circumcircle. The results of the study shows that he had difficulties to construct a circle whose three points were known. Although his deficient prerequisite knowledge, he constructed nine point circle and carried out some generalization processes.

Key Words: Concept construction, Geometers' Sketchpad, nine point circle, APOS.

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Pre-Service Mathematics Teachers' Mathematisation Processes In Modelling Activities

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ABSTRACT

Models are conceptual systems which consist of elements, relations, operations, and rules governing interactions. These systems are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other systems so that the other systems can be manipulated or predicted intelligently [1]. Mathematical modelling is defined as a dynamic process which facilitates the ability to see the relationships in everyday life problem, discover and write the relationships between them in mathematical terms, classify, generalize and draw conclusions. As an outcome of this process, a mathematical model is formed to solve real life problems. Furthermore, Mathematical modelling provide an opportunity to describe the problem situation, establish a bridge from real world to mathematical world, manipulate the model to make predictions related to the problem situation, translate the results gathered in mathematical world to real life, and verify the model [1]. During this process, problem solvers share their mathematical thoughts by using and creating representations. In this study it is aimed to explore the mathematical understandings and mathematisation processes that pre-service mathematics teachers used in constructing their models.

This study was conducted in the province of Rize during the 2016-2017 academic year. The sample of the study consisted of 10 group of pre-service teachers in the 4th year of elementary mathematics education programme. The data collection tool utilized in the study was four mathematical modelling activities designed by [2]: Foot Print, Which one is the best seat?, Summer Job Problem, and Jumping Ball.

The study employed the case study which is one of the qualitative research approaches. As for data analysis, the descriptive analysis method was utilized. The

pre-service teachers' models in the data collection tool were evaluated in terms of the nature of the problem factors that pre-service teachers chose to consider, the types of transformations they made through the operations they applied, and the representations they used. The frequency values for these solutions evaluated within this scope have been presented in a table.

Based on the findings, the pre-service teachers' solutions of mathematical modelling activities entailing daily life situations were found to be at a moderate mathematical thinking. They used varies representations such as table, graphics, written symbols, formula. They integrated these multiple representations effectively to solve these daily life problems. Furthermore, as creating their model, the pre-service teachers worked with some or all of the problem factors. For instance, in the summer job problem they focused on work load, months, earning money per hour, etc. Pre-service teachers' operations on these factors included variety of operations such as: finding units, using interval quantities, aggregating quantities, using informal measures of rate, etc. Based on these results, various recommendations have been made for further studies.

Key Words: Mathematical Modelling, Mathematical knowledge

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Pre-service Teachers' Views about the Solution Suggestions for Overcoming the Student's Mistakes and Misconceptions

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ABSTRACT

One of the components of pedagogical content knowledge that teachers should have is the knowledge of understanding the students (Shulman, 1986). It is expected from the well-trained teachers to identify the mistakes and misconceptions that students have and teach them the truth (MEB, 2017). In this study, it is aimed to reveal the pre-service teachers' solution suggestions for overcoming the student's mistakes and misconceptions about the subject of decimals. The case study method was carried out in this study. It was conducted with 65 pre-service elementary mathematics teachers being trained at the Faculty of Education at a state university in Black Sea Region of Turkey. The data was obtained from the test composed of six open-ended questions on the subject of decimals about density, operations, etc. The content analysis technique was used in analyzing the obtained data. The solution suggestions that the pre-service teachers expressed mainly gathered in the use of concrete materials and connecting the knowledge of fractions and digits of decimals for overcoming the student's mistakes and misconceptions. Furthermore they merely offered instructional expressions such as modeling and giving examples from the daily-life. The concept of decimal digits was explained by different methods such as decimal place value chart, writing decimals in expanded form and writing place values. Decimals related with fractions as converting, simplification, expansion and operations. Using the base ten blocks and the base ten blocks cards as a concrete material was stated almost all of the pre-service teachers.

Key Words: decimals, student mistakes, pedagogical content knowledge, pre-service teachers

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Primary School Teacher Candidates' Pedagogical Content Knowledge on Quadrilaterals

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ABSTRACT

The purpose of this study is to determine the level of primary school teacher candidates' pedagogical content knowledge about quadrilaterals. In line with the purpose of the study; the primary school teacher candidates' instructional statements towards determining, identifying and correcting the mistakes that primary school students made about quadrilaterals have been examined. Participants of the study is constituted of 83 4th grade primary school teacher candidates studying in the primary education department of education faculty of a university. The participants were selected by using purposeful sampling method which is among non-random sampling methods. People, events, objects, or situations that provide certain criteria for the detailed study of situations that are thought to have important and rich information are included in the study (Patton, 1987). Purposeful methods were used in this research because primary school teacher candidates were required to have mathematics instruction skill.In the research, a structured interview form consisting of six open-ended scenarios was used. The scenerios used in the study were developed by using the related literature (Ball,1988; McCoy, 2016; Stecher et al., 2006) and the experiences of the researcher. In developing data collection tool process, firstly a scenario pool consisting of 14 scenarios was composed. Then scenarios that were not suitable for the purpose of the study and measuring similar skills were removed from the interview form in the direction of two field experts' opinions and the data collection tool was finalized. Data obtained from the study were analyzed by content analysis method from qualitative data analysis techniques. Primary school teacher candidates' answers to the scenarios were analyzed in the context of describing the mistake, offering a solution to eliminate the mistake, and suggested teaching methods. Also, data were scored by two researchers to ensure

the scoring reliability. The scoring reliability calculated by using the formula of by Miles & Huberman (1994) (consensus / consensus + dissensus) was found as 0.93. At the end of the study, it was seen that primary school teacher candidates were partially successful in identifying and describing student mistakes related to quadrilaterals. Nevertheless, it has been seen that the primary school teacher candidates failed to offer a solution for correcting the mistakes. Also it was concluded that primary school teacher candidate's prefered to use mostly deductive methods in offering solutions for correcting mistakes. In addition, the inadequacies of primary school teacher candidates 'content knowledge were seen as a major obstacle to understanding and correcting student mistakes.

Key Words: pedagogical content knowledge, teacher candidates, quadrilaterals

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Problem Solving Approaches of Preservice Teachers

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ABSTRACT

Problem is the activity about which the person doesn't find a way out, for which he or she struggles to solve and is the thing that arouses the need to solve. According to Dewey, if the problem is the state of ambiguity in human brain, then, problem-solving is the elimination of these uncertainties. Generally, problem-solving is to make a conscious research to reach a target which is designed clearly but not easy accessible (Özsoy, 2005). As for problem-solving in Math, it is the elimination of the problems with mental processes by using necessary notions (Altun, 1995). This research aims to analyze the importance of the problem-solving in education and training, and to measure the prospective teacher's skill on this.

In order to determine the problem-solving ability of the math preservice teachers, a survey was implemented with 35 students, in total, of faculty of education, department of mathematics in the 3rd grade of 19 Mayıs University in 2016-2017 academic year- fall semester. This research employs case study of the quantitative research. For 13 weeks (39 hours) ongoing research, in order to investigate the problem-solving skills, the pupils were asked of the Polya's (2004) problem-solving phases that consist of four steps and they were also introduced by problem-solving strategies. Within the process, various applications regarding the problem-solving phases were implemented. As data gathering medium, a problem, which was written by Posaimenter (2010) and was previously translated by Turkish researchers, was used. The collected data were codified by two researchers. Then, the percent conforming were spotted. As a result, this research reveals that the problem-solving ability of the preservice teachers improved and it was clear that there happened a increment in their skills of choosing the suitable strategy and of implementing in positive way.

Key Words: Problem solving, problem solving stages, math preservice teachers.

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Processes of Solving Linear Figure Pattern Problems of Primary School Students at Different Learning Levels

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ABSTRACT

The concept of pattern, which is a regular ordering of mathematical objects such as numbers and shapes, is also the heart and essence of mathematics [1]. Pattern searching is one of the skills that is essential in mathematics learning, because it needs to determine or explain similar or different characteristics of concepts [2]. As it is important to examine the patterns and relationships that mathematics contain to understand the structure of mathematics, as in the curriculum of many countries, the pattern topic is in all the learning levels in the new elementary mathematics curriculum in our country. Because in all class levels "understanding patterns, functions and relationships" is a constant issue for algebra [3]. Indeed, mathematics education researchers have stated that pattern searching and generalization take place on the basis of algebraic reasoning and thinking and argued that in mathematics, especially in algebra, a generalization of patterns of everything [4]. However, at the primary school level in the international literature related to the pattern concept it made a lot of research, but there has been little study of patterns in primary level in Turkey. whereas, it is known how young children construct patterns, how they operate their cognitive processes in this structure, and how they have a mental structure, they are important to the development of algebraic thinking and reasoning skills. As a matter of fact, the necessity of learning the contribution concept and pattern concept that the students will bring to the development of algebraic thinking and reasoning skills should be learned from the pre-school education level. In this context, in addition to analysing the process of solving the pattern problems of the students at different levels of primary education, it is expected that it will contribute to the field such as examining the algebraic reasoning processes on the patterns of the students in this level, determining the problems occurring in the process and taking measures for solving these problems. The purpose of this study is to examine the process of solving linear figure pattern problem problems (in terms of different variables) of primary school students in different levels of education (2nd, 3rd and 4th grade) and to interpret reasoning skills through the findings obtained.

This research is a mixed methodology study involving both quantitative and qualitative research. The sample of the research consists of 62 students who attend a primary school in a province centre in the Eastern Black Sea Region. The data of the study were collected through a test and interviews. The data obtained from both data collections were evaluated in terms of different variables (problem solving, problem type, assistance questions, learning level, gender); quantitative data were analysed with descriptive and inferential statistics, and qualitative data were analysed with descriptive analysis.

From the obtained test data; it has been determined that as the learning level increases, the number of students reaching the correct result increases, the type of problem affects student performance, students are more successful in finding close terms, where students use more recursive or additive reasoning methods, functional or covariational reasoning methods are used less frequently. From the interview data obtained with this; it has been found that the students perform more successful solutions than the tests, students want less assistance in some problem types, the tendency to get assistance decreases as the level of learning increases, the increase in the number of students using functional or covariational reasoning methods through special assistance questions, and there is not much difference in terms of gender. It has also been determined that students who use functional or covariational reasoning methods perform more successful solutions and generalizations, and as the level of learning increases, these situations also increase.

Key Words: Pattern, generalization, reasoning, 2-4th grade

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Prospective Preschool Teachers' Pedagogical Content Knowledge on Quantity Concepts in Terms of Children's Mistakes

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ABSTRACT

In this study we examined the prospective preschool teachers' pedagogical concent knowledge in terms of identifying mistakes, determining the possible causes of mistakes and offering solutions for eliminating the mistakes. In other words, this study will determine the prospective preschool teachers' knowledge levels of understanding students and instructional strategies related to quantity concepts. The case study method, one of the qualitative research methods, was used in order to investigate prospective preschool teachers' pedagogical content knowledge on quantity concepts. The participants of the study were composed of 94 prospective teachers studying in the preschool education department of a university in Turkey. 52 of them were 2nd grade and 42 of them were 3rd grade graduate students. The participants were selected by using convenience sampling method which is among non-random sampling methods. In convenience sampling method, participants of a study can be selected on the basis of accessibility and expedient (McMillian& Schumacher, 2010). In this study, five open-ended vignettes were used as data collection tool developed by researchers. The data obtained from open-ended vignettes were analyzed through the summative content analysis technique. Summative content analysis enables us to categorize prospective teachers' answers according to the themes or categories already defined by researchers (Hsieh & Shannon, 2005). In this study, themes were created for identification of mistake as type of mistake, source of mistake, the way of teaching and elimination of mistake for each vignette. In this study, prospective teachers' written anwers to all vignette were analyzed by two researchers. Coding reliability was conducted because of two researchers coding the data independently from each other. In this context, we coded all data set and calculate a coding percentage (Yıldırım& Şimşek, 2011). We found coding reliability 0.91 by using formula of Miles & Huberman (1994). As a result of the study, it was observed that the prospective teachers partially succeeded in identifying the students' mistakes in many vignettes. However prospective preschool teachers was not at an adequate level in correcting students' mistakes on quantity concepts.

Key Words: Pre-school, prospective teacher, pedagogical content knowledge, quantity.

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Realistic Mathematics Education and Derivative Learning Opinions of 12th Grade Students about Realistic Mathematics Education and Derivative

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ABSTRACT

According to some mathematical thinkers, mathematics which is human activity can not be discovered, it is invented [1, 2]. Freudenthal, who has this idea, emphasizes that mathematics must be understood by putting forward the idea of realistic mathematics education. In other words, he stated that students should first discover mathematics in informal ways and then perform learning in formal ways. There are many studies related to realistic mathematics education in the literature but most of them are the level of primary education [3, 4]. The studies at secondary level are few and generally carried out quantitatively [5]. However, realistic mathematics education can provide positive results not only at the level of primary education but also at the level of secondary education and it is a method that will provide students with meaningful learning of mathematics. In this respect, it is important to evaluate the education of secondary school students with realistic mathematics education. This study was carried out to obtain the opinion of the students that learned the derivative using realistic mathematics education about realistic mathematics education and derivative. In this study, case study method was used from qualitative research designs. 9 students who are the 12th grade students of Anatolian High School in a province that located in northeastern Turkey participated to study. The data of the study were collected by a semi-structured interview form. We asked the questions such as "How did you find the training you received with realistic mathematics education?", "Have you ever done such a lesson before?", "Do you believe that this course adds something to you?" to students. The answers were analyzed by means of content analysis method. The findings of the study show that students have not taught with realistic mathematics education before and that they are satisfied with our work. As a result of the study, it was determined that realistic mathematics education attrach the attention of the students by giving them a lot of examples related to real life situations and the students who are learning with realistic mathematics education think that they can learn by this method more easily than other methods.

Key Words: Realistic mathematics education; derivative; secondary education.

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Reconfigurable Kalman Filter For Estimation Of Helicopter Flight Dynamics

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ABSTRACT

In this study, sensor fault detection, isolation and accommodation issues are discussed to detect the most suitable data from the sensors in the helicopter model. Fault detection and isolation methods based on the innovation sequence of the Kalman filter have been used to detect faults in the sensor, which are crucial for the aircraft's mission to perform successfully and that affect the helicopter dynamics [1]. The modeled data was first processed with Optimal Kalman filter. As a result of the statistical analysis of the innovation sequence of this filter, sensor detection and isolation were carried out [2]. Then the robust Kalman filter and reconfigurable Kalman filter helped to correct the fault. Sensor failure detection and isolation algorithms have been tested with Optimal Kalman filter. The robust Kalman filter and the reconfigurable Kalman filter are designed to correct the effects of sensor failures. Robust Kalman filter has been used to consider and counteract the situation that one of the measurement channels of the system is distorted [3]. When the Optimal Kalman filter algorithm fails, robust Kalman filter algorithm based on the scaling of the covariance matrix is used to ensure that the normalized innovation process is kept in the confidence interval [4]. The reconfigurable Kalman filter algorithm is used to continue operate in the presence of sensor faults [5]. The 12 states of the helicopter model that were obtained with simulations express the flight time of 10 seconds. In the developed algorithms, different sensor failure tests have been applied to correct the sensor faults. To test the algorithms, three different sensor fault conditions have been investigated in terms of continuous bias, noise increment and "0" sensor output [6]. Sensor fault simulations are presented for the angular velocity of pitch ‘q’. Simulation results show that both of Kalman filter algorithms estimate the helicopter state parameters with high accuracy. However, the error value of Robust Kalman

filter was found to be higher due to the use of fewer measurement channels. In the case of a faulty sensor has been investigated the estimation of state coordinates by simulation based on different Kalman filter methods and suggestions have been given to increase the estimation accuracy in this case. Simulations have shown that all three different fault conditions are detected in the sensors. Correction of the fault effect with the help of Robust and reconfigurable Kalman filter is shown and it is stated that it would be designed simply that is useful for algorithms. The results support previous work on sensor fault detection, isolation and accommodation and shed light on future work

Key Words: Kalman filter, Fault detection, Fault isolation, Fault tolerant estimation, Sensor faults, Robust Kalman filter, Reconfiguration.

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Reflection From Teaching Using Eba: The Case Of Triangle Basic Elements

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ABSTRACT

Developments in technology are affecting all aspects of daily life and leading to major changes in education as well. Along with the developing technology, many countries encourages the use of technology in their educational system and makes necessary infrastructure investments with various projects. Our country, along with the FATIH Project, has inaugurated one of the world's most comprehensive technology projects. This project aims effective use of technology in lessons by providing all classes with İnternet and interactive (smart) boards and each student per tablet/laptop. This situation will inevitably cause some differences in class practice. Based on these consideration, this study investigate how teacher-student interactions and teaching practices differ in a classroom where EBA activities are used with the help of interactive board and computers. It focuses on the teaching sequence of ‘triangle basic elements’ with 9th grade students. The data of the study, which was carried out with action research method, was collected through video records, researcher diary and observation notes. The results showed that teachers and classmates’ influence on a student's commenting and decision-making process has decreased as the students performed individually in interaction with computer. While in a classical learning environment, students are urged to do an activity or to solve problems as soon as possible, in such an environment every student maintains activities at his/her own learning speed. This caused students to face different number of problems, during the evaluation period. Besides, students were not afraid of making mistakes as they received feedback only from either the computer or the teacher. Lastly, computer aided learning environment; contrary to the classical learning environment, require the teacher to have more dialogue with less successful students.

Key Words: Technology enhanced teaching, student and teacher behaviours, FATIH Project.

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Reflections on Instruction of Inequality and Absolute Value: A Research on the Action

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ABSTRACT

It is reported that, in solving absolute value inequalities, students tend to use the methods they use in solving equations directly and this trend leads students to find a single value instead of finding a solution set in the solution of inequalities. (Demetgül, 2018). It focuses them on finding a value for the unknown. It is proved by students using equal symbols instead of inequality in their answers. Another misconception in the literature is the definition of absolute value. By making an excessive generalization, student accepts that absolute value will always be positive. Thus, not taking it into account that the absolute value can less than zero, students directly look for to the solution. One of the mistakes in the solution of inequality ıs that the symbol of inequality isn't reserved while unravelling the – symbol ın front of the unknown. (Orientation of inequality) For this purpose, it is considered as an action research in the teaching of absolute value and inequalities constitutes a learning environment. The research was conducted in a high school class.

Although there are three different definitions of the mathematical absolute value, the definition of distance is made primarily in textbooks. But when it is time to the solution of inequalities this causes the student to not make sense of the solution. It also makes it difficult the transition to the use of other definitions in the resolution of inequalities. One of the reasons why the students confuse the equation and inequality is that both are introduced as the process of finding the unknown. (Garuti, Bazzini ve Boero, 2001). Algebra learning area ıs a learning area that inequalities, equations and basic mathematical skills comes up again. The situation is the same for the 9th grade students. Of course, this special case study cannot be generalized for the whole country however, it can be thought that the obtained cross-section and

cross-section represent any Anatolian high school in the country and the students who are ın the band of 4.00-4.50 points in TEOG represent to some extent.

Clinical interviews with 8 (4 girls 4 boys) 9th grade students in Anatolian high school students were tried to be analyzed. 26 students ' solutions were divided into 2 groups, right, close to the right and incomplete, and then 8 students were selected from both groups ın order to implement clinical interview

Keywords: Absolute Value, Inequality, Learning Disability.

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Reflections on Pattern Generalization Processes of 8th Grade Students

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ABSTRACT

Generalization which is both an aim and a tool of thinking and communication, it is expressed as the focus of mathematical activities and based on the development of mathematical knowledge. Generalization which is finding pattern in a certain situation or event and collecting it with a thought; is an abstract at the same time as this and is the essence of doing mathematics [1]. Because patterns are the heart and soul of mathematics; even algebra, and indeed all of mathematics is about generalizing patterns. Although pattern generalization is at the focus of studies on this field, it is increasingly important in recent studies on justification, which is considered to be an inseparable twin of generalization. Justification; can be defined as justifications presented to convince oneself of an event or situation that has been decided or that the person has sufficient evidence to verify his claim [2]. At the same time, justification is the process that supports generalization [3] and engaging in justification effects a student's ability to generalize [4]. As a matter of fact, in recent years it is important to acquire skills such as generalization, justification and reasoning in mathematics courses and especially pattern activities can contribute to the development of these skills. Thus, many mathematics educators have dealt with patterns from different points of view and agreed on the idea that discovering and generalizing patterns are important for learning mathematics. They have also expressed that the study of patterns could improve students' algebraic concepts at early ages, and contribute to that algebraic thinking required in future learning. Many studies have been carried out at international and national level on the generalization of the patterns and patterns especially at primary and secondary school level.

However, a limited number of studies have been found that include beliefs about pattern generalization, generalization strategies, justification schemes, pattern type effects on generalization and generalization strategies. For this reason, the aim of the study is to examine the generalization strategies of the 8th grade students on the linear figural patterns, justification of how they form the generalizations, the effect of the pattern format on the rule sets and the opinions about what is the most useful generalization strategy in rule building.

This research is a mixed methodology study involving both quantitative and qualitative patterns. The sample of the research consists of 125 eighth grade students who study in two different secondary schools in a province center in the Eastern Black Sea Region. The data of the study were collected through two different tests, a questionnaire and interviews. Tests prepared by taking into account the literature and the curriculum are designed to determine students' generalization strategies and justification types. The survey adapted from the literature is designed to determine what beliefs students have regarding the most useful generalization strategies. In addition, interviews were conducted with nine students to determine the effects of the generalization strategies selected by the students in their rule-making. The data obtained from the tests and the questionnaire were analysed with descriptive and inferential statistical methods while the data obtained from the interviews were analysed with descriptive analysis method.

Approximately half of the learners were able to develop a functional rule in the generalization of the linear figural patterns. It was found that the students used notations, words and alphanumeric forms in the representation of functional rules and the students used strategies such as explicit, recursive, numerical, figural, guess and check. The reasoning schemes used by the students are; justifying without diagram, justifying with diagrams, justifying recursive rules with numeric values and justifying through explanation. In addition, students often believe that explicit, recursive, constructive and reconstructive approaches are the most useful strategies. With these interviews, however, the students were asked to form a rule with the strategies they found most useful, but half of the students (especially successful students) were able to establish a correct rule.

Key Words: Pattern, generalization, justification, 8th grade

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Secondary School Mathematics Teacher Candidates' Competence in Mathematical Proof Methods

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ABSTRACT

As a scientific discipline, Mathematics aims to enable people to reasonably answer to questions such as 'why' and 'how'. As a result of these questions, the need for proof has arisen. Although mathematical proof is regarded as an indispensable part of mathematics, it is observed that the importance and role of proof is not adequately emphasized in teaching programs, and mathematicians perceive the term as something demonstrating sufficient proof for the validation of mathematical statements [1]. For this reason, it is becoming increasingly important in advanced-level mathematics courses to focus on the proofs to mathematical propositions belonging to mathematical concepts.

The importance of proof in mathematics can be summarized as follows.

- In order to confirm the accuracy of a hypothesis, there is only one way: giving the mathematical proof.
- It is agreed that proving is a process leading to perceive and understand. Being able to prove means that the studied problem is understood and perceived. Here, however, there is more. The effort to prove a hypothesis may occasionally lead to in-depth understanding and perception of the theory within the question (problem). Even the proving fails, one can gain depth in knowledge and understanding [2].

Proof methods in mathematics are; direct proof (the method in which by assuming p is true, truth of q is shown in the proposition $p \Rightarrow q$), proof by contradiction (the method in which the statement to be demonstrated as true is accepted as false. Then, one can prove that it is false, and reach the contradiction) counterexample (in $p \Rightarrow q$ proposition, finding a case in which p is true and q is false), induction, deduction and test. This study aims to find out secondary school mathematics teacher candidates' competence in proof methods. The study was conducted in a university in Black Sea Region in 2017-2018 school year. In the study, a data collection tool consisting of 6 questions covering each proof method is used. The questions in the tool were written after being counselled by experts after literature review. As research pattern, being a kind of case study, one-case pattern was used. The analysis unit of the study is secondary school mathematics teacher candidates. Sub-units of this unit are consisted of a total of 75 teacher candidates; 27 of whom studying at second grade; 24 from the third grade; and 24 from the fourth grade of the department. For data analysis, being a qualitative research method, categorical analysis technique of content analysis was used. The aim of using this type of analysis is to subject the determined categories to a frequency analysis. The findings obtained from the study demonstrate that the teacher candidates are weaker at proof by contradiction method compared to other methods. The most successful class level was the third grade. A possible explanation for this success might be that they receive proof-oriented courses in that academic year. In this context, further proposals are made for later researches.

Key Words: Proof methods, secondary school mathematics teacher candidates, competence

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Secondary School Students' Attitudes Towards Mathematics And Its Relationship To Science, Technology, Mathematics And Engineering Professions

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ABSTRACT

The scientific and technological developments in the 21st century require well educated individuals. Therefore, developed and developing countries try to improve their aims related to science, technology, mathematics and engineering fields that contribute to science in their curriculum. Especially in recent years, studies in this field have been increasing in Turkey. Implementing technology-supported teaching environments in education by the Ministry of Education is an apparent example of the statement above. It is significant to identify students' attitudes in science, technology, engineering and mathematics and the factors that affect them. All these developments lead to the idea that the attitudes towards mathematics, which is fundamental to these sciences, determine the level of relation to these sciences. Therefore, it is important to establish the relationship of secondary school students' attitudes towards mathematics to science, technology, mathematics, and engineering professions. This research was conducted according to the quantitative research design. The purpose of this study is to anlyze the relationship of secondary school students' attitudes towards mathematics to science, technology, mathematics, and engineering professions. Accordingly, the relation between students' views on science, technology, mathematics and engineering and mathematics attitudes was compared. As part of the research, "Attitude Scale Towards Science, Technology, Mathematics and Engineering Professions" and "Attitude Scale Towards Mathematics" developed by Önal (2013) were used by Koyunlu Ünlü, Dökme and Ünlü (2016). The research was conducted in 2017-2018 academic year. Confirmatory and explanatory factor analyzes were carried out for the validity and reliability analyzes of the scale. The scale consists of four sub-dimensions and a total of 22

items. The internal consistency coefficient (Cronbach's alpha coefficient) for the whole scale was .90. The internal consistency coefficient (Cronbach's alpha coefficient) of the factors forming the scale was 0.89 (item number 10) for "Interest", 0.74 (number of items 5) for "Anxiety", 0,69 4) and 0.70 for "Necessity" (item number 3). The population of this research is composed of 6th, 7th and 8th grade students who are studying in secondary schools in Sivas city center. The sample of the study included 12 schools. The city center of Sivas is divided into four education regions. Three secondary schools were selected from each education region. The maximum diversity sampling method was used in the selection of the sample. The data were analyzed using the SPSS 22 program. Mean scores, standard deviation (SD) and Pearson product-moment correlation analysis technique were used to analyse the data. The results showed that students' mathematical attitudes, interest, anxiety, necessity, and working sub-dimensions are positively related to science, mathematics, and engineering interests. Contrary to this result, there is no relation between the anxiety and necessity sub-dimensions of technological interest and mathematics attitude.

Key Words: Science, Technology, Mathematics, Engineering, STEM, Attitude

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Self-Evaluations of High School Students Regarding to Own Metacognitive Behaviours In Problem Solving

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ABSTRACT

The aim of this study is to investigate metacognitive knowledge and skill situations through self-assessments in terms of metacognitive behaviors that high school students are experiencing in this process by observing their problem-solving experiences. The study was designed as a qualitative study and the obtained data were interpreted by frequency analysis and descriptive analysis. The working group constitutes a total of 94 students in the 9th and 10th classes in Akdağmadeni Anatolian High School located in the Akdağmadeni district of Yozgat province in the first semester of 2015-2016 academic year. To collect the data, the students were given 2 problems which were selected from the subjects that they were in the process during the semester. Students are asked to clearly solve these problems and write down what they think. After solving these problems, a questionnaire which prepared to evaluate students' metacognitive behaviors in 1991 by Fortunato, Hecht, Title and Alvarez. After solving these problems, in order to evaluate the metacognitive behaviors that occur in problem solving, the questionnaire developed by Fortunato, Hecht, Title and Alvarez in 1991 was applied to the students. This questionnaire consists of 21 articles and 4 sections. The whole application lasted approximately 1 lesson (40 minutes). When the data obtained from the questionnaire are analyzed and interpreted, it can be considered that the majority of the students have high metacognitive behaviors in problem solving. However, when we look at the solutions students make to the problems, it seems that they do not match these results. In other words, metacognitive behaviors that students say they have shown and metacognitive behaviors that arise from examining problem solutions do not usually coincide with each other.

Key Words: Metacognition, self-evaluation, high school students.

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Self-regulation skills in Pre-service math teachers' as learning and as teaching roles: A case of Algebra

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ABSTRACT

Self-regulation skills include individuals' ability to plan, evaluate, and organize their own learning processes, and these skills are important for improving individuals' academic success and in the development of lifelong learning. Many studies show that secondary school teachers' practice affects the development of their students' self-regulatory skills. Some studies show that teachers who have high self-regulation knowledge and high beliefs practice in the teaching process also improve the self-regulation skills of their students. Therefore, teachers must develop their self-regulatory skills so their students gain these learning skills as well [1]. In this context, teachers are expected to transfer self-regulating skills they exhibit during their learning process to their students through their teaching activities. Many studies examine the self-regulation skills of teachers, preservice teachers, or students. However, a limited number of studies examine teacher candidates' self-regulating skills during their learning processes and the self-regulating skills in their teaching processes [2]. This study aimed to examine and compare mathematics teacher candidates' self-regulatory skills in the algebra learning-teaching process.

The present study conducted with case study model that is a qualitative research design. Elementary math teacher in a university study to the east of Turkey to end the ongoing class participated in two pre-service math teachers. A purposeful sampling method has been used in the selection of the study sample. An interview and observation form was used to collect the data of the present study. In the data collection process, the first researcher gave a lecture on Algebra Instruction to the candidates of mathematics teachers as three class hours each week for 3 weeks. In these lessons, students were taught about what is algebra, the benefits of algebra

learning, the place of algebra in the curriculum, the historical development of algebra, and the transition process to algebra from arithmetic. This teaching process was done with the whole class, and individualized teaching was not done. In this process, the second researcher made observations as a non-participant observer. At the end of each application, pre-service math teachers were interviewed. After completing this application, pre-service mathematics teachers were asked to describe the students in the real classroom environment by giving a topic related to algebra from the middle school mathematics curriculum. These topics include "writing mathematical cues suitable for everyday life situations involving an unknown inequality from the first level" at the 8th grade level, "showing an unknown inequalities in the first degree to the number" and "solving an unknown inequalities in the first degree". Pre-service math teachers were observed in this process and interviewed with teacher candidates after the application. Data collected in the study were analysed through content analysis.

When pre-service mathematics teachers' self-regulation skills demonstrated in their learning process are examined, they are noted for their metacognitive importance and they are motivated by themselves (especially believing that they should learn thinking that they can come out in KPSS). On the other hand, mathematics teacher candidates were not able to monitor the learning process and determine their learning goals adequately.

Key Words: Self-regulation, metacognition, pre-service math teachers, algebra.

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Some Considerations in Teaching Concepts of Eigenvalues and Eigenvectors

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ABSTRACT

The concepts of eigenvalue and eigenvector are among important notions of modern mathematics. These concepts have numerous applications in engineering mathematics, statistics, economics and other fields of science.

In this study, we consider the issues in teaching eigenvalue-eigenvector concepts in undergraduate education. In some textbooks, eigenvalues and eigenvectors are defined as a feature of linear transformations on a vector space and if vector space is a real vector space only its real eigenvalues and eigenvectors are concerned. We know that if the vector space is finite-dimensional then each linear transformation defined on that space is expressed by a square matrix with respect to a chosen basis of vector space. It is said that “If there is no real eigenvalue of the real linear transformation, then the corresponding matrix has no eigenvalue”. To eliminate this problem, it is necessary to use the field of complex numbers in the definitons of vector spaces and matrices. That is, while defining eigenvalue, vector space defined over the field of complex numbers and matrices and vectors must have complex entries. In this case, since each real number is a complex number at the same time, complex numbers and complex vectors must be used in the eigenvalue-eigenvector definiton of real matrices.

In order to support the issues mentioned above, various questionnaires were made among interstudents of the Mathematics Education of Education Faculty and Mathematics Department of Science Faculty. In these questionnaires, the definition of the eigenvalues, the number of eigenvalues, the existence of non-zero solutions of linear homogeneous systems, etc. were asked. The results of surveys support the above-mentioned issues.

Key Words: Eigenvalue, eigenvector, vector space.

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Student Status Determination in Mathematics Course

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ABSTRACT

Mathematics is one of the important tools we use in solving the problems in our daily lives as much as we use in science (Baykul, 2009). In mathematics teaching, students develop many important skills such as how to draw a formula in mathematics, how to get the definitions, how to reach generalizations, how to verify generalizations and how to reason while they learn the meaning and relationships behind a formula (Olkun and Toluk Uçar, 2006). There is no doubt that teachers play a significant role in mathematics teaching. Taking into account the importance of mathematics and mathematics education, the opinions of secondary school students on mathematics teachers and their attitudes in mathematics classes form the basis of this study.

The aim of this research is to determine the relationship between the secondary school students and mathematics teachers by examining the attitudes of secondary school students in mathematics lessons and their opinions about mathematics teachers. The study was carried out with a total of 60 students, 30 girls and 30 boys in Grade 7 and Grade 8 in Atatürk Secondary School in Bafra District of Samsun Province during the first semester of 2017-2018 academic year. This study is based on the case study, one of the quantitative research methods. During the study conducted for 8 weeks, "Teacher Evaluation Survey" published by MoNE (Ministry of National Education) and "Mathematical Attitude Scale" developed by Baykul (2006) were applied to the students. The obtained data was analyzed by IBM SPSS 15.0 package program. As a result of the analyses, it was determined that there was no significant difference regarding opinions of students towards their teachers and the attitudes of students in mathematics in terms of gender and class variables. Moreover, it was found that there was no relation between the attitudes towards mathematics and the opinions about the teacher. Students expressed positive

opinions about their teachers such as teachers are allowing different ideas, encouraging learning, providing effective mathematics teaching and establishing good communication.

Key words: Mathematics teaching, attitude, teacher evaluation.

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Students' Performance in Linear Functions with Different Representations: A Cross-Age Study

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ABSTRACT

Representation is seen as a "useful tool both communicating information and understanding" (National Council of Teachers of Mathematics, 2000, p.64). Linear functions which are one of the important topics in mathematics because of their real life applications include many problem solving tasks with the using of different representations. But several research suggest that linearity and linear functions are complex topics and students have some difficulties in making connections among various representations of linear relationships and functions and with the *x*- and *y*-intercepts. There are not enough studies which demonstrate students' performance in linear functions with different representations in Turkey. Therefore, the research is thought to make contributions to the literature. Within this context, the aim of this research is to explore Turkish high school students' performances in the various forms of representations of linear functions.

The participants in this study were 458 high school students from a public school in Ordu, which is a city in the middle and eastern part of the Black Sea region of Turkey. Of these 458 students 113students were in Grade 9, 111 were in Grade 10, 153 were in Grade 11 and 81 were in Grade 12. The final form of the task includes five open ended questions (Q1, Q2, Q3, Q4 and Q5) pertaining to tabular representation (TR), geometrical representation (GeoR), symbolic representation (SR), verbal representation (VR) and graphical representation (GraR) of linear functions, respectively.

Findings showed that 12th grade students' performance were highest in all representation modes. On the other hand while 9th grade students' performances were in GeoR and SR, 10th grade students were found least successful in TR and VR.

Key Words: linear functions, different representations

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Teachers' Mathematical Modeling Competencies: Task Dimension[4]

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ABSTRACT

Mathematical modeling (MM) can be defined as a mathematical representation of non-mathematical phenomenon [1]. The individuals do not have to comply with predefined standard rules but they have to decide what the problem is, interpret the situation and make mathematical operations in accordance with their interpretation [2]. MM is so important since it not only offers better understanding the world and life which is full of mathematics but also may support the understanding of mathematical content and motivate students to engage with it [3].

Teachers have the most effective role in gaining MM competency. Student learning depends on the quality of teaching which depends on teacher competencies [3]. Teachers' content knowledge and pedagogical content knowledge have implications for the teaching and learning of mathematical modeling. For this reason, there are some teacher competencies for teaching modeling. Ferri [3] describes these competencies in four dimensions: 1)Theoretical dimension: aims and perspectives of modeling, modeling cycles, types of modeling tasks 2)Task dimension: solving modeling tasks (multiple solution), analyses of modeling tasks, development of modeling tasks 3)Instruction dimension: planning a lesson with modeling tasks, carrying out the lessons, interventions 4)Diagnostic dimension: recognising phases in modeling process, recognising difficulties and marks modeling tasks. In this study, task dimension is discussed with the aspect of "analyses of

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modeling tasks". Analyses of modeling tasks can be described as knowing features of modeling tasks and distinguishing modeling tasks from traditional problems.

This study was conducted with 6 secondary mathematics teachers. The participants involved in a workshop that focused on MM. The programme comprised of two parts: the theoretical part that includes aims and efficacy of MM and modeling cycles and the practical part that includes solving modeling tasks, distinguishing them from traditional problems and developing modeling tasks. In order to determine the competence of teachers to analyze modeling problems, three problems were identified; a MM problem (Antique Theater Problem [4]), a transition problem (Elevator Problem [5]) and a word problem (Toy Shopper Giapetto [6]. Then, teachers were asked to explain if these problems are modeling problems and provide their reasons. The results show that the first criteria for determining MM tasks are "real life situation" for all teachers. Then they take into consideration that the problem is thought-provoking, complex, and the emergence of different models. All teachers agree that involving just real life situations is not enough to be a MM problem. Such that, although the three problems involve real-life situations, the Toy Shopper Giapetto has been identified by all the teachers that are not a MM problem. However, most teachers believed that having thought-provoking or complicated features is enough for being MM problem. Four of them thought the Elevator Problem is modeling problem because of its thought-provoking and complicated structure. Furthermore, teachers' consideration of the emergence of different models supports this interpretation. In summary, findings reveal that teachers have similar difficulties in analyzing MM tasks and also results of this research overlap with Ferri's [3] study. Strengthening teachers' theoretical competencies will be effective in eliminating these difficulties.

Key Words: Mathematical modeling, task competency, teacher education.

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Teaching Maths To The Gifted Students Through Differentiated Teaching Method

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ABSTRACT

Gifted and talented students need educational opportunities and environments adequate to their cognitive characteristics during their education. Differentiation of math curriculum is a vital topic regarding the importance of mathematics as a science which has been playing an important role during the development and presence of humanity. Besides, math education is a field in which individual and cognitive differences ocularly stand out.

The purpose of this study is to study the impact of a mathematics program that has been modified to feed the needs of gifted students on their mathematical learning skills, their behavior and the effectiveness of these changes. The "Geometric Shapes" unit that is being taught in 6th and 8th grade Mathematics class, has been differentiated by using the "Paralell Syllabus Model" and "Grid Model" that are used in the education of gifted students. The subjects of this experiment were five 8th grade students that are part of the "Special Skills Development Program" (SSDP) in BILSEM, a government organization that provides education for gifted children in Samsun. In order to gather data in the scope of this research, interviews, a qualitative research technique, have been used. To increase the reliability of this method, tests with open ended questions regarding the unit, have been made before and after the changes. Then, the modified math problems in the "Geometric Shapes" unit were completed by the students. Finally the two grades that were acquired from the tests were compared. After the activity, an interview was made by each student recording their thoughts on their mathematics learning skills, their learning skills in the specific unit, their behavior and the effectiveness of the differentiation. A descriptive analysis method has been used to study the answers. From the results, we can observe that the differentiation of the syllabus has impacted the mathematical learning skills and the learning skills in the specific unit as well as their approach in a positive way.

When the results of the tests that were held before and after the activity were compared, an increasing success rate has been observed.

Key Words: Mathematic education, gifted student, scientific creativeness.

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Teaching Mathematics and Creative Drama: A Sample Lesson Plan

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ABSTRACT

In the study, an implementation aiming at the acquisition of "Unit fractions show and range on the number line" with the creative drama teaching method of the 5th grade secondary school students was carried out. The study group is formed of 27 students studying in the 5th grade of a secondary school in Samsun province. A workshop for teaching the subject of fractions was performed by using creative drama teaching method. At the end of the implementation, students were asked three open-ended questions in order to take their opinions on the learning outcome.

Creative drama is a teaching method that can be used in various fields (Fulford and et. al., 2001). It has also been started to be used as a method in education-training process in recent years. Creative drama has a significant place in programs due to the fact that understanding of conventional education is insufficient and putting the individual in the center of teaching.

After teaching of natural numbers in elementary school classes, especially when the topic of fractions starts to be explained, the students have difficulties in understanding the subject and this situation has a negative effect on the mathematical success of the students and their attitude towards the course (Soylu and Soylu, 2005). For this reason, teaching and learning of knowledge especially about the fractions is important in terms of mathematical education in primary education (MEB, 2005). The studies conducted have revealed that not learning the topic of fractions fully negatively affects the learning of other mathematical topics (Doğan and Yeniterzi, 2011). The reason is that the topic of fractions is the basis of many subjects such as decimal numbers, rational numbers and ratio & proportion (İpek, Işık and Albayrak, 2005).

According to the findings obtained it has been revealed that students learned what they pay attention to while putting the fractions in order and how they compare the fractions according to their greatness in the result of the implementation of creative drama carried out in mathematics class. As a result it has been concluded that, the creative drama teaching method has been effective in teaching fractions and the students acquired new outputs.

Key Words: Creative drama, fractions, mathematics education.

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The Analysis On Scientific Studies About Realistic Mathematics Teaching Approach

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ABSTRACT

The main aim of this study is to analyze the national thesis studies about Realistic Mathematics Teaching and the articles published in Park Academy Magazine and to evaluate their general situation. For this purpose, 34 thesis study published in the years between 2007 - 2018 from high education council national thesis center and 12 articles published in Park Magazine have been examined. Paper examination model has been used in the research. Most of the studies examined consists of post graduate thesis and articles and also there are 7 Phd. thesis. When publishing years have been examined, it has been found out that the publishing mostly were made in 2015 and 2016. When research group has been examined, it has been clear that most of them were students. It has been confirmed that there are 3 studies conducted with both students and teachers and there is only 1 study conducted with teachers. As for stages of learning, they have been examined according to 3 stages; elementary education first stage, elementary education second stage, and secondary stage. It has been seen that the studies towards to Realistic Mathematics Approach were mostly applied mostly for 6^{th} - 8^{th} grades, then 3^{rd} - 4^{th} and little for the 9^{th}-12^{th} grades. Not only quantitative method has been applied for the most of the studies but also there have been some studies which includes qualitative and mixed method. The number of the thesis in which the experimental model is used as the research model has been high. When the studies have been examined in terms of the subjects; ıt has been learned that while the large part of the studies have consisted of practices which try to identify the impact of realistic mathematics approach to the students' success, their behaviors, permanency and motivation and at the same time the impact on visual mathematics literacy and problem solving behaviors, the beliefs related to the realistic mathematics teaching approach have been identified. When mathematics have been examined in terms of subjects, it has been found that the practices have been mostly on the fractions, probability and statistic, ratio and proportion, and length-liquid- time measurement units. According to the findings of the research it is thought that the findings might contribute to the following researches that may be held in the future to see pros and cons of those researches related to the realistic mathematics approach.

Key Words: Realistic mathematics education, attitude towards mathematics course, mathematics success

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The Criteria That Mathematics Teachers Pay Attention While Creating Written Exam Questions and Determining Awareness of Misconceptions in This Context

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ABSTRACT

The aim of this study is to determine the misconceptions in the context of the criteria that elementary education mathematics teachers pay attention to when they formulate written exam questions during the assessment and evaluation process. The participants of the study are 20 elementary mathematics teachers who provide services within the Ministry of National Education and the convenience sampling is chosen from the purposive sampling methods. In order to determine the criteria that the teachers pay attention to when preparing the written exam questions, interviews are conducted with the teachers about the exam questions they prepared for the 7th graders on the basis of the question "Why did you choose this question as an exam question?". With the aim of determining teachers' awareness of the misconceptions, a misconception test is created and applied to the the teachers' students. With the help of the students' answers with misconceptions, the semi-structured interviews are conducted through the question of "What do you think your students might consider while giving this answer?". Content analysis is applied to the data obtained from the research conducted as a case study, one of the qualitative research designs. In the findings obtained as a result of the research; the criteria that the teachers pay attention to while creating the written exam questions respectively are; Conformity to learning levels of learners, appropriateness of learning outcomes, suitability to mathematical thinking skills and conformity to the examination system. The teachers' perspectives towards the students' misconceptions and their awareness levels of misconceptions are concluded as low levels.

Key Words: Scholl exam questions, misconceptions, teacher awareness,

The Discovery of an Educational Mathematics Game: "Let's Put a Number with Minus Here"

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ABSTRACT

Learning begins in the social life that one takes place in. Numbers, counting, and relations such as more/less develop through our daily life experience before the beginning of formal education. In other words, our experience in daily life is the basis for an intuitive learning. These intuitive learnings are sometimes seen at children's games, too.

Games can be used as an educational tool [1] [2]. Also, games provide active learning environments to children, who participate in games voluntarily. Children both have fun and learn while playing games. When the game is played again and again, learning becomes stronger and students become more interested in the topic that they learn through the games.

Except the games that the educators created, the games developed by children can also be educational. The game to be examined and analysed in the context of this paper is developed by a fourth-grade student (Esin). This game, which researchers discovered while Esin and her friends are playing, is basically based on the negative-positive numbers and the addition of integers. This paper will focus on how a student who has not yet achieved a learning of the whole number concept develops the game, the mathematical content of the game, and the learning that the game players have gained.

This research is a qualitative research which consists of two-phase. In the first phase of the research, Esin was asked to teach this game to the researchers and then the game was played with her. Later on, an interview was held with Esin on how she developed the game. In this phase, the mathematical content of the game and the parameters that develop the game were examined.

In the second phase of the research, learning which has been gained through this mathematical game has been analysed. For this, three friends of Esin who played this game with her, were also involved in the research and they played the game. Students were asked to explain all their moves while playing the game. The mathematical knowledge that the students used and explained while playing the game was examined. Also, an individual interview was held with each student to see what they have learned about the concept of the integer. In these interviews, the students were asked about ordering, comparing and addition of integers.

All the phases were video recorded, and then the games and interviews were transcribed. Based on the data obtained, inferences on intuitive learnings that primary school students have developed regarding the concept of integers through this game will be provided.

Key Words: Game, integers, intuitive learnings, elementary school students.

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The Effect of 5E Learning Model Based Activities on The Process of Creating the Concept of Congruency and Similarity of 8th Grade Students

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ABSTRACT

The purpose of the study is to determine the effect of 5E learning model-based activities used in the instruction of eighth grade congruency and similarity unit in geometry course on students. For this purpose, a guiding activity set was developed based on learning model and the effectiveness of the set during the process was evaluated.

The study was carried out with 9 eighth grade students in a primary school students in Ankara city during 2016-2017 education perriod. data collection tols of the study 5E learning model-based activities, structured questionnare consisting of fifteen open ended questons, interwiev from consisting of eight semi-structured open-ended questions, video recordings and researcher methods. during the process of the study's implemention, 5E learning model based activities used.

The research process, the students structured the 5E learning model based activities, accompaniment and similarity concepts in a meaningful way and reached the findings that the activities performed according to the evaluation results obtained the permanence in the students minds.

In order to determine the effect of 5e Learning model based activities on students geometric skills, the answer of the students in the problem and observation forms were analyzed. as aresult of the research , it was the positive effects on the students ability and also 5E model based activity also determined information is determined to provide permanentity.

Key Words: congruency, similarity, 5E model-based learnig

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The Effect of Realistic Mathematics Education on the Academic Achievement and Attitudes of the 12th Grade Students on Derivative

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ABSTRACT

This study was conducted to the students in the 12th grade high school to investigate the effect of realistic mathematics education on the academic achievement and attitudes about the derivation learning. In this study single group pretest-posttest from quantitative research designs was used [1]. 26 students who are the 12th grade students of Anatolian High School in a province that located in northeastern Turkey participated to study. Derivative achievement test consisting of open-ended questions developed by researchers as a data collection tool and "Derivative Attitude Scale" developed by [2] were used in the study. In the research process firstly, achievement and attitude tests were applied to the students in a lesson hour. Then, derivative activities that were designed in accordance with realistic mathematics education were given to students for 4 hours (2 days, 2 lessons per day). The activities are designed for the conceptual understanding of the derivative and examples of different application areas are presented. Academic achievement and attitude tests applied again end of this process. Descriptive and inferential statistics were applied to the collected data in the study. We present information about central tendency and diffusion measures and normality in the descriptive statistics section. Dependent sample t-test was used because of the data approximates normal distribution. Results show that the derivative applications that are designed according to realistic mathematics education have a significant positive effect on the academic achievement and attitude on the learning of derivative of the 12th grade students.

Key Words: Realistic mathematics education; derivative; experimental success; attitude.

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The Effect of Sudoku, Futoshiki and Kakuro Puzzles on 8th Grade Students' Achievement on Equality and Inequality Subjects

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ABSTRACT

Examining the effect of number placement type puzzles on eighth grade students' academic achievement on equations and inequalities is the purpose of the present study. Although puzzles and solving puzzles has a very long history, Zbigniew Michalewicz and Matthew Michalewicz were the first to use the concept and approach of puzzle-based learning. The Puzzle-based Learning that is currently being developed is based on 3 main rules; first one is totally understanding the puzzle, second one is making concreate calculations and last one is building a suitable model to solve problem.

The design of the study was randomized pre-test - post-test control group design, using matched subjects. This study was conducted with convenience 34 eighth grade students in the 2015 - 2016 academic year. These students were placed to experimental and control groups in accordance with their scores on the Level Placement Test (LPT). Students were matched and paired based on the LPT results (i.e. first pair contains first and second rank students in LPT, third and fourth rank students is second pair, etc.). One of pair was assigned treatment group and other one was assigned to control group, randomly. Before the study, achievement test (AT) was administrated as a pre-test. Validity and reliability analysis of AT was done with a pilot work.

In the implementation phase, both control and experiment groups were taught by the same teacher however working sheets were given to only treatment group students additional to lecturing. Working sheets were daily and they included sudoku, futoshiki and kakuro puzzles. On the other hand, control group students were just

lectured and they did not receive any addtitinoal activity or working sheets. Sudoku, Futoshiki and Kakuro puzzles were given as extracurricular activities throughout the research. At the end of 25 days, AT was administrated as a posttest. After 6 weeks, AT was implemented as a permanency test.

Results show that when Sudoku, Futoshiki and Kakuro puzzles are given as an extracurricular activity, they have no effect on academic achievement and permanency of achievement.

Key Words: Sudoku, Futoshiki, Kakuro, Mathematics Achievement, Equality and inequality, Extracurricular Activities.

Acknowledgement. This study is a part of the M. Sc. Thesis of the first author.

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The Effects of Discovery-Based Teaching Strategy on 8th Grade Students' Mathematical Reasoning and Connecting Skills

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ABSTRACT

It is thought that mathematics teaching, grounded on discovery way of learning approach, improves students' skills of assimilating cognitive knowledge and interpretation in case of problems. It is also a known fact that using the mathematical language, constructing models, making deductions and making connections will also increase students' mathematics success (MEB, 2013). In the secondary school mathematics course, it is aimed at students' understanding mathematics concepts, making connections between concepts, reaching a new concept with reasoning skill, solving daily problems, making connections between mathematical concepts with the models they establish. In this study, investigating the effect of the discovery based approach mathematics learning on the 8th graded students' reasoning and connecting skills in terms of the teaching of inequalities.

This study was conducted with the composite method in which qualitative and quantitative research were used. The composite method focuses on the gathering, analysis and blending of both qualitative and quantitative data in a single and set of researches (Creswell & Clark, 2014). Experimental research method, as quantitative, and situational research, as qualitative, were used. Mathematical reasoning evaluating scale of Pilten (2008) was applied as pre-testing and post-testing to total 60 students from the experiment and control group. Within this study, while teaching based on discovery way of learning approach was conducted to experimental group, the process was recorded. Quantitative data were analysed with statistical tests and t-test was applied to qualitative data.

In the experiment group, in which mathematics teaching based on discovery way of learning approach was conducted, it was observed that there was an increase

in mathematical reasoning of students while in the control group, in which traditional teaching was conducted, there was no increase in mathematical reasoning (t=3,515 , p=0,001). It was seen that the students of experiment group, where discovery way of learning was conducted, discovered the new knowledge by making connections between concepts. Students, with the directed questions, realized that they needed to use the pre-learned concepts to discover new concepts. Before starting the new lesson, they were asked about what they had learnt in the previous lesson, they were made to remember the previous lessons and by using the daily life examples, they were attracted to the lesson. It was observed that students were encouraged to answer the questions and participate in the lesson as the questions were open-ended questions and as they were allowed to answer the questions with their own thoughts. It was seen that the questions that guided the students of experimental group led the students to think and it was also seen that the students could give examples based from daily life, from other lessons and other subjects. It was concluded that discovery way of learning approach encouraged students to make connections, discover on their own and reasoning.

Key Words: Discovery-Based Teaching Strategy, Reasoning, Connecting

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The Impact of Reading on Students' Problem Solving Skills and Their Mathematics Success*

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ABSTRACT

In the age of information, the requirement's increasing for the people who is thinking, questioning, analyzing and solving problems. Reading can play a big part of it to have and contributes to development of these abilities (Tanju, 2010). In this study, it was tried to investigate the reflections of reading books, which are known to contribute to development of knowledge and linguistic skills, to students' mathematical knowledge. Accordingly, it is aimed to observe the effect of students' reading levels on their success rates in the Mathematics course and on their mathematical problem solving skill.

74 students from two different secondary schools in Giresun were chosen as the samples for this study. They were separated into levelled reading groups after determining the number of books they read so far and then their academic success rates in Mathematics course and mathematical problem solving skills were examined. A problem solving success test consisting of 20 questions which were prepared with the help of a specialist, a form to determine the number of books read so far, and students' Mathematics course written exam scores were used as data gathering tools.

SPSS was used in analysing the correlations of gathered data. Kolmogorov-Simornov test was applied to decide whether the data had a normality distribution or not and also a homogeneity test was applied. For the purpose of analysing the data, one way independent sample variant analysis (one way ANOVA) was used. Descriptive analysis test, ANOVA and Tukey were made use of in the process of data analysis. At the end of the research it was found out that extensive reading had no impact on students' Mathematics success rates and on their problem solving skills.

Key Words: Reading, success rate in mathematics, problem solving skills, problem solving steps.

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The Influence of Mathematics History on Students' Mathematical Attitudes*

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ABSTRACT

Mathematics It is regarded by the students as merely numerical, lifeless and impenetrable science. The fact that mathematics consisting solely of numbers and formulas is often regarded as boring and repetitive by students is a major problem for mathematics teaching [1]. Using approaches such as mathematical history to overcome this problem is important in terms of mathematical thinking and perception. Placing the history of mathematics in mathematics lessons will reveal the perception that mathematics is a science that is open to development, living and always interesting [2]. The most special aspect of teaching mathematics is the development of specific concepts of mathematics in one's mind. It should not be emphasized that mathematics is based solely on symbols and formulas in the process of teaching mathematics. At the same time, approaches should be focused on how and why the questions are included and how to develop a positive attitude towards mathematics [3].

The National Council of Teachers of Mathematics (NCTM) in the United States emphasized that the history of mathematics should be integrated into mathematics teaching. This organization has stated that mathematics is a great success for human history and that the cultural factors behind this success must be revealed. In this context, the effect of applying the history of mathematics on the mathematics attitudes of the students was investigated in this study. In the process of obtaining the data, a pre-test / post-test unequal control group model, which was evaluated as a semi-experimental design, was used. Convenience sampling method has been adopted with the research group determined. In the experimental group, activities related to algebra and numbers issues were used in related places of the course. A scale aimed at examining the mathematics attitudes of middle school students developed by [4] was used as a data collection tool. The obtained data were analyzed using the Statistical Package for Social Sciences (SPSS 22) program. When the results were examined, it was found that the scores of the experimental group were significantly higher than the scores of the control group in terms of interest, necessity factor and attitude total scores in the attitudes of the groups towards the mathematics lesson. There was no significant difference in the scores of the study and anxiety factors.

Keywords: History of Mathematics, Secondary School Students, Mathematics attitude.

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The Influence of Mathematics History on the Academic Achievement of Middle School 8th Grade Students *

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ABSTRACT

There is a need for approaches that will isolate students' psychological prejudices against mathematics and make mathematics more fun and engaging [1, 2, 3]. At this point, mathematical history can be used as an important adjunct. The use of mathematical history is also mentioned in the Secondary School Mathematics Teaching Program (OMDPP), which was renewed in 2013. Under the heading of the program's learning-teaching approach, the heading "Use of Information on the Development of Mathematics in the Program", which includes the use of the history of mathematics in teaching mathematics, is included. Therefore, in this study, the effect of the use of mathematical history in the teaching environment was investigated for the academic achievement of the students. In the process of obtaining the data, a pre-test / post-test unequal control group model, which was evaluated as a semi-experimental design, was used. The study was conducted with 39 (20 experimental, 19 control) eighth grade students in two different branches in a secondary school in Erzincan. Adequate sampling method has been adopted with the research group determined. During the course of the application, activities related to algebra and numbers were used in the course. As data collection tool, achievement test was used for the subjects of algebra and numbers developed by the researcher. The obtained data were analyzed using the Statistical Package for Social Sciences (SPSS 18) program. When the findings were examined, no significant difference was found between the final test success point average of the experimental group and the final test success point average of the control group. It is seen that the average scores of the achievement scores of the experimental and control groups are very

* This work consists of a part of the first author's doctoral thesis.

close to each other. In this context, it can be said that the application of the history of mathematics does not have a meaningful effect on the achievements of students in terms of algebra and numbers.

Key words: History of Mathematics, Academic Achievement, Grade 8 Student.

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The Investigation of Algorithmic Thinking Skills of 5th and 6th Graders According to Different Variables

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ABSTRACT

Informally, computational thinking describes the mental activity in formulating a problem to admit a computational solution. The solution can be carried out by a human or machine, or more generally, by combinations of humans and machines (Wing, 2006). Though the idea of computational thinking was first introduced by Seymour Papert (1980), the discussions with regards to the teaching of this concept became widespread with the notion of Wing (2006) suggesting that every student should be taught computational thinking as one of the fundamental areas such as reading, writing and arithmetic. ISTE (2015) indicates that computational thinking skill is an expression of creative thinking, algorithmic thinking, critical thinking, problem solving, cooperative learning and communication skills and underlines that it cannot be described independently of these skills. Being an important component of computational thinking skill, algorithmic thinking is defined By Brown (2015) as the ability to understand, implement, assess and design algorithms to solve a range of problems. As for Futschek (2006), it is an ability that is necessary at any stage of problem solving process whereas Olsen (2000) indicates that this ability is one of the most important abilities that students should develop in education environment. Nevertheless, the studies carried out on algorithmic thinking are fairly limited, in the literature. Thus, the present study aims to investigate the algorithmic thinking skills of secondary school students according to different variables. In this regard, the *Algorithmic Thinking Test* developed by the author as a data collection tool was administered to 138 students in total studying at 5th and 6th grades at public secondary schools in the province of Ordu. For determining the schools that would take part in the study, the TEOG (Transition from Primary to Secondary Education) exam results carried out in 2017 were taken into account, in line with the consensus of mathematics teachers and school principals across the province. In this regard, the students studying at schools that ranked in the middle group according to success rating participated in the study. As a result of the study, the algorithmic thinking skills of the students were assessed taking into account the variables of gender, grade and mathematics achievement and recommendations were presented for relevant studies that can be carried out in the future.

Key Words: Algorithmic thinking, 5^{th}-6^{th} graders, computational thinking

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The Investigation of Algorithmic Thinking Skills of 5th and 6th Graders at a Theoretical Dimension

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ABSTRACT

Informally, computational thinking describes the mental activity in formulating a problem to admit a computational solution. The solution can be carried out by a human or machine, or more generally, by combinations of humans and machines (Wing, 2006). Though the idea of computational thinking was first introduced by Seymour Papert (1980), the discussions with regards to the teaching of this concept became widespread with the notion of Wing (2006) suggesting that every student should be taught computational thinking as one of the fundamental areas such as reading, writing and arithmetic. ISTE (2015) indicates that computational thinking skill is an expression of creative thinking, algorithmic thinking, critical thinking, problem solving, cooperative learning and communication skills and underlines that it cannot be described independently of these skills. Being an important component of computational thinking skill, algorithmic thinking is defined By Brown (2015) as the ability to understand, implement, assess and design algorithms to solve a range of problems. As for Futschek (2006), it is an ability that is necessary at any stage of problem solving process whereas Olsen (2000) indicates that this ability is one of the most important abilities that students should develop in education environment. Nevertheless, the studies carried out on algorithmic thinking are fairly limited, in the literature. Thus, the present study aims to investigate the algorithmic thinking skills of secondary school students at a theoretical dimension. In this regard, the *Algorithmic Thinking Test* developed by the author as a data collection tool was administered to 138 students in total studying at 5^{th} and 6^{th} grades at public secondary schools in the province of Ordu. For determining the schools that would take part in the study, the TEOG (Transition from Primary to Secondary Education) exam results carried out in 2017 were taken into account, in line with the consensus of mathematics teachers

and school principals across the province. In this regard, the students studying at schools that ranked in the middle group according to success rating participated in the study. As a result of the study, the algorithmic thinking skills of the students were assessed considering the sub-dimensions of these skills and recommendations were presented for relevant studies that can be carried out in the future.

Key Words: Algorithmic thinking, 5^{th}-6^{th} graders, computational thinking

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The Investigation of Pre-service Teachers' Knowledge of Teaching Strategies for Comparing Decimals

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ABSTRACT

Shulman (1986) claims that teacher knowledge should include subject-matter knowledge, pedagogical content knowledge and curriculum knowledge. Consisting of the key elements of teaching strategies and student understanding, pedagogical content knowledge (PCK) is defined by Shulman (1986) as teachers' ways of representing and formulating the subject-matter knowledge in the context of facilitating student learning taking into account the learning difficulties and prior knowledge of students and teaching concepts. The knowledge of teaching strategies refers to the ways of representation and explanation used in teaching concepts and ideas (Shulman, 1987) and it involves teachers' knowledge of how to provide concepts and principles for facilitating student learning (Magnusson, Krajcik, & Borko, 1999). The present study aims to investigate the knowledge of pre-service teachers about teaching strategies to be used in comparing decimals. In the study, the case study method was used. The study group consisted of 75 pre-service secondary school mathematics teachers studying at 3rd year at a public university in the spring semester of 2017 – 2018 academic year. The pre-service teachers had taken Special Teaching Methods-I lesson before and were taking Special Teaching Methods-II at the time of study. The data were collected through the data collection tool developed by the authors. According to the results, the pre-service teachers suggested methods such as using decimal place value chart, writing decimals in expanded form, converting to fractions, giving examples from the daily-life, using models etc. On the other hand, it was found out that the pre-service teachers mostly carried out activities based on using models; however, they made wrong explanations because of taking different-sized referent whole.

Key Words: knowledge of teaching strategies, comparing decimals

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The Investigation of the Relationship Between Mathematical Connection Skill and Self-Efficacy Belief

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ABSTRACT

One of the general aims of mathematics teaching is to help individuals gain mathematical knowledge that they will need in their lives and fundamental skills they might be able to use in different areas (Baki, 2014, p.34; The Ministry of National Education [MoNE], 2013, s.1; National Council of Teachers of Mathematics [NCTM], 2000, p. 4). In order to fulfil this aim, it is essential that students gain fundamental mathematical skills such as understanding and interpreting mathematical concepts and the relations among these concepts and using them in their lives as well as being able to connect mathematics with different areas and disciplines (Ball, 1990; Kinach, 2002; Vale, McAndrew & Krishnan, 2011; NCTM, 2000; Skemp, 1978; Van de Walle, 2013). In this regard, mathematical connection is explicitly highlighted as one of the most important skills in mathematics learning and practice processes in updated mathematics curriculums or standard documents (Chapman, 2012; MoNE, 2013).

Self-efficacy is a concept of judgments about how well individuals can perform the actions needed to cope with possible situations. The self-efficacy belief is a factor affecting the learning process of all areas (Bandura, 1995), especially of mathematics. Therefore, it is expected that self-efficacy belief in mathematical connection will be effective on performance related to the skill. From this point of view, this study aims to investigate the relationship between self-efficacy beliefs and performances related to students' mathematical connection skills.

The study is a case study and was conducted with secondary school students. The Connection Skill Test developed by the researchers and the Mathematical Connection Self-Efficacy Scale developed by Özgen and Bindak (2018) were used as data collection tools. The theoretical basis of the Connection Skills Test is the

study carried out by Yavuz Mumcu (2018). Accordingly, the test consists of 4 sub-dimensions: *Connection Between Different Representations, Connection Between Concepts, Connection with Real Life, and Connection with Different Disciplines.* As a result of the application, the correlation between the students' performances of using subject skill and the self-efficacy beliefs were tested and interpreted as significant. The results of the study were elaborated and discussed in association with the literature and recommendations were presented with regards to the results obtained. Moreover, information was given about different relevant studies that can be carried out in the future.

Key Words: Mathematical connection skill, self-efficacy belief, secondary school students

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The Nature of Prospective Middle School Mathematics Teachers' Contextual Problems about Linear Graphs

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ABSTRACT

Problem posing involves generating new problems based on a given situation or reformulating an existing problem. For this reason, it provides important details about learners' mathematical knowledge, support their conceptual understanding and foster their ability to reason and communicate mathematically, and capture their curiosity and interest [1]. Although the concept of functions, particularly functional relations of the form of *y=mx+n* is a fundamental topic in middle school mathematics, the number of studies about linear functions and problem posing is limited and has quantitative nature [2, 3]. Posing problems using linear graphs in real life situations is complex issue for prospective teachers because it requires knowledge about dependent/independent variables, covariation, linearity, slope, first degree equation and logic about contextual cases [3, 4, 5]. Considering the importance of problem posing and linearity, we aimed to investigate the nature of prospective middle school mathematics teachers' problem posing about linear graphs. Furthermore, this study presents information about what kinds of errors and difficulties prospective teachers had when they posed problems with the possible reasons of these difficulties within a qualitative perspective.

The study was conducted with 27 prospective teachers in the scope of school practice course. All participants posed two contextual problems based on two linear graphs individually (see Figure 1) into one week. They also reported the solutions of the problems, the aim of each problem, predictions about how students can solved the problems, and in which problems they had difficulty while posing and the reasons of these difficulties. At the end of reporting, we conducted a group discussion with the participants about their problems. For the data analysis, types of proper and improper

problems were grouped in different themes created based on data and related studies.

The results indicated that all problems (n=52) posed by prospective teachers were improper. Improper problems involved some errors such as (i) giving graphs into the problems, (ii) inadequate wording in problem statements, (iii) determining unsuitable dependent and independent variables, (iv) inadequate subject matter knowledge about linearity and covariation, (v) confusing with cumulative and noncumulative data, (vi) lack of contextual knowledge in problems, and (vi) not paying attention to the slopes in the graphs.

Key Words: linear graphs, problem posing, contextual problems, prospective middle school mathematics teachers

Problem Posing Task
Numerical data and what axes represent in graphs are not given specifically. Write real life problems where these linear graphs might be the answer.

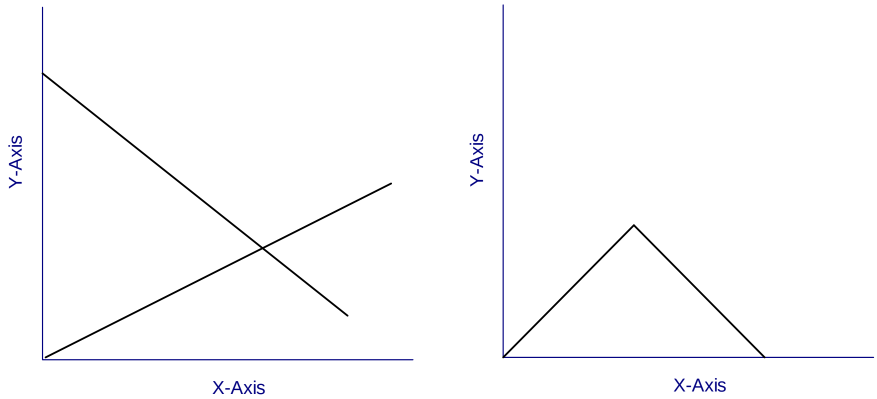

Figure 1. Problem Posing Task

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The Peer Assessment of Mathematics Games Designed by Scratch Programming Language

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ABSTRACT

Technology appeared first in calculators in school mathematics in the 1970s to facilitate four operations. After this date, the importance of technology-supported mathematics education has increased (Goos, 2010). It can be said that the use of computer technologies in an appropriate way increases the quality of mathematics education, this situation affects the student achievement positively (Kul & Birişçi, 2017). The Ministry of National Education made changes in the curriculum and quality of the Information Technologies and Software course in the 2013-2014 academic year. The learning domains of the framework program published by the Turkish Education Board were arranged, the Scratch Programming Language was started to be taught to bring skills determined within the scope of "4. Problem Solving, Programming and Original Product Development" learning domain (MoNE, 2013). In this research, it was aimed that the 7th grade students' mathematics games designed using Scratch Programming Language would be evaluated by the peers of them. The participants of the study consisted of 15 7th grade students in a private school in the province center of Bartın in the academic year of 2017-2018. A purposive sampling method was used for the selection of the participants, it was taken into account that the sample was made up of the students who took education in mathematical game designing in the Scratch Programming Language. In the study, the case study method was used as a qualitative research method. The forms were rearranged by the researchers taking into consideration the assessment forms of Durak (2016) and Alan (2017) as data collection tools, two different forms as Educational Computer Game Material Assessment Form-I (ECGMAF-I) and Educational Computer Game Material Assessment Form-II (ECGMAF -II) were used.

In the process of data collection, the students were asked to design a mathematics game in 6 class hours in the computer class with the use of the processing block without an achievement restriction after the students were reminded of the codes in the Scratch Programming Language in 2 class hours. Each stage of the designed game projects was uploaded to the section for the projects that were not published in the class studio in the Scratch-MIT Media Lab website. At the end of the study, the game projects were assessed according to ECGMAF I-II forms. The assessment of the projects was carried out with the participation of the students in groups in the computer class. After the students individually designed their game projects, the other students assessed the projects. As a result of the study, the students were asked to write their positive or negative opinions about the application. The data were analysed qualitatively. The students stated that the peer assessment put responsibility on them and increased their self-confidence. In addition, the process allowed the students socialized with each other in the classroom and enjoyed game design, as a result the students took a positive attitude in peer assessment. As a negative aspect of peer assessment, a small number of the students stated that they could act emotionally in the process of evaluating other students because of close friendships. In conclusion, it was observed that most students behaved objectively during the assessment and saw the deficiencies in their own games more clearly when they evaluated others' games.

Key Words: Mathematics game, peer assessment, scratch programming language.

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The Relationship Between The Misconceptions Eight Graders Have About Algebra And Their Metacognitive Awareness

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ABSTRACT

This study aims to review the relationship between the misconceptions eight graders have about algebra, and their metacognitive awareness. The study was carried out with 48 students enrolled in the 8th grade of a secondary school in Güce district of Giresun province. “Algebra Test [1]” was applied to identify the misconceptions the students have about algebra, while “Junior Metacognitive Awareness Inventory [2]” was applied to assess their metacognitive awareness levels. The algebra test is a 34-questions test developed to make an assessment regarding potential harboring of nineteen misconceptions identified in the literature. The “Junior Metacognitive Awareness Inventory”, in turn, is composed of two major components: “knowledge of cognition” and “regulation of cognition”. The students were first subjected to “Junior Metacognitive Awareness Inventory”, followed by the application of the “Algebra Test” during a class. Then the scores each student got in both the test and the survey were calculated. The relationships between the algebra test results with the results concerning the knowledge of cognition and regulation of cognition components of the “Junior Metacognitive Awareness Inventory”, not to mention the relationship between the metacognitive skills and the algebra test results, were analyzed through three correlation tests. The correlation tests revealed a medium level of correlation (correlation factor $r=.617$, $p<.01$) between the knowledge of cognition component of the junior metacognitive awareness inventory and the algebra test results of the participating students. The correlation between the regulation of cognition component of the junior metacognitive awareness inventory and the algebra test results was also found to be at a medium level (correlation factor $r=.515$, $p<.01$). The correlation between the junior metacognitive awareness

inventory and the algebra test was, again, found to be at a medium level (correlation factor r=.607, p<.01).

Key Words: Misconceptions, algebra content domain, metacognitive awareness

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The Relation Between the Number Sense and Self-Efficacy towards Number Sense of the Sixth Grade Students

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ABSTRACT

The concept of number sense can be mentioned where flexible thinking is required on numbers, quantities or processes like reasoning about numbers and quantities, making an estimation, testing the relevance of the result of an operation done or doing a mathematical operation mentally. Yang (1995), has defined number sense as the person's ability to use the numbers and operations responsively while doing mathematical reasoning and having tendency, and also being able to develop useful and practical strategies about the mathematical situations. While the individuals whose number sense is developed can approach to numbers and operations more flexibly, the individuals whose number sense isn't developed have difficulty to go out of rules that they learnt previously while tackling with numbers and operations. Individuals being able to use the numbers and operations independently of the algorithmic rules they have learnt flexibly can depend on not only their number senses but also their beliefs that they can use the numbers and operations flexibly. Any result in literature hasn't been found on, what sort of relation can be contacted between the beliefs of individuals that they can use the numbers and operations flexibly and how much they have number sense.

Bandura (1997) has defined the self–efficacy which is one of the basic concepts of Social Cognitive Theory as the beliefs of the individuals directing the events that affect their lives and their necessary capacity to show a certain level performance. While Zimmerman (1995) was mentioning about the features of self-efficacy concept, stated that self-efficacy was multi-dimensional, and that it could be low in another field when an individual's self-efficacy could be high in a field. This situation has been a convenient basis for the studies performed on self-efficacy to be able to

become more specific. Accordingly, appropriate definitions of self-efficacy have been done for different fields, and the studies have been conducted in that way. For example, Pajares and Miller (1995) defined self-efficacy towards mathematics as a measure of the confidence that the individual feel according to the situation or problem for overcoming a specific task or a mathematical problem.

Like the number sense that we need to be able to do a reasoning about the number and operations more flexible and more practical, our beliefs on using our number sense and the confidence we feel about our efficacy on this subject is also as important as using our number sense. This study has been prepared in the light of the foresight on this importance in order to determine the efficacy towards number sense levels of the middle school students.

This research is a quantitative research. As the method of research, the descriptive research method has been chosen. In this scope, “Self-Efficacy Scale Towards Number Sense” developed by Alkaş Ulusoy and Şahiner (2017) have been applied to a group consisting of 300 students attending the 6., 7., and 8th grade of an elementary public school in Ankara. The data collected have been analysed by SPSS programme. The findings relating to the study and results, will be shared during the congress.

Key Words: Number sense, self-efficacy, self-efficacy towards number sense.

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The Relationship between Success of Abstract Mathematics Lesson and Metacognitive Awareness and Attitude toward Proof

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ABSTRACT

Abstract mathematics subject aims to establish a link between the mathematical knowledge students learn in secondary school and the mathematics knowledge that students will learn in mathematics departments in universities. The abstract mathematics subject mainly consists of five learning areas as symbolic logic, set theory, natural numbers, whole numbers and rational numbers. First of the learning areas is symbolic logic and it consist of based of proof and proving. Proof is the basis for the other four learning areas listed above in abstract mathematics. In this context, it is clear that proof for abstract mathematics is important. [1] emphasized that students need to work their cognitive and metacognitive skills after proving their lack of knowledge to prove it. Because proof is based on reasoning and justification requires explaining what individuals do and why. Knowing why individuals perform an operation requires metacognitive skills. For this reason, we think that the proof is associated with metacognitive awareness.

In the light of the reasons explained above, this study was conducted to examine the relationship between the success of abstract mathematics teaching and the attitude toward metacognitive awareness and proof.

This study conducted by correlational research model from quantitative research design. The study was conducted in Turkey with students studying in elementary mathematics education department of a university located in the northeast. The data of the study were collected through abstract mathematics subject achievement test, metacognitive awareness scale for teachers and attitude scales for

proof. The abstract mathematics achievement test was prepared as a multiple-choice test by the researchers. We use to determine the metacognitive awareness of elementary math teacher students in the study and the teacher form adapted by [2] to the metacognitive awareness scale adapted to Turkish by [3] developed by [4]. We used to determine the attitudes of the participants towards proof and proving, “Scale of attitude towards proof and proving" developed by [5]. The scales used in the study were prepared in likert style. The reliability analyzes show that the data collected with the tests and scales used in the study were reliable. The data collection process was carried out in two lesson hours. Achievement test of abstract mathematics subject was applied to the students during the first class hour. In the second lesson hours, the students were administered the metacognitive awareness scale and attitude scale for proving. Descriptive and inferential statistics were used in the analysis of the data.

The result of this study, which examines the relation between the achievement of abstract mathematics lesson and the attitude towards proof and metacognitive awareness of the students of elementary math teacher education, show that the academic achievement of abstract mathematics subject of elementary math teachers were low and that their attitudes towards proving and metacognitive awareness were moderate. Significant correlations were found between some of the variables.

Key Words: Pre-service math teachers, abstract mathematics, metacognitive awareness, proof, proving.

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To What Extent is the Westside Test Anxiety Scale (WTAS) Reliable?

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ABSTRACT

Assessment ways (exams, tests, etc.) are being used all over the world to distinguish successes from failures in any educational process. However, they are not good enough to reveal such distinction as the psychological factors are out of scope in these tests. That is to say each individual has different state of mind at the time that he or she is to be assessed with such kind of methods.

Anxiousness known as a psychological disorder is one of those factors needed to be taken into account before any exam is applied. To measure the levels of such anxiety researchers refer some other instruments which we call them as psychological scales like the Westside Test Anxiety Scale (WTAS). Such scales should have some characteristics in it like reliability and these characteristics are highly dependent on the sample of which the scale is applied. This means that an estimate of any reliability index of a scale can vary from one sample to another.

In this regard, the aim of this study is to provide a meta analytic reliability generalization for the (WTAS) in order to find the overall reliability and to expose the amount and the sources of variability among reliability estimates. To do this, a literature scan between the years 2007 (first introduced date of the scale) and 2018 was conducted to find every empirical study that referred the Westside Test Anxiety Scale on the databases of “Google Scholar, ScienceDirect and Jstor”. “Westside”, “Westside Test”, “Westside Test Anxiety” and “Westside Test Anxiety Scale” were used as the keywords in advanced research tabs of each data base. Of the 143 studies found only sixty reported an estimation of reliability with coefficient alpha. Statistical analysis was performed with “metafor” package of R. The results suggest that the (WTAS) is a reliable instrument with an acceptable level as the pooled alpha

is 0.73. The total amount of heterogeneity was found to be 95.2%. To explain such variability among studies four variables were extracted from each study which we call them as moderators, however, none of these succeeded to explain such heterogeneity. Therefore, we kindly alert all researchers to report their own statistical results with all possible demographic information in their future studies.

Key Words: Reliability Generalization, Test Anxiety.

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Understanding The Role of Data Collection In Doing Statistics: A Case Study with Middle School Preservice Teachers

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ABSTRACT

Data collection plays a significant role in statistical thinking. It helps researchers to transform research questions into pieces that can be worked on. What kind of data to be collected, from whom and how, what data could be excluded are some issues that researchers have to cope with while collecting data. These questions not only shape the data collection process, but also impact analyzing and interpreting the data. However, data collection is often the most neglected part of doing statistics in the classroom [7]. Teachers usually present data without providing much information about it and ask students to analyze and interpret. Therefore, students usually had difficulty to make sense in selecting and employing data analyses methods (e.g., deciding measures of central tendency and variation, organizing data by using grahps) and in interpreting the findings [7]. The GAISE report emphasize all aspects of doing statistics (i.e., formulating research questions, collecting data, analysing data, interpreting findings) and encourage teachers to design their instruction by focusing on all aspects of doing statistics [1]. The current middle school mathematics program [3] also recommend that students should participate in all aspects of doing statistics in the classroom. However, research has shown that this is a challenging task for teachers [5,8]. Several professional development models are offered in order to develop in-service and preservice teachers' knowledge and skills for teaching statistics [4,5]. Lesson study is one of the effective professional development program for preservice teachers in terms of developing statistical competence [2,6]. In this study, we focused on three middle school preservice teachers and investigate their knowledge and skills about collecting data when doing statistics. They participated in three lesson study. We collected data by means of lesson plans they designed, video-taped meetings for planning, assessing and revising the lesson

plans. Results showed that participants began to notice the importance of data collection in doing statistics towards the end of the study. They began to ask questions about the meaning of data collection while doing statistics as they teach. Also, they provided students opportunuties to think about data collection process (e.g., how data could have been collected). To conclude, findings showed that preservice teachers developed deeper understandings about the role of data collection in doing statistics as they engage with lesson study practices. They would benefit from collaborative work in designing lesson plans, implementing them in classroom environments, and reflecting on the plans. Hence, lesson study practices could be integrated into teacher education programs.

Key Words: Data Collection, Lesson Study, Preservice Teachers

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Use of Mental Games in Mathematics Lesson Plans

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ABSTRACT

Mental Games Course is offered as an elective at both middle school level and university level in Turkey. According to the curriculum of Mental Games Course in middle school, identifying and improving students' intelligence potentials and developing their problem-solving strategies are among the purposes of this course (MoNE, 2013). It is also seen that processes such as problem solving, reasoning, and communication in the mental game curriculum overlap with basic skills in the middle school mathematics curriculum (MoNE, 2018). Literature review presented that researchers who are aware of these commonalities between mental games and mathematics education conducted studies regarding the effects of mental games on mathematics success, problem solving and reasoning (e.g. Bottino, Ferlino, Ott, & Tavella, 2007; Bottino & Ott, 2006; Bottino, Ott, & Benigno, 2009; Kurbal, 2015).

Considering the context of research related to mental game and the potential of this context in terms of mathematics teaching, research questions of the present study were shaped and listed as follows:

- What are the learning areas pre-service middle school mathematics teachers focused while preparing lesson plans by using mental games?
- Which mental games (matching game and crossword-puzzle game) do they prefer to use while preparing lessons plans?
- What are the reasons for their selection of the particular mental game?
- What are their opinions about using mental games in middle school mathematic course when they become in-service teachers?

Participants of the study are six triads of pre-service middle school mathematics teachers who took Mental Games Course in university level. In the study, participants were asked to prepare a lesson plan as a group and to answer an instrument involving 3 questions individually. The lesson plans can be prepared based on the

desired learning area or learning outcome. However, at least one of two games (crossword-puzzle game and matching game) should be used.

When the lesson plans prepared were examined, it was found that four groups prepared a lesson plan for geometry and measurement, two groups prepared for numbers and operations as the learning area. While three groups were using in the matching game, and two groups used the crossword-puzzle game. Only one groups used in both the matching game and the crossword-puzzle game.

Groups who used matching game stated that this game is appropriate to objective, feasible to visualization related concepts, and easy to prepare as reasons for selection. Also, they expressed that crossword-puzzle game is hard to prepare, knowledge-based, and not feasible for visualization related concept and group work.

On the other hand, groups who used crossword-puzzle game stated that this game is appropriate to objective, and particular concepts and feasible for the assessment section of the lesson. Their reason for not selecting matching game is not appropriate to the chosen objective and concept.

The opinions of participants on using these games in their future mathematics lessons were positive since they considered that these games are useful in terms of increasing participation and motivation of students and creating a collaborative learning environment.

Key Words: Mental game, lesson plan, pre-service teacher.

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Views and Designed Activities of Mathematics Teacher Candidates Who Got Trained About STEM

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ABSTRACT

It is known by everyone that the need for information to keep pace with our era is rapidly developing day by day. Because we live in a time when knowledge is antagonized and acknowledged as power. For today's more global, technologically and competitive economy, it is important to strengthen in the disciplines of science, technology, engineering and mathematics [1]. STEM education, which has gained importance in the world in recent years, includes the integration of four important disciplines such as science, technology, engineering and mathematics. The search for solutions of students by referencing existing knowledge in a new encountered situation is expressed as another advantage of STEM-based instruction [2]. In expanding STEM education throughout the countries, teachers with the necessary training and competence in the STEM field have a key proposition [2]. In this study, it was aimed to examine the ideas of mathematics teacher candidates about stem education and the activities they designed. Semi-structured interview form and activity card were used as data collection tool in the study. Semi-structured interview form was used for teacher candidates' views on stem education. On the activity card, teacher candidates were asked to design activity for the stamper and the activities they designed were analysed.

34 mathematics teacher candidates who attended the last grade of elementary mathematics teacher education participated in the study. The obtained data were analysed using the content analysis method.

As a result of the study, the majority of the mathematics teacher candidates expressed that STEM education is useful, and can be practiced in middle schools. In the activities designed by the teacher candidates, mostly modelling was used and the science-mathematics relation was preferred. Some activities are prepared so that they are not suitable for the STEM.

Key Words: Stem education, mathematics teacher candidate, design activity

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Views of Prospective Teachers About The Teaching Mathematics With Games

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ABSTRACT

The anxiety, fear and failures of the students towards the mathematics lesson are some of the important problems of the Turkish education system. Many researchers have suggested that the use of game to develop a positive attitude towards students against mathematics and improve academic success [1, 2]. In this context, it is important for teachers and prospective teachers to experience games as a method for teaching mathematics.

In this special case study, it is aimed to determine the experiences of prospective mathematics teachers on teaching mathematics with game and their views on this subject. A total of 8 prospective teachers, 5 of which are female and 3 of which are male having education in 3rd year of elementary school mathematics teaching program participated to the study. During the spring semester of 2017-2018, the prospective teachers designed 4 games in groups of two within the scope of the Community Service course. The games they prepared were applied for 8 weeks, two hours a week in two middle schools located in Kars province center. Prospective teachers wrote a report about their practice every week. At the end of spring semester, views of prospective teachers about mathematics teaching with games were taken by using interview form. Student reports and interview forms were analyzed using content analysis and MAXQDA 12 package program.

Under the headline of suggestions, opinions of prospective teachers, positive and negative aspects of using games in math teaching, difficulties faced while preparing games, difficulties faced in application, contribution of study to prospective teachers were presented. Prospective teachers stated that positive aspects of teaching math with games are learning with amusement, permanent learning. They also stated that it ingratiates the lesson, it increases participation to the lesson and it materializes the lesson. In addition to this, they also expressed that negative aspects

of teaching mathematics with game is time consumption and also it makes class management difficult. While preparing the games, they stated that they sometimes have difficulties in preparing materials that attract attention at student level, and when they are applying games, they have difficulties in creating heterogeneous groups in class and in classroom management. Prospective teachers stated that the study is a contribution to their professional experience and that they will integrate game to the lessons in their classes when they start their profession.

In line with the results of the study, it may be advisable to arrange training programs that will provide experience to teachers and prospective teachers in teaching mathematics with game in order to convert the fear towards math into love and to increase the success of students in this field.

Key Words: Teaching with game, teaching mathematics, prospective mathematics teacher

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What is Open-Ended Question? What is It Not?

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ABSTRACT

It is important to describe what open-ended question is not before making a right definition of it. Unlike multiple-choice questions, open-ended questions do not have only one correct answer. However, the fact that there can be more than more correct answer does not mean that any answer can be accepted. Unlike other types of question, open-ended questions ask for a more comprehensive and systematical thinking as well as creative answers. Thus, open-ended questions can be defined as the questions that might have more than one correct answer and where different ways of reasoning might lead to different answers and that require individuals to interpret data, think critically, consider different criteria for the solution and take some decisions (Thomas and Badger, 1991; Thompson, 1998; Anderson, 2003). This type of problems might lead to several solutions and the correct answer might be reached through different ways.

Most of the questions that are conventionally used in mathematics education require students to answer in the form of a number, figure or mathematical object. As the answers are predetermined and specific, these type of questions are called 'close-ended questions'. They are also called well-defined problems as there is only one correct answer and answers can be evaluated as correct or wrong. As for open-ended questions, they give both high-achieving students the chance to engage in different operations and low-achieving students to participate according to their skills. Also, students can choose different strategies to answer and use the ones they feel comfortable with (Mihajlović and Dejić, 2015).

Nevertheless, these characteristics are not taken into account while using the concept of open-ended problems/questions in the literature and they are referred to as classic exams where the questions are given in the written form and students are expected to answer in writing. In the present study, open-ended questions are handled and the examples that can be employed by teacher trainers, teachers and program developers are introduced. Thus, it constitutes a theoretical study. As a result of the study, it is aimed to eliminate the conceptual ambiguity about open-ended problems/questions through a literature analysis concerning open-ended question types and sharing examples.

Key Words: open-ended question, open-ended problem

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What Paper Folding Can Do For The Concepts of Parallelism and Perpendicularity

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ABSTRACT

Parallel and perpendicular lines and construction of them are one of the basic concepts in geometry. These concepts have an important role in learning some geometric concepts in later years like altitude of triangles. Although there is no much study in accessible literature about parallelism and perpendicularity, the current ones showed that students have some misconceptions about parallelism and perpendicularity of two lines (Ubuz, 1999), and have difficulties in construction of them (Mitchelmore, 1992a). Also, student generated examples about the concepts are limited (Ulusoy, 2016). Inadequacy in definitions of parallelism and perpendicularity can be the reason of limitations in examples of the students (Ulusoy, 2016). Other studies about different concepts supports the belief of Ulusoy (Tsamir, Tirosh, and Levenson, Barkai & Tabach, 2015). Therefore, some pedagogical approaches should be developed for the development of the definitions of the concepts (Edward & Ward, 2008) in order to eliminate students' difficulties and misconceptions.

To improve students' geometric thinking, some techniques like cutting or folding can be very useful (Baykul, 2002). Origami which means paper folding both make learning more concrete and enhance understanding of geometry concepts (Georgeson, 2011). In addition, with origami, opinions and attitudes of students regarding mathematics develop considerably (Cornelius & Tubis, 2006).

In accordance with these findings, in this study, we aimed to analyze the effect of paper folding activities on geometry achievement of sixth grade students regarding the concepts of parallelism and perpendicularity.

The most appropriate design type to show cause and effect relationship are experiments (Fraenken & Wallen, 2006). Therefore, for analyzing effect of the activity

on geometry achievement of students, one group pre-test post-test design, a type of experimental design, was used in this study. Participants of the study were 46 sixth grade students selected from a public middle school in Ankara.

Paper folding activities were prepared by the researchers depending on the studies in the literature to eliminate students' difficulties and misconceptions regarding the concepts of parallelism and perpendicularity. In the activities, firstly students constructed perpendicular line segments through folding their waxed paper. Afterwards, they explored construction of parallel line segments. All the constructions then were passed to the squared paper as suggested in the mathematics curriculum. Different from the paper folding activities in the literature, in this activity, teacher did not give directions to students for constructions. Instead, with proper questions, students are provided to explore the steps in constructing parallel and perpendicular line segments.

Results of the paired samples t-test showed that there is a significant effect of paper folding activity on geometry achievement of students about the concepts of parallelism and perpendicularity. In addition, effect size, d=1.56, indicates a large effect. Explorations that students discover during the activity and their continuous engagement with constructions on squared paper following waxed paper was thought to be a possible reason of reaching that result. Therefore, it is suggested to give a place to such kind of activities in teaching of parallelism and perpendicularity concepts.

Key Words: Paper folding, parallelism and perpendicularity, middle school mathematics

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STATISTICS

A Brief Overview on The Geometry of The Uncentered Coefficient of Determination

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ABSTRACT

In this study we will investigate the geometrical properties of the uncentered coefficient of determination in linear regression analysis. As is known, regression analysis needs information about the coefficient of determination and the partial correlation coefficient. Researchers in many disciplines using regression analysis need knowledge about the coefficients of determination and partial correlation. Geometry is a very useful tool for understanding regression analysis. The geometric description of the coefficients of determination and partial correlation provides a better understanding of these notions. For this reason, the geometric description of the coefficients of determination and the partial correlation provides a better understanding of these coefficients. Recent studies in this respect have focused on the geometric properties of the coefficients of determination in the linear regression analysis. This study especially focuses on the uncentered coefficient of determination. In this paper, by giving concrete examples we will try to introduce some of these notions we consider important. In addition to these above, an application of Euclidean space geometry is given since it is an important tool for a linear regression study. To simplify most of the ideas of the regression theory, their n-dimensional geometry has to be understood.

Key Words: Coefficient of determination, uncentered coefficient of determination, coefficient of partial correlation, regression analysis, geometric properties.

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A Comparison of Variation Coefficients of Biochemical Reaction Model for Several Probability Distributions

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ABSTRACT

In this study, we add random effects with Laplace distribution (also known as the double exponential distribution) to the parameters of the Biochemical Reaction Model. The resulting equation system becomes a system of nonlinear differential equations with random parameters which enables the investigation of the randomness in the biochemical reaction process. The parameters of the model, which are obtained by using the backward and forward reaction rates, are transformed into random variables with Laplace distribution. The coefficients of variation for the compartments of the random model are analysed to interpret the amount of randomness in these model components. Normal (Gaussian), Symmetrical Triangular and Generalized Beta distributions are also used to investigate the random procedure of the biochemical reaction. The coefficients of variation for these probability distributions are compared to comment on the similarities or differences in the results. It is aimed to show that all of these probability distributions produce almost identical coefficients of variation for the compartments if some conditions are satisfied for the random parameters of the model. The random model is simulated in MATLAB to examine its numerical characteristics such as the expected values, standard deviations and coefficients of variation for these four probability distributions. Results show that the random model produces similar results to the deterministic model while providing additional information on the random behaviour of the system.

Keywords: Random Differential Equation, Biochemical Reaction Model, Coefficient of Variation, Simulation, Laplace Distribution.

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A Generalization of the Hermite-Hadamard's Inequality for Convex Stochastic Processes on the co-ordinates

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ABSTRACT

Convex stochastic processes play a vital role in the theory of inequalities. Many inequalities are consequences of inequalities hold for convex stochastic processes. It is well-known that one of the most fundamental and interesting inequalities for classical convex stochastic processes is that associated with the name of Hermite-Hadamard inequality which provides a lower and an upper estimations for the integral average of any convex stochastic processes defined on a compact interval, involving the midpoint and the endpoints of the domain. These inequalities are also necessary to compare some values of a stochastic process with its expected value. These concepts may be particularly interesting from optimization view point, since it provides a broader setting for studying optimization and mathematical programming problems.

In recent years, the concept of convexity has been extended and generalized in various directions. In this regards, very novel and innovative techniques are used by different authors (see, [1-8]). In [1] Set et all defined the concept of convex functions on the coordinates in a rectangle from the plane and established the Hermite-Hadamard inequality for it.

The main subject of this study is initially to consider a generalization of the Hermite–Hadamard's inequality for convex stochastic processes on two-dimensional interval. Besides, generalized Hermite-Hadamard type inequalities are obtained for mean-square integrable convex stochastic processes on the co-ordinates. As special cases, one can obtain several new and correct versions of the previously known results for various classes of these stochastic processes. Applying some type of

inequalities for stochastic processes is another promising direction for future research.

Actually, using the paper]2} titled "Generalized Hermite–Hadamard's inequality for functions convex on the coordinates", we apply it on convex stochastic processes on the co-ordinates. Differently from its method, it is only used mean-square integrability throughout this study.

Key Words: Hermite-Hadamard inequality, convexity, mean-square integrability, stochastic processes, co-ordinates.

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A Look at the Generalized Mathematical Model Named as Koya-Goshu Model for Biological Growths

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ABSTRACT

In this study, the generalized mathematical model named as Koya-Goshu biological growth model is represented by using the ordinary differential equation or the rate-state ordinary differential equation $f'(t) = r_t f(t)$ where $f(t)$ is representing growth function and r_t is relative growth rate at time t. Widely used growth models such as Generalized Logistic, Richards,Generalized Weibull, Particular Case of Logistic, Weibull, Gompertz, Classical Logistic, Brody (Monomolecular, Mitscherlich), and Von Bertalanffy functions are shown in the special cases of Koya-Goshu biological growth model. All the parametric relationships identified in the models above have been exhibited through the flow chart. By using this chart, the relationships of the models above will be better understood. Some properties of the Koya-Goshu function are given. Both increasing and decreasing growths are shown by using this function. The Koya-Goshu function covers some parameters. One of them is v. The sign of this parameter influences increasing or decreasing of the growth. While for v>0 the function is increasing, for v<0 the function is decreasing. Furthermore; since the inflection point of a growth curve determines the sigmoidally of that curve, inflection point of Koya-Goshu model is investigated. It can be seen that the inflection points of Koya-Goshu function match with the cases of some growth models

Keywords: Growth models; Koya-Goshu Function; Brody; Von Bertalanffy; Richards; Weibull.

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A New Estimator for Ergodic Distribution of a Semi-Markovian Renewal Reward Process when Demand Distributions Belongs to the L∩D Subclass of Heavy Tailed Distributions

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ABSTRACT

In this study we investigate an estimation problem for an ergodic distribution of a semi Markovian inventory model of type (s,S). The ergodic distribution and the moments of the ergodic distribution, which represent the semi-Markovian inventory model, are not easily obtained in some cases. In such cases, these characteristics of semi Markovian inventory model can be expressed through renewal functions. Different estimators for the renewal function are available in the literature, when the distribution which defines the time between renewals is not known precisely. One of the most important study on this problem is the publication by Frees [2] in which he proposed a statistical estimator for the renewal function.

The study of Frees [2] allows different estimators to be obtained for the characteristics of these models. Hence, statistical estimator problem for an inventory model of type (s, S) have been considered in the literature with different type of interference of chances, when demand distributions are light tailed (see for example [1], [3], [4], [5]). The difference of our study from current literature is, we consider mentioned semi-Markovian renewal reward process with a L∩D subclass of heavy tailed distribution. The study by Kamislik et. al. [1] is our main motivation. In that study they provided asymptotic expansion for ergodic distribution of an inventory model of type (s, S) with uniform distributed interference of chance, when demands belongs to the L∩D subclass of heay tailed distributions using asymptotic expansion provided by Embrechts and Omey [7]. Based on the main results of Frees [2] and Kamislik et. al. [1] we obtained a statistical estimator for ergodic distribution of an

inventory model of type (s, S) and some statistical properties such as consistency, asymptotic unbiasedness and asymptotic normality were investigated for the obtained estimator, respectively.

Key Words: Estimators, Renewal Reward Process, Heavy Tails, L∩D Subclass.

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A Parametric Estimator for the Geometric Function in Geometric Process with Exponential Interarrival Times

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ABSTRACT

Let $\{N(t), t \geq 0\}$ be a counting process and X_k be the interarrival time between $(k-1)th$ and kth event of this process for $k = 1,2, \ldots$. The counting process $\{N(t), t \geq 0\}$ is said to be a GP with the ratio parameter a if there exists a real number $a > 0$ such that $a^{k-1}X_k, k = 1,2, \ldots$ are independent and identically distributed random variables with a distribution function F_1. Let $\{N(t), t \geq 0\}$ be a GP with the ratio parameter a and F_k be the distribution function of X_k, $k = 1,2, \ldots$. Then, it is obvious that $F_k(x) = F(a^{k-1}x)$ for $k = 1,2, \ldots$. The mean value function of a GP, which is also called the geometric function, is given by

$$M(t) = E\big(N(t)\big) = \sum_{k=1}^{\infty} F_1 * \ldots * F_k(t), \quad t \geq 0, \tag{1}$$

where * denotes Stieltjes convolution. It is well known that the geometric function $M(t)$ satisfies the following integral equation.

$$M(t) = F(t) + \int_0^t M(a(t-x))dF(x), \quad t \geq 0. \tag{2}$$

Let $\{N(t), t \geq 0\}$ be a GP with ratio parameter a and the first interarrival time of this process has the distribution function F_1. Assume that the functional form of the distribution F_1 is known and let $\theta_1, \theta_2, \ldots, \theta_r$ be the unknown parameters of $F = F(\theta_1, \theta_2, \ldots, \theta_r)$ where $F_1 = F$. Also, the ratio parameter a is unknown. From the definition of the GP, the distribution of X_k is $F_k(x) = F(a^{k-1}x)$ for $k = 1,2, \ldots$. Thus, it is clear that the parameters $a, \theta_1, \theta_2, \ldots, \theta_r$ are the unknown parameters of $F_k = F_k(a, \theta_1, \theta_2, \ldots, \theta_r)$. Now, let $\{X_1, \ldots, X_n\}$ be a data set which comes from a GP. The random variables in this data set are independent but not identically distributed. Let $\hat{a}, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_r$ be the estimators of the parameters $a, \theta_1, \theta_2, \ldots, \theta_r$ based on this data set. By writing the estimators $\hat{a}, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_r$ instead of the unknown parameters $a, \theta_1, \theta_2, \ldots, \theta_r$ into the distribution functions F and F_k, we can obtain the resulting estimators of F and F_k as $\hat{F} = F(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_r)$ and $\hat{F}_k = F_k(\hat{a}, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_r)$ for

$k = 2,3,\ldots$, respectively. Then, we can define a parametric estimator of the geometric function $M(t)$ for each fixed $t \geq 0$ as

$$\widehat{M}(t) = \sum_{k=1}^{\infty} \hat{F}_1 * \ldots * \hat{F}_k(t), t \geq 0, \tag{3}$$

by replacing the distributions F and F_k in equation (1) with their estimators $\hat{F}$ and $\hat{F}_k$.

In this paper, we deal with the problem of estimating the geometric function in the GP with exponential interarrival times. The unbiasedness, asymptotic unbiasedness and consistency properties of the parametric estimator $\widehat{M}(t)$ given in (3) are investigated. Further, a method for the computation of this estimator is given depending on the integral equation (2). The performance of the estimator $\widehat{M}(t)$ is evaluated for small sample sizes and different parameter values by a simulation study.

Key Words: Geometric process, geometric function, parametric estimator, consistency, asymptotic unbiasedness.

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A Study Over Determination Of Asymptotic Deceleration And Absolute Acceleration Points In Logistic Growth Model

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ABSTRACT

Since growth models has generally upper horizontal asymptote, they do not have a maximum point. We wonder about after which point growth can be considered constant, that is, after which point the curve of the growth function is too close to its asymptote. That point is called maximum deceleration point. After this point the deceleration is very slow and the second derivative of the growth function goes to zero as time tends to infinity. After this point it is considered that the amount of the growth is quite small. Moreover, we wonder about which point is an absolute acceleration point so that before that point acceleration is very slow and after that point actual acceleration starts. So we could say that after this point actual growth starts. In this study, the logistic growth model was used to investigate these points, asymptotic deceleration and absolute acceleration points in addition to the other critical and important points such as inflection point, maximum acceleration point, maximum deceleration point. The graphs of the logistic growth model which show all these points mentioned above are also given by using a data set.

Key Words: Logistic model, asymptotic deceleration point, absolute acceleration point, inflection point, maximum deceleration point.

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A Study Over The Effect Of Mean Curvature And Arc Length Values In Selecting The Appropriate Growth Model

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ABSTRACT

In this study after given the definition of the curvature, it is searched the effect of mean curvature and arc length values on the choice of appropriate growth model by using two separate data sets. For this purpose, mean curvature and arc length values are compared with some model selection criteria such as coefficient of determination, error sum of squares and Akaike information criterion. For two separate data sets, Gompertz, Logistic and Bertalanffy models were used. The results of mean curvature and arc length values are found to be in accordance with these known criteria. For two data sets, it is seen that Bertalanffy model is the best model in according to both the known criteria, coefficient of determination, error sum of squares and Akaike information criterion, and the prospective criteria, mean curvature and arc length values. Actually, it is seen that the best fit order is the same according to both the known criteria and the prospective criteria. Thus, it is considered that the mean curvature and arc length values can be regarded as the appropriate model selection criteria. In addition, the compliance of the mean curvature and arc length values with other model selection criteria can be investigated for getting the best model choice.

Key Words: mean curvature; arc length values; model selection criteria

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An Application of Smooth Goodness of Fit Tests for Gamma Response Model

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ABSTRACT

Reliability engineering, survival analysis and other disciplines mostly deal with positive random variables, which are often called lifetimes. As a random variable, a lifetime is completely characterized by its distribution function. Gamma distribution is one of the most common distribution used to model lifetime data and the distribution can be used in a range of disciplines including reliability, survival analysis, hydrology, climatology [1-4].

Generalized linear models provide a flexible modelling framework encompassing many commonly used models including the normal linear model, logistic regression model, Poisson regression model and Gamma regression model.

Goodness of fit tests how well a set of observations fits to a statistical model. Smooth goodness of fit tests are applied to many distributions in the literature such as Gamma, Inverse Gaussian, Nakagami, Poisson and etc [5-6]. It has been investigated how the smooth testing concept can be used to test the distributional assumption in a generalized linear model and applied smooth testing approach to the generalized linear modelling framework to derive a test of the distributional assumption of response by [7]. The application of this theory to Poisson, negative binomial, binomial, normal and gamma regression models has been demonstrated by [7]. In this study smooth tests are summarized in the presence of Gamma response model and an application to real data is carried out.

Key Words: Gamma distribution, generalized linear model, smooth tests.

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Asymptotic Expansions for the Moments of the Random Walk with a Normal Distributed Interference of Chance

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ABSTRACT

It is well known that many problems of queuing theory, stock control, reliability, insurance, physics and etc., can be expressed by random walk and its modifications. In the literature, there exists many significant papers dealing with these problems (for example, Alsmeyer (1991), Anisimov (1999), Borovkov (1976), Feller (1971), Gihman and Skorohod (1975), Lotov (1996), Khaniyev and Mammadova (2006) and etc.). However, these studies are generally theoretical and they are not exactly helpful for solving concrete real-world problems because of the complexity of their mathematical structure. Because of these reasons, in this study, a semi-Markovian random walk process (X(t)) with a normal interference of chance is considered and investigated by means of asymptotic methods. Firstly, under some assumptions, it is proved that the process X(t) is ergodic. Then the exact and three terms asymptotic expressions for the first four ergodic moments of the process are derived by using basic identity for random walk, Dynkin principle, methods of renewal theory and Milne theorem. The obtained asymptotic expansions with three terms are very easy and useful in terms of implementation. Moreover, we tested how close the approximated expressions are to the exact formulas, using Monte-Carlo simulation methods. It is observed that the approximation values obtained for the ergodic moments are more than 98% close to the simulation results, even not large values of parameter *a*.

Key Words: Random walk, discrete interference of chance, moments of ergodic distribution, asymptotic expansion, Monte Carlo simulation method.

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Calculating the Moments of Circular Data Using Angular Distance

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ABSTRACT

The obtained data from observation can be existed in various measurement spaces. One of the measurement spaces is an angular space in which data are expressed as the angular. For instance, a biologist may be measuring the orientation of an animal depending on a factor in the nature or a geologist may be interested in the direction of the earth's magnetic pole. Such directions may be univariate as in the first two examples or bivariate like the last one. In general, data identifying in angular space are referred to as directional data. Data showing univariate angular change are called as circular data; data showing bivariate angular change are called as spherical data. If data show more than two angular changes are called as hyper-spherical data.

The conventional statistical techniques are not applied to circular data because of the special nature of circular data. The mainstay of many studies on this subject are calculation of the trigonometric moments of circular data. But these approaches bring along the approximate results according to the conventional statistical methods. In this study, direct angular distance is used to calculate the moments of circular data. In order to eliminate these approximate results, the circular moments are proposed by using direct angular distance. The performance measurement of trigonometric moments are defined to compare the proposed method with the jackknife method.

Key Words: Circular data, Sample trigonometric moments, Iterative circular mean, Jackknife method.

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Comparison Of Goodness Of Fit Tests For Lomax Distribution

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ABSTRACT

The Lomax or Pareto II (the shifted Pareto) distribution was proposed by Lomax. This distribution has found wide applications such as the analysis of business failure life time data, income and wealth inequality size of cities, actuarial sciences, medical and biological sciences, engineering, lifetime and reliability modeling. So it is important to test whether the data set comes from Lomax distribution. For this purpose, goodness of fit test are used for checking whether the data set is compatible with Lomax distribution. Goodness of fit test are as a method that tests the sample data is drawn whether or not from a specific theoretical distribution. In this study some goodness of fit tests are investigated for Lomax distribution with estimated parameters. Extensive tables of goodness of fit critical values for the Lomax distribution are developed through simulation for the Zhang, modified Anderson Darling, Esteban and Mann test statistics. Moreover, type 1 error rates and the power of the test statistics are compared by a Monte Carlo simulation study. These test statistics are compared under Weibull distribution, Pareto distribution, Gamma distribution and Normal distribution. Finally a real data set is analyzed for comparison of goodness of fit tests that used in the study.

Key Words Lomax distribution, Goodness of fit test, Critical values, Power Comparison

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Comparison of Page-type Tests in Repeated Measures Designs

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ABSTRACT

Let $n\,x\,b$ observations are grouped into b blocks each containing n treatments / time points. The effect of the treatments are observed, and let X_{ij} denote the observation in i^{th} block $i=1,...,b$ at the j^{th} $j=1,...,n$ treatment / time condition. This design can be either a balanced complete block design or a repeated measures design. When the assumptions of two-way ANOVA are justified, parametric F test is appropriate to determine whether the treatment / time effects are all the same. Testing treatment / time effects with the knowledge of priori ordering is the main focus of this study for the situations where the assumptions of parametric F test are violated and randomized blocks have repeated measures over time points or conditions. The null hypothesis and the alternative hypothesis in increasing order are respectively $H_0 : \tau_1 = \tau_2 = \cdots = \tau_n = 0$ and $H_1 : \tau_1 \leq \tau_2 \leq \cdots \leq \tau_n$. Page test is well-known test for testing ordered alternative hypotheses for two-way layout. Recently, several non-parametric tests based on Page test were developed using orthogonal contrasts [1]. The key assumption of these non-parametric tests is that randomized blocks have independent observations. Since the randomized blocks with repeated measures are considered in this study, this assumption is violated for these non-parametric tests. Therefore, approximate distributions of the Page-type tests mostly have no-closed forms and depend on variance of the correlated error terms. Circular bootstrap method is used for obtaining empirical distributions of Page-type tests for the case where randomized blocks have repeated measures. In the simulation study, several Page-type tests are compared empirically based on circular bootstrap method in terms of type 1 error and power values.

Key Words: repeated measures, ordered alternative, circular bootstrap.

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Detection of the Periodicity and Forecasting: A Study for Air Pollution in Ankara

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ABSTRACT

Air pollution can be defined as the fact that the various substances that need to be found in the air are out of the specified standards. Substances, which are out of the standards, disturbs human health. Forecasting of air pollution rates is very important in terms of taking preventive measures.

In this study, monthly particulate matter (PM10), which has aerodynamic diameter less than 10, data is used. Monthly PM10 values, which obtained from eight different stations in Ankara during 1993:01-2017:12 period, are examined.

Air pollution data usually behaves some periodic structure in time. One of the main goal of this study is to search such periodicities in PM10 data, monthly observed in capital city of Turkey, Ankara. Particulate matter quantities are observed monthly and the data is modelled using time series techniques. Stationarity of the series is checked by periodogram based unit root test in addition to traditional unit root tests. The air pollution data is modelled by using ARIMA method and its periodicity is detected by using periodograms.

Periodograms are estimators of the spectral density function is used for testing of stationarity of the data and detection of periodicity of the data. Periodogram based test has some advantages against standard unit root tests and because of this reason its results are considered in this study.

Key Words: Air Pollution, Forecasting, Periodicities

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Estimation of the Mean Value Function for Gamma Trend Renewal Process

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ABSTRACT

A stochastic process $\{N(t), t \geq 0\}$ is called counting process if it counts the number of the events that occur as a function of time. The sequence of interarrival times in accordance with this process uniquely determine the counting process. For example, if the interarrival times are independent and identically distributed random variables with a distribution function F, then a renewal process can be used in modelling of this counting process. However, in many maintenance, replacement applications and some analysis in reliability theory, the data set comes from a counting process includes random variables that alter in some systematic way. Systematic changes mean that there is a trend in the pattern of the data set and the interarrival times are not identically distributed. In such cases, a trend-renewal process (TRP) can be used as a model. The TRP is defined as follows.

Let $\lambda(t)$ be a non-negative function on $t \geq 0$, and write $\Lambda(t) = \int_0^t \lambda(u)du$. The counting process $\{N(t), t \geq 0\}$ is a $TRP(F, \lambda)$ if the time-transformed random variables $\Lambda(S_1), \Lambda(S_2), \dots$ constitute a renewal process with an interrenewal time distribution function F. That is, if $\Lambda(S_1), \Lambda(S_2) - \Lambda(S_1), \Lambda(S_3) - \Lambda(S_2), \dots$ are independent and identically distributed with the distribution F. F is called the renewal distribution, and λ is called the trend function of the TRP .

Let $\{N(t), t \geq 0\}$ be a $TRP(F, \lambda)$. The mean value function of TRP is defined by $M(t) = E\big(N(t)\big), t \geq 0$. Some statistical applications of TRP need knowledge of the mean value function $M(t)$.

Let define $\widetilde{N}(t) = N\big(\Lambda^{-1}(t)\big), t \geq 0$. Then, by the definition of TRP, we see that the stochastic process $\{\widetilde{N}(t), t \geq 0\}$ is a renewal process with interrenewal time

distribution function F. For the mean value function $\widetilde{M}(t)$ of the renewal process $\{\widetilde{N}(t), t \geq 0\}$, it is clear that

$$\widetilde{M}(\Lambda(t)) = M(\mathrm{t}), \quad t \geq 0, \tag{1}$$

In this study, we consider the distribution F and trend function λ as Gamma distribution with shape parameter α and scale parameter $\beta = 1/\alpha$ and $\lambda(t) = abt^{b-1}, t \geq 0; a, b > 0$, respectively. The parameters α, a and b are estimated based on a data set $\{X_1, \dots, X_n\}$ which comes from TRP. Then, a parametric estimation $\widehat{M}(t)$ of $M(t)$ for each fixed $t \geq 0$ is proposed based on the estimation of the renewal function $\widetilde{M}(t)$ by using the equation (1). Further, some asymptotic properties of this estimator is investigated and its small sample properties are evaluated by a simulation study.

Key Words: Gamma distribution, mean value function, trend function.

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Evaluating Fuzzy System Reliability Using Weibull Hazard Function

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ABSTRACT

Weibull hazard function is quite popular in the reliability literature. There are many situation that component reliability is not crips (fuzzy). Bell shaped, Triangular, Trapezoid, Gaussian etc. memberships are a kind of fuzzy sets that used in fuzzy reliability literature [1,3,4]. In this study, triangular fuzzy membership is selected for its simplicity. Therefore, we calculate fuzzy component reliability and system reliability, when components have a Weibull hazard function and a fuzzy membership using triangular fuzzy membership.

Reliability is one of the most important parameters for engineer and system designers. It is the probability that a product or service will operate propely for a specified period of time under the without failure [3].

Hazard function measures the conditional probability of a failure given the system is currently working.

$$\mathrm{R(t)} = \exp[-\int_0^t h(x)dx\ ]$$

formula shows relation between reliability and instantaneous hazard(failure) function [2]. Hazard function has three phases: early failure, useful life and wear-out failure. Weibull hazard model can be used for all phases.

Weibull model is used when the hazard rate function can not be represented linearly with time. A typical expression for the hazard function (decreasing or increasing or constant) under this condition is

$$h(t) = \frac{\gamma}{\theta} t^{\gamma-1}$$

where θ is positive and characteristic life, γ is shape parameter of the distribution. This model is referred to as the Weibull model. For γ=1, Weibull model is also used for Exponential model and for γ=2, it returns Rayleigh model [2].

Consequently, we found component reliability as a function of α, β, θ and γ parameters and membership degree u and time t, that is,

$$\mu_{\tilde{R}(t)}(u) = \begin{cases} \dfrac{ln(u) + \dfrac{t^\gamma}{\theta} + \beta t}{\beta t}, \exp\left(-\dfrac{t^\gamma}{\theta} - \beta t\right) \leq u \leq \exp\left(-\dfrac{t^\gamma}{\theta}\right), \beta > 0, 0 < t < \infty \\ -\dfrac{ln(u) + \dfrac{t^\gamma}{\theta} - \alpha t}{\alpha t}, \exp\left(-\dfrac{t^\gamma}{\theta}\right) \leq u \leq \exp\left(-\dfrac{t^\gamma}{\theta} - \alpha t\right), \alpha > 0, 0 < t < \infty \end{cases}$$

Key Words: Fuzzy System Reliability, Weibull Fuzzy Membership, Triangular Membership Function.

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Forecasting of Daily Electricity Consumption for Turkey

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ABSTRACT

Electricity is one of the most important parts of everyday life. Since electric energy is a type of energy that can not be stored, energy providers cut off the electricity if they can not meet the needs. In the case of excess production, they export electric energy. On the one hand, many countries export the electricity, on the other hand these countries would import the electricity. Therefore, forecasting of electricity consumption quantities is important in terms of planning.

Turkey's electricity needs have increased significantly in recent years and therefore, given the importance of nuclear power plants and renewable energy sources has also increased.

In this study, daily electricity consumption quantities are examined for Turkey, from 2012 to 2016. Since the data is observed at a specific time interval, time series techniques are used when performing analyses. In order to make statistical inference in time series, stationarity is an important assumption. In this study, the stationarity of the data is checked by periodogram based unit root test, ADF and Phillips-Perron tests. According to unit root tests results, the data is stationary. Then, the electricity consumption is forecastable. In order to forecast the values for future data, the data is modelled with ARIMA models. The reason that the electricity consumption is forecastable is that we used daily data for only five years period. On the other hand, when we use larger data, the consumption turns out to be nonstationary. The Akaike's İnformation Criterion (AIC) value is considered in model selection. Then, the electricity consumption for January 2017 is forecasted and the forecasted values are compared with actual values.

Key Words: Electricity Consumption, Forecasting, Periodogram

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Generalized Inverses of Linear Transformations in Hilbert Spaces and Some Applications in Statistics

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ABSTRACT

It is well known that the linear estimation in statistics is some kind of orthogonal projection in Hilbert spaces. The orthogonal projection operator can be constructed by using generalized inverses. A generalized inverse of an integral operator, called "pseudoinvers", was firstly defined by Fredholm in 1903. Generalized inverses of differential operators, discussed by Hilbert in 1904, are consequently studied by numerous authors. Generalized inverses of differential and integral operators had appeared before the generalized inverses of matrices, first noted by E.H. Moore at a meeting of the American Mathematical Society in 1920. Interest in the subject in the 1950s centered around the least squares properties of generalized inverses and the relationship of generalized inverses to solutions of linear systems [1]. In 1955 Penrose showed that Moore's inverse, for a given matrix A, is the unique matrix A satisfying the four equations: $1. AA^+A = A$, $2. A^+AA^+ = A^+$, $3. (A^+A)' = A^+A$, $4. (AA^+)' = AA^+$.

Thousands of papers on various aspects of generalized inverses and their applications have appeared. A natural definition of generalized inverses of linear operators is the one given by Tseng [1]. Let H_1 and H_2 are Hilbert spaces and $A: H_1 \to H_2$ is a linear operator, briefly denoted as $A \in \mathcal{L}(H_1, H_2)$. An operator $A^g \in \mathcal{L}(H_2, H_1)$ is a Tseng inverse of A if, $R(A) \subset D(A^g)$, $R(A^g) \subset D(A)$ and $A^g A(x) = P_{\overline{R(A^g)}}(x)$ for $x \in D(A)$, $AA^g(y) = P_{\overline{R(A)}}(y)$ for $y \in D(A^g)$. Here, D stands for the domain and R for the range. $P_{\overline{R(A^g)}}$ is the orthogonal projection operator on the closure of $R(A^g)$, and $P_{\overline{R(A)}}$ is the orthogonal projection operator on the closure of

$R(A)$. An operator A has a Tseng inverse if and only if the null space $N(A)$ is closed.

In this study, we will focus on the generalized inverses defined by Tseng. Our primary aim is visualization of some geometrical aspects of generalized inverses. Some applications of generalized inverses in statistics will also be considered.

Key Words: Hilbert space, orthogonal projection, linear estimation.

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Hermite-Hadamard Type Inequalities for Convex Stochastic Processes on the n-co-ordinates

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ABSTRACT

Convex stochastic processes play a vital role in the theory of inequalities. Many inequalities are consequences of inequalities hold for convex stochastic processes. It is well-known that one of the most fundamental and interesting inequalities for classical convex stochastic processes is that associated with the name of Hermite-Hadamard inequality which provides a lower and an upper estimations for the integral average of any convex stochastic processes defined on a compact interval, involving the midpoint and the endpoints of the domain.

In recent years, the concept of convexity has been extended and generalized in various directions. In this regards, very novel and innovative techniques are used by different authors (see, [1-8]). In [1] Set et all defined the concept of convex functions on the coordinates in a rectangle from the plane and established the Hermite-Hadamard inequality for it.

The main subject of this study is initially to generalize it for n-coordinates by using the definition of convex stochastic processes on the co-ordinates in [1]. Namely, we consider the convex stochastic processes on n-dimensional interval. Besides, Hermite-Hadamard type inequalities are obtained for mean-square integrable convex stochastic processes on the n-co-ordinates.

Actually, using the paper]2} titled "Hermite-Hadamard's inequalities for s-convex functions based on the n-co-ordinates", we apply it on convex stochastic processes. Differently from its method, it is only used mean-square integrability throughout this study.

Key Words: Hermite-Hadamard inequality, convexity, mean-square integrability, stochastic processes, n-co-ordinates.

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Hermite-Hadamard Type Inequalities for Mean-Square Differentiable r-Convex Stochastic Processes Using Fractional Integral Operators

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ABSTRACT

Convex stochastic processes play a vital role in the theory of inequalities. Many inequalities are consequences of inequalities hold for convex stochastic processes. It is well-known that one of the most fundamental and interesting inequalities for classical convex stochastic processes is that associated with the name of Hermite-Hadamard inequality which provides a lower and an upper estimations for the integral average of any convex stochastic processes defined on a compact interval, involving the midpoint and the endpoints of the domain. These inequalities are also necessary to compare some values of a stochastic process with its expected value. These concepts may be particularly interesting from optimization view point, since it provides a broader setting for studying optimization and mathematical programming problems.

Meanwhile, fractional integrals and derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. It draws a great application in nonlinear oscillations of earthquakes, many physical phenomena such as seepage flow in porous media and in fluid dynamic traffic model. For more recent development on fractional calculus, one can see the monographs. Due to the widely application of Hermite-Hadamard-type inequalities and fractional integrals, many researchers turn to study Hermite–Hadamard-type inequalities involving fractional integrals not limited to integer integrals. Recently, some inequalities involving fractional integrals have been obtained for different classes of stochastic processes [1-8].

Motivated and inspired by the recent activities in this area, r-convex stochastic processes which are an extensions of convex stochastic processes are investigated in this study. Besides, Hermite-Hadamard type inequalities for r-convex stochastic

processes and some estimates for these inequalities are obtained. Moreover, some new Hermite–Hadamard-type inequalities for mean square differentiable r-convex stochastic processes involving mean square fractional integrals are derived.

Actually, using the paper]2} titled "Hermite-Hadamard-type inequalities for r-convex functions based on the use of Riemann-Liouville fractional integrals", we apply it on r-onvex stochastic processes. Differently from its method, it is only used mean-square integrability throughout this study.

Key Words: Hermite-Hadamard inequality, r-convexity, stochastic processes, mean-square continuity, mean-square fractional integrability.

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Investigation of Different Output Probability Distribution in Hidden Markov Model

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ABSTRACT

In many studies in literature, it is assumed that random variables are independent. But this assumption does not always happens in applications. In many of real world problems, Markov chains are used to model the problem. A Markov chain is useful when we need to compute a probability for a sequence of events that we can observe in the world. In many cases, however, the events we are interested in may not be directly observable in the world. In Markov models, random variables as assumptions are considered to be dependent on various orders. These models are used to predict the future behavior of the system from the current behavior of the system. In a Markov model, the states of the system can be observed. In the case of the hidden Markov model, the states of the model are not directly observable or measurable. However, this model can be analyzed by the help of another observed stochastic process. One of the most important elements in the analysis of a hidden Markov model is the probability distribution existing between observed values and model of the states. In this study hidden Markov is given and different probability distributions for a Hidden Markov models are examined.

Key Words: Stochastic processes, Markov chains, Hidden Markov models, Probability distribution.

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Investigation of A Model For Polio By Using Random Differential Equations and Parameters with Gaussian Distribution

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ABSTRACT

In this study, the deterministic compartmental model given by Nkamba et al. 2017 is used to model the random dynamics of Polio transmission. The deterministic model analyses the effects of vaccination in controlling the spread of the disease. We use parameters with Gaussian distribution in the random model to investigate the random behaviour of disease transmission. Monte-Carlo simulations are used to obtain the numerical characteristics of disease transmission such as the expected value of infected population, standard deviation of disease recovery and etc. Random Differential Transformation Method is used to obtain the approximate analytical solution for the random model and approximate numerical characteristics are calculated by using the approximate solution. The approximate numerical characteristics and the simulation results are compared to investigate the error of the approximation method. Results show that using random parameters in the deterministic equation system provides additional and valuable information about the random dynamics of disease transmission. The expected values of the compartments provide similar results to the deterministic case whereas the variations, standard deviations, coefficients of variation and confidence intervals provide useful information on the randomness of results within the investigated time interval. The results for the randomness of transmission cannot be analysed by using the deterministic model therefore the implementing the random effects into the equation system is vital for fully understanding the spread of the disease.

Keywords: Random Differential Equations, Poliomyelitis, Simulation, Random Differential Transformation Method.

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Kalman Filter Based Sensor Fault Detection And Isolation Algorithms Applied To Helicopter Dynamics

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ABSTRACT

Fault detection, isolation and accommodation algorithms are applied to increase the reliability of helicopters with appropriate flight data and reduce costs associated with hardware redundancy in flight system design. The problem of deriving the most acceptable flight data for the helicopter is discussed in the study. The derivation of the most suitable sensor data based on the Kalman filter technique has been investigated [1]. It is important to examine faults that affect helicopter dynamics to estimate states [2]. In the proposed study, this important sensor failure issue has been examined in detail. Two important approaches have been used to diagnose faults in sensors. These are fault detection and isolation [3]. Fault detection is the determination of the presence of an error in the sensor which is based on a comparison of a statistical function and a threshold value. It has been performed depending on the changes in the innovation sequence of the Kalman filter [3]. After the fault is detected, faulty sensor information is provided in the fault isolation step. Two different methods based on the expected value of the normalized innovation sequence sample and change of sample variance have been used for fault isolation of sensors [4]. It is important that the location of the fault is known when a sensor is faulty [5]. Algorithms have been developed for the detection and isolation of sensor data for this purpose [3]. Using these algorithms, diagnostic tests have been performed for different sensor fault cases. Tests are based on the statistical properties of the normalized innovation sequence of the Kalman filter. To test the algorithms, three different sensor fault conditions have been investigated in terms of continuous bias, noise increment and "0" sensor output [6]. It has been proved that the simulations detect three different fault conditions by using the mentioned tests for sensor fault detection However, it is determined that the sensor failure scenarios

show different characteristics and improvements are made in algorithms used for diagnosis. It has also been shown that faults can be determined by the residues found. This is one of the most notable features of the study. The 12 states of the helicopter model that were obtained with simulations express the flight time of 20 seconds. Fault detection and isolation are presented for pitch and roll angular velocities through Kalman filter based results. In the study, the simulations were used to detect and isolate sensor faults and it has been shown that all three fault conditions examined were detected. The results obtained show the effectiveness of the proposed fault diagnosis methods.

Key Words: Kalman filter, Fault detection, Fault isolation, Fault tolerant estimation, Sensor faults.

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L Moment And TL-Moment Estimation For Two Parameter Lindley Distribution And Its Application

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ABSTRACT

Lindley distribution was introduced by Dennis V. Lindley in 1958. Then, Ghritany, et. al. (2008) have discussed various properties of this distribution and showed that in many ways provides a better model for waiting times and survival times data than the exponential distributions. Its moments, failure rate function, mean residual life function and stochastic orderings have been discussed. Th Lindley distribution belongs to an exponential family and it can be written as a mixture of exponential and gamma distribution.

The probability density function (pdf) and cumulative density function (cdf) of the two-parameter Lindley distribution with α and θ are defined as follows:

$$f(x) = \frac{\theta^2}{\theta+\alpha}\left(1+\alpha x\right)e^{-\theta x} \quad x>0,\ \theta>0,\ \alpha>-\theta,$$

$$F\left(x\right) = 1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x} \quad x>0,\ \theta>0, \alpha>-\theta,$$

respectively. Here is defined as α shape parameter and θ scale parameter.

The main objective of this study is to determine the best estimators of the unknown parameters of the Lindley distribution. Therefore, estimating the model parameters precisely and efficiently is very important. The estimation methods under consideration are maximum likelihood estimators (MLEs), moment estimators (MEs) L- moment estimators (LMEs), Trimmed L- moments estimators (TLMEs). Furthermore, the performances of the obtained estimators AIC, BIC and mean square errors are taken into consideration. A real data set taken from the wind speed

data is analyzed at the end of the study for better understanding of methods presented in this paper.

Key Words: Parameter estimation, L-Moments, Tl- Moments, Lindley Distribution

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Limit Theorem for a Semi-Markovian Random Walk with Two Barriers

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ABSTRACT

It is well known that many problems of queuing theory, stock control, reliability, insurance, stochastic finance, mathematical biology, physics and etc., can be expressed by the random walk and its modifications. In the literature, there exists many significant papers dealing with these problems (for example, Alsmeyer (1991), Anisimov (1999), Borovkov (1976), Feller (1971), Gihman and Skorohod (1975), Lotov (1996), Khaniyev and Kucuk (2004), Khaniyev et al.(2001) and etc.). However, these studies are generally theoretical and they are not exactly helpful for solving concrete real-world problems because of the complexity of their mathematical structure. Because of these reasons, in this study, a semi-Markovian random walk ($X(t)$) with two barriers is considered and investigated by means of asymptotic methods. Firstly, under some assumptions, it is proved that the process $X(t)$ is ergodic. Then, by using the basic identity for random walk, the characteristic function of the ergodic distribution of the process $X(t)$ is expressed in terms of the characteristics of the boundary functionals $N(z)$ and $S_{N(z)}$. Finally, limit theorem for the characteristic function of the ergodic distribution of the standardized process is proved for the case in which the components of the random walk have a bilateral exponential distribution.

Key Words: Semi-Markovian random walk with two barriers, characteristic function, ergodic distribution, bilateral exponential distribution.

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Linear Prediction in Hilbert Spaces

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ABSTRACT

Let (Ω, U, P) be a probability space. The vector space of square-integrable random variables $\mathcal{H} = \left\{ X : E(X^2) = \int_\Omega X^2 dP < \infty \right\}$ is an inner product space under $< X, Y >= E(XY) = \int_\Omega X(\omega)Y(\omega)dP(\omega)$. For $P\{\omega : \omega \in \Omega, X(\omega) = Y(\omega)\} = 1$, it is said that the random variables X and Y are almost equal. Almost equality is an equivalence relation. Let a random variable, also represent its equivalence class. Then, the set of square-integrable random variables is a Hilbert space, that is, a complete inner product space. Denote by $\mathcal{H}_0$ ($\mathcal{H}_0 \subset \mathcal{H}$) the Hilbert space of zero mean random variables. $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_0^{\perp}$ and $\mathcal{H}_0^{\perp} = \mathcal{L}(1)$. Here, $\mathcal{L}(1)$ denotes the one-dimensional subspace spaned by the identity function.

Let $\{Y_t : Y_t \in \mathcal{H}_0, t \in [0, \infty)\}$ be a quadratic mean (q.m.) continuous stochastic process, i.e. $E(Y_t - Y_s)^2 \overset{t \to s}{\rightarrow} 0$, and $\mathcal{H}_s^Y = \mathcal{L}\left(\{Y_t : 0 \le t \le s\}\right)$ consists of all linear combinations $\sum_{t_i \le s} \alpha_i Y_{t_i}$ and q.m. limits of such combinations. Then, $\mathcal{H}_s^Y$ is a subspace of $\mathcal{H}_0$ and $\mathcal{H}_0 = \mathcal{H}_s^Y \oplus \mathcal{H}_s^{Y\perp}$. The ortogonal projection of any $X \in \mathcal{H}_0$ on $\mathcal{H}_s^Y$ is $P_{\mathcal{H}_s^Y} X = \arg\min_{Z \in \mathcal{H}_s^Y} \|X - Z\|$, where $P_{\mathcal{H}_s^Y}$ denotes the orthogonal projection operator. $P_{\mathcal{H}_s^Y} X$ is the linear least squares predictor of the random variable X given $\{Y_t : 0 \le t \le s\}$. But, how can we find $P_{\mathcal{H}_s^Y} X$? When, $\{Y_t : Y_t \in \mathcal{H}_0, t \in [0, \infty)\}$ is a q.m. continuous stochastic process with orthogonal increments,

then $P_{\mathcal{H}_s^Y} X = \int_0^s \frac{d}{dt} E(XY_t) dY_t$. The integral is a Wiener integral. In the finite dimensional case, there exists one-to-one inner product preserving map from $\mathcal{H}_0$ to Euclidian space R^m and the projection operator can be easily obtained using matrix generalized inverses. As an application, we will consider the state prediction in Linear State-Space Models and the famous Kalman Filter from geometrical point of view.

Key Words: Hilbert space, orthogonal projection, linear prediction.

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Parameter Estimation In Semi-Geometric Process With Exponential Interarrival Times

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ABSTRACT

The geometric process (GP) is widely used in many practical applications since its introduction. This process is defined in the following way. Let $\{N(t), t \geq 0\}$ be a counting process and X_k be the interarrival time between $(k-1)th$ and kth event of this process for $k = 1,2, \ldots$. The counting process $\{N(t), t \geq 0\}$ is said to be a GP with the ratio parameter a if there exists a real number $a > 0$ such that $a^{k-1}X_k, k = 1,2, \ldots$ are independent and identically distributed random variables with a distribution function F. Let $\{N(t), t \geq 0\}$ be a GP with the ratio parameter a and F_k be the distribution function of X_k, $k = 1,2, \ldots$. Then, it is obvious that $F_k(x) = F(a^{k-1}x)$ for $k = 1,2, \ldots$. If $a < 1$, then $\{X_k, k = 1,2, \ldots\}$ is stochastically increasing. If $a > 1$, then $\{X_k, k = 1,2, \ldots\}$ is stochastically decreasing. If $a = 1$, then GP reduces to a renewal process.

One of the assumptions of a GP is that the times between events are independent. This assumption is rather restrictive and can limit its applications in the real world. To overcome this limitation, one may expand the definition of the GP to a new stochastic process as follows.

Definition: Let $\{N(t), t \geq 0\}$ be a counting process with the sequence of interarrival times $\{X_k, k = 1,2, \ldots\}$. If $P(X_k \leq x | X_1 = x_1, \ldots, X_{k-1} = x_{k-1}) = P(X_k \leq x | X_{k-1} = x_{k-1})$ and the marginal distribution function $F_k(x)$ of X_k is given by $F(a^{k-1}x)$ for $k = 1,2, \ldots$, where F is a distribution function and a is a positive real constant, then $\{N(t), t \geq 0\}$ is called a semi-geometric process (SGP).

In other words, the SGP is a counting process such that the corresponding sequence of interarrival times $\{X_k, k = 1,2, \ldots\}$ is a Markov process with $F_k(x) = F(a^{k-1}x)$. To investigate the probabilistic and statistical properties of the SGP, we first need to obtain the joint distribution function of $X_1, X_2, \ldots, X_k$. For this

purpose, a simple method is to use copulas. The copulas are a tool for constructing multivariate distributions and formalizing the dependence structures between random variables. Now, let us denote the copula between X_{k-1} and X_k as $C_{k-1,k}(F_{k-1}(x_{k-1}), F_k(x_k); \boldsymbol{\theta})$ and its corresponding density as $c_{k-1,k}(F_{k-1}(x_{k-1}), F_k(x_k); \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the parameter vector. Assume that a data set $\{X_1, X_2, \dots, X_n\}$ comes from a SGP. Then, the log-likelihood function is derived based on the copula functions as

$$l(\boldsymbol{\theta}) = \sum_{k=1}^{n} \log(f_k(x_k)) + \sum_{k=2}^{n} \log\left(c_{k-1,k}(F_{k-1}(x_{k-1}), F_k(x_k); \boldsymbol{\theta})\right), \qquad (1)$$

where $\boldsymbol{\theta} = [\beta, \phi]$, β is the parameter vector of marginal distributions and ϕ is the parameter of the copula. In order to obtain the maximum likelihood estimates of parameters, a computational method proposed by Joe and Xu (1996), called “inference function for margins (IFM)”, can be used. According to this method, the log-likelihood function (1) is divided into two parts in the form of

$$l(\boldsymbol{\theta}) = L_m(\boldsymbol{\theta}) + L_c(\boldsymbol{\theta}),$$

where $L_m(\boldsymbol{\theta})$ and $L_c(\boldsymbol{\theta})$ are the first and second summation in equation (1). In fact, these functions denote the log-likelihood functions for margins and copula, respectively. The log-likelihood function L_m is maximized to get estimates $\hat{\beta}$ and then, the log-likelihood function L_c is maximized by replacing β with $\hat{\beta}$ to get estimate $\hat{\phi}$.

In this study, a parameter estimation problem for SGP is considered when the marginal distributions of SGP are assumed to be an exponential distribution. Under Clayton copula family, the parameters of the SGP are estimated based on the likelihood function given in (1) by using the IFM method. The performance of the estimators is evaluated by a simulation study.

Key Words: Semi-geometric process, geometric process, copula, maximum likelihood, inference function for margin.

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Performance of Goodness of Fit Tests of Nakagami Distribution against Generalized Nakagami Distribution

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ABSTRACT

In mobile communications, a profound understanding and improvement of propagation channel models are very important for system design and related performance analysis. A great number of channel models have been proposed to describe the statistics of the amplitude and phase of multipath fading signals such as Nakagami, Rayleigh, Rician, Weibull, Generalized Nakagami, Generalized Gamma. However, Nakagami distribution provides better matching to experimental data than the other distributions. Applications of Nakagami distribution have also been carried out in many scientific fields such as engineering, hydrology and medicine [1-3]. In this study, Nakagami distribution is considered as a special case of Generalized Nakagami distribution and some goodness of fit tests are constructed to decide which distribution will have better fitting. Goodness of fit tests are statistical tools evaluating distributional adequacy or discrepancy of the hypotheses. Goodness of fit of Nakagami distribution is examined in the literature [4-5]. The aim of the study is to adapt Neyman's $C(\alpha)$, Rao's Score and Likelihood ratio goodness of fit test statistics to check whether the Generalized Nakagami distribution for a given data set is statistically superior to the Nakagami distribution. The goodness of fit tests are then compared in terms of Type I errors and power of tests by using a simulation study.

Key Words: Nakagami distribution, Generalized Nakagami distribution, Likelihood ratio, $C(\alpha)$, score

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Recently Some Important Studies Delivered on the Least Squares Fitting of Quadratic Curves and Surfaces

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ABSTRACT

In the regression analysis by using linear and non-linear least squares method, the problem of quadratic curves and surfaces the best fit to a set of points on a plane is an important issue. In a nonlinear regression, the least squares fit has many nice features. It is invariant under translations, rotations, and scaling, i.e., the fitting contour does not depend on the choice of the coordinate system.

For best fit in non-linear regression, the most popular contours are lines, circles, and ellipses. In 3D space, one fits planes, spheres, or more complex surfaces (such as ellipsoids). The most advanced fitting are reviewed and they are extend to quadratic curves and surfaces. First, it begins with 2D fitting problem. In order to minimize the sum of the squares of the given points, several algorithms are presented that calculate the ellipses, circles, spheres and planes. These algorithms are compared with classical, simple and iterative methods.

For the solution of the best fitting problem, the algorithms used in ordinary optimization methods are refered. Solutions developed by these algorithms put forth the best fit curves and surfaces in a certain accuracy rating. In this paper, we introduce some of recent studies on this issue.

Key Words: Minimization problem, Least squares the best fit method, Quadratic curves and surfaces.

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Some Bounds for the Mean Value Function of the Remaining Life Process Based on an Integral Equation

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ABSTRACT

Assume that $X_1, X_2, \ldots$ be independent and identically distributed nonnegative random variables with distribution function F having positive mean μ and variance σ^2. Let $S_0 = 0, S_n = X_1 + \cdots + X_n, n = 1,2, \ldots$ and $N(t) = \sup\{n: S_n \leq t\}, t \geq 0$. Here $N(t)$ denotes the number of renewals in the time interval $(0, t]$ for each fixed $t \geq 0$. Define $Z(t) = \sum_{n=1}^{N(t)+1} X_n - t, t \geq 0$. The stochastic process $\{Z(t), t \geq 0\}$ is called a remaining life process. $Z(t)$ denotes the remaining life at t for each fixed $t \geq 0$. The distribution function of $Z(t)$ is given by

$$F_{Z(t)}(x) = F(t + x) - \int_0^t (1 - F(t + x - y) dM(y),$$

where M is renewal function of the renewal process $\{N(t), t \geq 0\}$ with interrenewal time distribution function F. If F is a non-arithmetic distribution then the asymptotic distribution of $Z(t)$ is

$$\lim_{t\to\infty} F_{Z(t)}(x) = \frac{1}{\mu} \int_0^x \big(1 - F(y)\big) dy.$$

The mean value function of the remaining life process $\{Z(t), t \geq 0\}$ is defined by $H(t) = E(Z(t))$. $H(t)$ is known as mean remaining life of an item in use at time t. Now, define $R(t) = E(X - t | X > t)$ where X is considered as generic random variable with same distribution as all of the X_n. $R(t)$ denotes the mean remaining life of an item having distribution F at age t. It can be obtained a renewal-type integral equation for the mean value function $H(t)$ as

$$H(t) = R(t)\big(1 - F(t)\big) + \int_0^t H(t - x)\, dF(x), t \geq 0. \quad (1)$$

In this study, some bounds for the function $H(t)$ are found based on the monotone convergence of the obtained sequence of the functions by the help of the

renewal integral equation (1). The boundary comparisons between the functions $H(t)$ and $R(t)$ are made.

Key Words: Renewal function, integral equation, remaining life process, renewal process.

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Some Equalities For Covariance Matrices Of Blups Under General Linear Mixed Model

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ABSTRACT

The general linear mixed models involving fixed and random effects are used in many statistical applications in a variety of fields and they play an important role in data analysis. A general linear mixed model can be expressed as

$$y = X\beta + Zu + \varepsilon, \qquad (1)$$

where y is an observable random vector, X and Z are known matrices, β is a vector of fixed effects, u is a vector of random effects, and ε is a vector of random errors. One of the main topics in the general linear mixed model is to predict or\and estimate all unknown parameters. Best Linear Unbiased Predictors (BLUPs) of unknown parameters are commonly used for statistical inferences from the model. BLUPs of unknown parameters under linear mixed models are based on minimum covariance matrices structure according to Löwner partial ordering among all linear unbiased predictors. Matrix algebra can be used for the characterization of predictors and comparison of their properties under linear mixed models. Especially, some formulas related to ranks and inertias of matrices are simplified some matrix operations for the characterization and they also provide building some inequalities and equalities occurred in the comparison of the covariance matrices of predictors.

In this study, a linear mixed model as given in (1) is considered with assuming uncorrelated random effects u and ε. Some equalities based on ranks and inertias of the covariance matrices for the BLUPs of the predictable mixed effects under the model are given to compare them with the other types of predictor There is an extensive literature on linear mixed models which include the BLUPs of random effects and their properties, see, e.g., [1,2,4], and for some results related to ranks

and inertias see, e.g., [3], rank and inertia formulas for covariance matrices of predictors/estimator see, e.g., see, e.g., [5,6].

Key Words: BLUP, General linear mixed model, Inertia, Predictors, Rank, Covariance matrix.

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Some Fractional Integral Inequalities for Mean-Square Twice-Differentiable r-Convex Stochastic Processes

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ABSTRACT

Stochastic convexity and its applications are very important in mathematics and probability. It is well-known that one of the most fundamental and interesting inequalities for classical convex stochastic processes is that associated with the name of Hermite-Hadamard inequality which provides a lower and an upper estimations for the integral average of any convex stochastic processes defined on a compact interval, involving the midpoint and the endpoints of the domain.

Meanwhile, fractional integrals and derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. It draws a great application in nonlinear oscillations of earthquakes, many physical phenomena such as seepage flow in porous media and in fluid dynamic traffic model. For more recent development on fractional calculus, one can see the monographs. Due to the widely application of Hermite-Hadamard-type inequalities and fractional integrals, many researchers turn to study Hermite–Hadamard-type inequalities involving fractional integrals not limited to integer integrals. Recently, some inequalities involving fractional integrals have been obtained for different classes of stochastic processes [1-8].

The authors' findings led to our motivation to build our work. The main subject of this study is initially to present new some integral inequalities for mean-square twice-differentiable r-convex stochastic processes using mean-square fractional integrals. Besides, estimates for abovementioned inequalities are obtained using the same materials.

Actually, using the paper]2} titled "Hermite-Hadamard-type inequalities for r-convex functions based on the use of Riemann-Liouville fractional integrals",

we apply it on r-onvex stochastic processes. Differently from its method, it is only used mean-square integrability throughout this study.

Key Words: Hermite-Hadamard inequality, r-convexity, stochastic processes, mean-square continuity, mean-square fractional integrability.

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Some Integral Inequalities of the Hermite-Hadamard Type for s-Convex Stochastic Processes on n-co-ordinates

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ABSTRACT

Stochastic convexity and its applications are very important in mathematics and probability. It is well-known that one of the most fundamental and interesting inequalities for classical convex stochastic processes is that associated with the name of Hermite-Hadamard inequality which provides a lower and an upper estimations for the integral average of any convex stochastic processes defined on a compact interval, involving the midpoint and the endpoints of the domain.

In recent years, the concept of convexity has been extended and generalized in various directions. In this regards, very novel and innovative techniques are used by different authors (see, [1-8]). In [1] Set et all defined the concept of convex functions on the coordinates in a rectangle from the plane and established the Hermite-Hadamard inequality for it.

The main subject of this study is initially to consider s-convex stochastic processes in first sense and second sense on n-dimensional interval. Besides, Hermite-Hadamard type inequalities are obtained for mean-square integrable s-convex stochastic processes in first sense and second sense on n-co-ordinates.

Actually, using the paper]2} titled "Hermite-Hadamard's inequalities for s-convex functions based on n-co-ordinates", we apply it on s-convex stochastic processes. Differently from its method, it is only used mean-square integrability throughout this study.

Key Words: Hermite-Hadamard inequality, s-convexity in first sense and second sense, mean-square integrability, stochastic processes, n-co-ordinates.

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TOPOLOGY

∂-Connected Object In The Category Of Proximity Spaces

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ABSTRACT

Baran [1] defined separation properties at a point *p* i.e., locally (see [2]) and then generalized this to the point free definitions by using the generic element, [3] p. 39, method of topos theory for an arbitrary topological category over sets. One reason for doing this is that, in general, objects in a topos may not have points, however they always have a generic point. The other reason is that the notions of “closedness” and “strong closedness” on arbitrary topological catogories is defined in terms of $T0$ and $T1$ at a point, p. 335 [1]. The notions of “closedness” and “strong closedness” in set based topological categories are introduced by Baran [1] and it is shown in [4], that these notions form an appropriate closure operatör in the sense of Dikranjan and Giuli in some well-known topological categories. The notion of proximity on a set X was introduced in 1950 by Efremovich [5]. He characterized the proximity relation “A is close to B” as a binary relation on subsets of a set X. All our preliminary information on proximity spaces and more information can be found in [6]. There are various generalization of (strongly) connected objects in a topological category were introduced and compared ([7, 8]) . The main objective of this paper is to characterize ∂-connected object in the category of Proximity spaces ([8]) . Finally, we investigate the relationships between ∂-connected object and (strongly) connected object in this category [8].

Key Words: Topological category, Proximity space, closedness, connectedness.

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A Note on Hausdorffness and Compactness in the Category of Proximity Spaces

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ABSTRACT

Various generalizations of the usual separation properties of topology and for an arbitrary topological category over sets separation properties at a point p are given in [1]. Baran defined separation properties first at a point p, i.e., locally, then they are generalized this to point free definitions by using the generic element, method of topos theory. One of the uses of local separation properties is to define the notions of closedness and strong closedness on arbitrary topological categories in set based topological categories. These notions are introduced by Baran and they are used to generalize each of the notions of compactness, connectedness, Hausdorffness, and perfectness to arbitrary set based topological categories.

The notion of proximity on a set X was introduced by Efremovich [2]. He characterized the proximity relation "A is close to B" as a binary relation on subsets of a set X. The set X together with this relation was called a proximity space. A proximity space is a natural generalization of a metric space [3].

The main goal of this paper is to characterize the (strongly) compact objects and various notions of Hausdorff objects in the topological category of proximity space. Moreover, we investigate the relationships among these characterizations.

Key Words: Topological category, Proximity space, Hausdorff, Compact.

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Hosoya Polynomial of Graphs Belonging to Twist Knots

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ABSTRACT

There is a deep relationship between knot theory and graph theory. Graphs are effective tools for studies into knot theory. It is possible to switch between the two theories by forming the graphs of knots. Thus, the study areas of the two theories meet on a common ground. In graph theory, there are a lot of invariants for a graph like polynomials. These polynomials are related by structure of a graph, which is invariant under graph automorphisms. Polynomials have been quite handy when dealing with knots and links. What is a graph? It takes place a set of points called vertices and a set of edges which connect vertices. The Hosoya polynomial of a graph was defined by H. Hosoya in 1988. The polynomial was thereafter independently described by Sagan, Yeh, and Zhang under the name Wiener polynomial of a graph. Both names are still used for the polynomial but the term Hosoya polynomial is nowadays used by the majority of researchers. Among others, the Hosoya polynomial has been by now investigated on trees, composite graphs, benzenoid graphs, tori, zig-zag open-ended nanotubes, certain graph decorations, armchair open-ended nanotubes, zigzag polyhex nanotorus, TUC4C8(S) nanotubes, pentachains, polyphenyl chains, as well as on Fibonacci and Lucas cubes and Hanoi graphs. In this study, we firstly introduce Hosoya polynomial and we work out the Hosoya polynomials for graphs of twist knots which are one of interesting families of knots and try to get a generalization for them.

Key Words: Hosoya polynomial, twist knots, knot graph.

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Internal Categories In The Category of Semi-Abelian Algebras

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ABSTRACT

A Lawvere Theory [2] **T** is defined to be a category with a certain object *T* such that all objects are some multiple copies of **T**. A *model* of a theory **T,** which is also called a T-algebra, is a product preserving functor from **T** to the category of sets. A topological T-algebra is defined to be same type of functor from the category **T** to the category of topological spaces. An algebraic theory **T** whose category Set(**T**) of the models is semi-abelian is called *semi-abelian theory* and a model of such a theory is called *semi-abelian algebra*. According to [1] some properties of topological groups such as being Hausdorff, compact, connected and etc. can be generalized to the topological semi-abelian algebras.

In this work for a semi-abelian theory **T** we define the internal category in the category Set(**T**) of semi-abelian algebras and prove that the fundamental groupoid of a topological T-algebra *A* is an internal groupoid in Set(T). That leads a functor from the category of topological semi-abelian algebras to the category of internal groupoids. We extend some results about topological groups and internal groupoids in the category of groups to topological semi-abelian algebras and the internals in the category of semi-abelian algebras. We obtain a criterion for the lifting of internal groupoid structure to the covering groupoids considering the internal groupoid structure in the category Set(**T**).

Key Words: Topological T-algebra, Internal groupoid, Covering map

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Local Pre-Hausdorff Constant Filter Convergence Spaces

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ABSTRACT

In 1991 [2], M. Baran introduced a local pre-Hausdorff topological space (where a topological space *X* is local pre-Hausdorff if given any fixed point and any distinct point from this fixed point if there is a neighbourhood of one missing the other, then the two points have disjoint neighbourhoods. Pre-Hausdorff objects [3,4] are used to characterize the decidable objects [5] in a topos [6,7], where an object *X* of E, a topos, is said to be decidable if the diagonal Δ is a complemented subobject. Furthermore, local pre-Hausdorff objects are used to define various forms of each of local Hausdorff objects and local T_3 objects, and local T_4 object in arbitrary topological categories.

Let *A* be a set, *F*(*A*) set of all filters on *A* and *K* be a function from *A to F*(*A*)*.* If *K* satisfies the following two conditions:

1) $[x] \in K$ for each $x \in A$, where $[x] = \{ B \subset A \mid x \in B \}$

2) If $\alpha \subset \beta$ and $\alpha \in K$ implies $\beta \in K$ for any filter β on *A,* then (*A*, *K*) is called a constant filter convergence space.

A map $f : (A, K) \to (B, L)$ between constant filter convergence spaces is called continuous if and only if $\alpha \in K$ implies $f(\alpha) \in L$ (where $f(\alpha)$ denotes the filter generated by $\{ f(D) \mid D \in \alpha \}$ i.e., $f(\alpha) = \{ U \subset X : \exists D \in \alpha$ such that $f(D) \subset U \}$). The category of constant filter convergence spaces and continuous maps is denoted by ConFCO which is introduced by Schwarz in 1979 [1].

The aim of this paper is to characterize local pre-Hausdorff constant filter convergence spaces and give some invariance properties of them.

Key Words: Topological category, local pre-Hausdorff spaces, constant filter convergence spaces.

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Local T_2 Constant Filter Convergence Spaces

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ABSTRACT

Sequences are sufficient to describe topological properties in metric spaces or, more generally, topological spaces having a countable base for the topology. However, filters or nets are needed in more abstract spaces. Nets are more natural extension of sequences but are generally less friendly to work with since quite often two nets have distinct directed sets for domains.

In 1937 [5], H. Cartan introduced filters were introduced. In 1954 [6], H.J. Kowalsky gave a filter description of convergence. A complete reliance on filters for the development of topology can be found by H.J. Kowalsky in 1961 [7]. In 1964 [1], D.C. Kent, in order to capture also order convergence within the theory, proposed yet another weakening of the axioms, thus defining what are known as convergence structures.

Filters play a fundamental role in the development of fuzzy spaces which have applications in computer science and engineering. Filters are also an important tool used by researchers describing non-topological convergence notions in functional analysis.

In 1979 [2], F. Schwarz introduced the category ConFCO of constant filter convergence spaces and showed that ConFCO is isomorphic to GRILL, the category of grill spaces.

In 1991 [3], M. Baran introduced local T_2 axioms [4] of topology to topological category which are used define the notion of local T_3 and T_4 object of topological category.

The aim of this paper is to characterize local T_2 constant filter convergent spaces and investigate the relationships among these various form of local T_2 constant filter convergent spaces.

Key Words: Topological category, local T_2 objects, constant filter convergence spaces.

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Local T_0 Filter Convergence Spaces

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ABSTRACT

The notion of a filter which can be viewed as a generalization of sequences was introduced by Henri Cartan [1] in 1937 and has been used as a valuable tool in the development of topology and its applications. Concepts such as points of closure and compactness that are extremely important in general topology theory cannot be described using sequences, but can be described using general filter theory. In 1954, Kowalsky [2] gave a filter description of convergence.

Let A be a set, $F(A)$ set of all filters on A and K be a function from A *to* $F(A)$. If K satisfies the following two conditions:

(1) $[x] \in K(x)$ for each $x \in A$, where $[x] = \{B \subset A \mid x \in B\}$

(2) If $\alpha \subset \beta$ and $\alpha \in K(x)$ implies $\beta \in K(x)$ for any filter β on A, then (A, K) is called a filter convergence space.

A map $f : (A, K) \rightarrow (B, L)$ between filter convergence spaces is called continuous if and only if $\alpha \in K(x)$ implies $f(\alpha) \in L(x)$ (where $f(\alpha)$ denotes the filter generated by $\{f(D) \mid D \in \alpha\}$). The category of filter convergence spaces and continuous maps is denoted by FCO which is a topological category.

Baran [3] defined local T_0 objects in a set based topological category that can be used to define a notion of closed subobject of an object in a topological category and it is shown in [4, 5, 6] that this notion forms an appropriate closure operator in the sense of Dikranjan and Giuli [7] in some well-known topological categories.

In this paper, we characterize local T_0 filter convergence spaces and investigate the relationships between local T_0 filter convergence space and the usual T_0 filter convergence space.

Key Words: Topological category, filters, filter convergence spaces, local T_0 objects.

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Local T_1 Preordered Spaces

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ABSTRACT

There is a connection between topology and order. A topological space defines an preordered (reflexive and transitive) relation and given a preordered relation on a set one can get a topology. Domain theory which can be considered as a branch of order theory studies special kinds of partially ordered sets, namely, directed complete partial orders of a domain, i.e., of a non-empty subset of the order in which each two elements have some upper bound that is an element of this subset has a least upper bound. The primary motivation for the study of domains, which was initiated by Dana Scott in the late 1960s, was the search for a denotational semantics of the lambda calculus, especially for functional programming languages in computer science [1-2].

In 1991, Baran [3] introduced a local T_1 object in a topological category which was used to define the notion of strongly closed sub-object of an object in a topological category. The other use of a local T_1 property is to define the notion of local completely regular [5] and local normal objects [4] in set-based topological categories.

A preordered space [6] is a pair (B,R), where B is a set and R is a reflexive and transitive relation on B. A map $f: (B,R)\rightarrow(B_1,R_1)$ between preordered spaces is said to be continuous if aRb, then $f(a)R_1 f(b)$ for all $a,b \in B$.

The aim of this paper is to characterize local T_1 preordered spaces and strongly closed subsets of a preordered space as well as to investigate the relationships between this closedness and the up-and down- closedness.

Key Words: Topological category, local T_1 objects, preordered spaces, strongly closed sub-objects.

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Monodromy Groupoids for topological group-groupoids and Topological Double Groupoids

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ABSTRACT

The notion of monodromy groupoid was indicated in [1] by J. Pradines to generalize the construction of a simply connected Lie group from a Lie algebra to a corresponding construction of a Lie groupoid from a Lie algebroid and it was developed by Mucuk in [3] to construct a topological groupoid structure on it. Monodromy groupoid structure is more general than universal covering and fundamental groupoid. The concept of crossed module over groups was introduced by Whitehead [2] in the investigation of the properties of second relative homotopy groups for topological spaces. However, the structure of crossed module is inadequate to give a proof of 2-dimensional Seifert-van-Kampen Theorem. Hence, one needs the idea of double groupoid which can be expressed as a groupoid object in the category of groupoids [6]. The categorical equivalence of crossed modules over groups and special double groupoids was proved in [4].

In this paper, we define topological versions of special double groupoids by the categorical equivalence given in [4] and we relate the monodromy groupoids of topological group-groupoids with double groupoids. A motivation point of the relating monodromy groupoids, crossed modules and double groupoids is to produce more examples of monodromy groupoids and double groupoids.

Key Words: Crossed module, double groupoid, monodromy groupoid.

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Multiple Neutrosophic Topolojical Spaces

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ABSTRACT

Theory of Fuzzy set, Multiset, Intuitionistic fuzzy set, Neutrosophic set and their combinations are very useful Mathematical tools to deal with uncertainty, imprecision and vagueness. Especially Neutrosophic sets and its applications are took our attention as many others. There are many works on Intuitionistic Topological Spaces and Neutrosophic Topological Spaces but we did not come across with a work on Multiple Neutrosophic Topological Spaces.

In this work, at first we gave fundamental definitions and some properties of Fuzzy set, Multiset, Intuitionistic Fuzzy set, Neutrosophic set and Neutrosophic multiset. Then, we built union and intersection of two Neutrosophic multisets and complement of a Neutrosophic multiset.

As Multiple Neutrosophic Topological Spaces are generalizations of Neutrosophic Topological Spaces, we come across same difficulties to built topology on Neutrosophic multisets. The main difficulty we encountered building a topology on Neutrosophic multisets is, there are not only one type for union or intersection of two Neutrosophic multisets and there is not only one type for complement of any Neutrosophic multiset.

We attained several topologies on Neutrosophic multisets for different types of union or intersection of Neutrosophic multisets.

At the end of this work we gave some basic properties of Multiple Neutrosophic Topology with some examples.

Key Words: Neutrosophic Multiset, Neutrosophic topology , Multiple Neutrosophic Topology.

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On Disoriented Knot Theory

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ABSTRACT

The disoriented knot theory was first introduced in [1] as a generalization of the theory of the oriented knot and link. In this work, we introduce the notion of a disoriented crossing and explanation how to adapt same fundamental concepts and invariants of the knot theory to a setting in which we have disoriented crossing.

A disoriented knot is an embedding of a disoriented circle in three dimensional space $\mathbb{R}^3$ (or S^3). A disoriented link of *k*-components is an embedding of a disjoint union of *k* circles in $\mathbb{R}^3$, where at least on of circles is disoriented.

A crossing of a disoriented knot *K* is disoriented if the overpass and underpass arcs of the crossing have opposite orientation. In other words, if A_1 and A_2 are the arcs of which *K* is an embedding, then one of the underpass and overpass arcs is A_1 and the other is A_2. If a crossing of a disoriented knot *K* is not disoriented, we say that it is a disoriented knot with zero disoriented crossing.

Let *L* be a link with exactly two components K_1 and K_2. Denote the two arcs of K_1 by A_1^1, A_1^2 and denote the two arcs of K_2 by A_2^1, A_2^2. We say that a crossing of *L* is disoriented if either of the following holds:

1. One of the underpass and overpass arcs of the crossings is A_1^1 and the other is A_2^1 or A_2^2.
2. One of underpass and overpass arcs of the crossings is A_1^2 and the other is A_2^1 or A_2^2. Otherwise, we say that the crossing is oriented.

In this work, we give a brief description of disoriented diagrams and the Reidemeister moves on disoriented diagrams. We illustrate some examples of the disoriented diagrams and all Reidemeister moves that are permissible on disoriented diagrams. We also define the linking number of a disoriented links, prove that the linking number of a disoriented link is its invariant and give two examples. We see that the disoriented linking number is always equal to the oriented linking number, regardless of the choice of disorientation.

Key Words: Disoriented knot, linking number, complete writhe.

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On Ordered Hyperspace Topologies in The Setting Of Čech Closure Ordered Spaces

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ABSTRACT

In many branches of mathematics and computer science, assorted generalizations of topological spaces are used. One of this generalizations is done by using closure operator. Closure operators obtained from Kuratowski closure by omitting some axioms and they have numerous applications. Closure operators, which are, grounded, extensive and monotone were studied by E. Čech. Then, lots of researchers have studied it. In [2], closure operators were employed for solving problems related to digital image processing and in [3], the relation between Čech closure space and structural configuration of proteins were studied. In [1], hyperspaces of Čech closure spaces were introduced and in [5] Vietoris-like topologies were studied and these results were generalizing the well known topological results. The cooperation between topology and order was studied by Leopolda Nachbin [4] in the 1950's and he developed the theory of topological ordered spaces and it has an application in mathematical economics and topology. In [6], the concept of an ordered Hyperspace was introduced and in [7] the relationships between an ordered topological space and its ordered hyperspace was investigated. In this work we study the relationships between Čech closure ordered space and its ordered hyperspace. Our results will generalize the well known results which were obtained in [1] and [7].

Key Words: hyperspace,closure,preorder.

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On Soft Topological Subspace

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ABSTRACT

In 1999, Molodtsov [1] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. He showed that the soft sets have several applications solving many practical problems in economics, engineering, social science, medical science, etc. Studies on the topological structures on the soft sets started with Shabir and Naz [2]. We have based our work on elementary soft topology. We introduce a soft topological space whose topology is defined the by the elementary union and intersection operations over an initial universe with a fixed set of parameters, which is different from soft topological space due to both Shabir and Naz [2] and Hazra et al [3]. We call this topology as elementary soft topology. We also investigate the notions of soft open sets, soft closed sets, soft neighbourhoods of a soft element, soft basis and their basic properties in elementary soft topological spaces. Since the most important fact is that the elementary union and intersection operations are not distributive in general, it is not always possible to obtain an elementary soft sub-topology of an elementary soft topology via these operations. We give conditions to solve this problem and investigate some properties of elementary soft topological space and elementary soft topological subspace which are fundamental for further research.

Key Words: Soft element, elementary soft topology, elementary soft topological subspace.

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Some Fixed Point Results For Fuzzy (h,β)-Contractive Mappings in Non-Archimedean Fuzzy Metric Spaces

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ABSTRACT

The concept of fuzzy metric space was introduced in different ways by some authors and the fixed point theory in this kind of spaces has been intensively studied. Gregori and Sapena introduced the notion of fuzzy contractive mapping and gave some fixed point theorems for complete fuzzy metric spaces in the sense of George and Veeramani, and also for Kramosil and Michalek's fuzzy metric spaces which are complete in Grabiec's sense. Recently, Mihet enlarged the class of fuzzy contractive mappings of Gregori and Sapena, considered these mappings in fuzzy metric spaces in the sense of Kramosil and Michalek and obtained a fixed point theorem for fuzzy contractive mappings. For more details on fixed point theory for contraction type mappings in fuzzy metric spaces. One most popular fixed point theorem is Banach Fixed Point Theorem. By using this theorem, most authors have proved several fixed point theorems for various mappings. Recently, Dinarvand introduce the new concepts of fuzzy β-ϕ-contractive mapping via triangular and β-admissible mappings.

In this work, we prove some fixed point results in non-Archimedean fuzzy metric spaces. Motivated by Dinarvand, we introduce the new concepts of fuzzy fuzzy (h,β)-contractive mapping via triangular and (h,β)-admissible mappings. Later, we derive several sufficient conditions which ensure the existence and uniqueness of fixed points for these classes of mappings in the setup of complete non-Archimedean fuzzy metric spaces. Some examples are supplied in order to support the useability of our results. We present some fixed point results in G-complete fuzzy metric spaces and some cyclic results. Our main results substantially generalize and extend some known results in the existing literature.

Key Words: Triangular (h,β)-admissible mapping, fuzzy (h,β)-contractive mapping, fixed point.

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Some Fixed Point Theorems in Modular Spaces

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ABSTRACT

It is well known fixed point theorems have important roles and applications in mathematics analysis, particularly in differential and integral equations. One most popular fixed point theorem is Banach Fixed Point Theorem. By using this theorem, most authors have proved several fixed point theorems for various mappings. Such as, Dutta and Choudhury proved (ψ, ϕ)-contractive mappings in complete metric space. Samet et al. introduced the concept of (α, φ)- contractive type mappings and established various fixed point theorems. Recently, Salimi et al. modified the concept of (α, φ)- contractive type mappings. Later, Alizadeh et al. developed a new fixed point theorem in complete metric spaces. They proved the concept of cyclic (α, β)-admissible mappings and $(\alpha, \beta) - (\psi,\phi)$-contractive mapping and established some fixed point results in metric spaces.

On the other hand, some authors introduced a new concept of modular vector spaces which are natural generalizations of many classical function spaces. Firstly, Nakano initiated the concept of modulared spaces. Later, some authors proved new fixed point theorems of Banach type in modular spaces.

In this work, we present some fixed point results as a generalization of Banach's fixed point theorem using some convenient constants in the contraction assumption in modular spaces. Some examples are supplied to support the usability of our results. As an application we show that the existence of solutions for an integral equation.

Key Words: cyclic (α, β) –admissible mapping, modular space, fixed point.

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Topological R-Module Groupoids

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ABSTRACT

The theory of covering spaces and covering groupoids are important topics in algebraic topology. It is well known that if X is a topological group whose underlying space has a simply connected cover, then the category $TGCov/\pi_1 X$ of topological group covers of *X* is equivalent to the category $GpGdCov/\pi_1 X$ of group-groupoid covers of $\pi_1 X$ [4, 8]. A *R*-module groupid is a *R*-module object in the category of groupoids [1]. In [1] it is proved that if *R* is a topological ring with identity 1_R and *M* is a topological *R*-module then the fundamental groupoid $\pi_1 M$ becomes a *R*-module groupoid. And also in [1], Alemdar and Mucuk proved that if *R* is a topological ring with identity 1_R and *M* is a topological R-module whose underlying space has a universal covering then the category $TModCov/M$ of topolocical R-module coverings of *M* is equivalent to the category $GdModCov/\pi_1 M$ of *R*-module groupoid coverings of the *R*-module groupoid $\pi_1 M$. In this paper first we define the topological R-module groupoid. Then we show that if *R* is a discrete topological ring with identity 1_R and *M* is topological R-module whose underlying space has a universal covering then the fundamental groupoid $\pi_1 M$ is a topological R-module groupoid. Finally we prove that if R is a topological ring with identity 1_R and M is a topological R-module whose underlying space has a universal covering then the categories $UT_d ModCov/M$ and $UT_d ModCov/\pi_1 M$ are equivalent.

Key Words: Group-groupoid, Covering groupoid, Topological R-Module groupoid.

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Topological T- algebras

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ABSTRACT

An algebraic theory T in the sense of Lawvere [3] is a category with a certain object *T* such that all objects are some multiple copies of *T*. For such a theory T a product preserving functor from T to the category of sets is called a *model* of the theory or a T-algebra and a model of T in the category of topological spaces is called topological T-algebra. Natural transformations between T-algebras are called T-homomorphisms. Hence a T-homomorphism is a map between sets commuting with all operations of the theory. An algebraic theory T whose category Set(T) of the models is semi-abelian is called *semi-abelian theory* and a model of such a theory is called *semi-abelian algebra*. Some properties of topological groups such as being Hausdorff compact, connected and etc. have been generalized to the topological T-algebras for a semi-abelian algebra.

On the other hand we know from [5] that if X is a connected topological space which has a universal cover and G is a subgroup of the fundamental group of X at identity e, then there is a covering map with characteristic group G. In this paper we apply this method to topological T-algebras including topological groups and some others.

Key Words: Topological T-algebra, Topological group, Covering map

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Tutte Polynomial for Graphs of Twist Knots

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ABSTRACT

Actually almost all of us encounter knots at some point in our lives. We are always trying to either tie a knot or solve a knot. Formally, a mathematical knot is a one-to-one continuous mapping K: $S^1 \rightarrow S^3$. Knot theory is subfield of algebraic topology, which is mainly concerned with the problem of classification mathematical knots. Knot theorists have developed their own instruments for this aim. One of the most useful is a knot invariant. The Tutte polynomial is a two-variable polynomial that is connected by a graph, a matroid or a matrix. The Tutte polynomial has a lot of exciting applications in different areas for example combinatorics, probability, knot theory, algebra, statistical mechanics, computer sciences, chemistry and biology. It was indicated by W. T. Tutte. We transport the Tutte polynomial to knot theory. Because each knot have a corresponding graph. We study the Tutte polynomial for graphs of twist knots. We introduce the Tutte polynomial and we compute the Tutte polynomial of graphs belonging to twist knots and the Tutte polynomial of signed graphs belonging to twist knots. We find some general forms for the Tutte polynomial of graphs belonging to twist knots and the Tutte polynomial of signed graphs belonging to twist knots. Twist knots are significant class of knots to take into account especially in contact geometry.

Key Words: Tutte polynomial, twist knots, signed graph.

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ABSTRACTS OF POSTER PRESENTATIONS

A Note On Bounded Engel Elements In Groups

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ABSTRACT

Let G be a group, an element g∈G is called a (left) Engel element if for any x∈G there exists a positive integer n=n(x,g) such that [x,n_g]=1, where the commutator [x,n_g] is defined inductively by the rules [x,1_g]=[x,g] and for n=2 or n>2, [x,n_g]=[[x,(n-1)_g],g]. If n can be chosen independently of x, then g is called a (left) n-Engel element or more generally a bounded (left) Engel element. The group G is an Engel group if all its elements are Engel. A subset X of a group is commutator closed if [x,y]∈X for any x,y∈X. In this study, we deal with groups generated by commutator closed set of bounded Engel elements. Our main result is to show that a residually finite group which satisfies an identity and is generated by a commutator closed set X of bounded left Engel elements is locally nilpotent. Moreover, we extend such a result to locally graded groups, if X is a normal set. Consequently, we obtain that a locally graded group satisfying an identity, all of whose elements are bounded left Engel, is locally nilpotent. Recall that a group is locally graded if every nontrivial finitely generated subgroup has a proper subgroup of finite index. The class of locally graded groups contains locally groups as well as residually finite groups.

Key Words: Engel element, Residually finite group.

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Almost Generalized Derivations

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ABSTRACT

It is very interesting and important that the similar properties of derivation which is the one of the basic theory in analysis and applied mathematics are also satisfied in the ring theory. The commutativity of prime rings were investigated with derivations.

In [1], Bresar defined concept of generalized derivation. An additive map $d: R \to R$ is called generalized derivation if there exists a derivation α of R such that $d(xy) = d(x)y + x\alpha(y)$ for all $x, y \in R$. In [1], Bresar showed that if R has the property that $Rx = \{0\}$ implies $x = 0$ and $h: R \to R$ is any function, $d: R \to R$ is any additive map satisfying $d(xy) = d(x)y + xh(y)$ for all $x, y \in R$, then d is uniquely determined by h and moreover h must be derivation.

In [3], Maksa defined bi-derivation in ring theory mutually to partial derivations and examined some properties of this derivation. A map $D: R \times R \to R$ is said to be symmetric if $D(x,y) = D(y,x)$ for all $x, y \in R$. A map $d: R \to R$ defined by $d(x) = D(x,x)$ is called the trace of D where $D: R \times R \to R$ is a symmetric map.

In [5], Öztürk defined permuting tri-derivation in a ring. A map $D: R \times R \times R \to R$ is called permuting if

$D(x,y,z) = D(x,z,y) = D(z,x,y) = D(z,y,x) = D(y,z,x) = D(y,x,z)$

hold for all $x, y, z \in R$. A map $d: R \to R$ defined by $d(x) = D(x,x,x)$ is called trace of D where $D: R \times R \times R \to R$ is a permuting map. A permuting tri-additive map $D: R \times R \times R \to R$ is called permuting tri-derivation if

$D(xw,y,z) = D(x,y,z)w + xD(w,y,z)$ for all $x, y, z, w \in R$.

In [2], Hvala gave a relation, using generalized derivation defined by Bresar, between prime rings and its extended centroid in ring theory. Many authors have investigated comparable results on prime or semi-prime rings with generalized derivations.

In this study, we prove some results about that what happens if we take trace of symmetric bi-derivation or permuting tri-derivation instead of derivation in definition of generalized derivation. Also we apply these results to very well-known results in [2].

Key Words: Prime ring, generalized derivation, symmetric bi-derivation, permuting tri-derivation.

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Ambarzumyan Theorem on a Time Scale

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ABSTRACT

Inverse spectral problems consist in recovering the coefficients of an operator from some given data. Although the literature for inverse Sturm–Liouville problems on a continuous interval is vast, there are no studies on the general time scales. Ambarzumyan's theorem is known as the first result on inverse Sturm-Liouville problems on a continuous interval (see [1]). He prove that if q is a continuous function on (0, 1) and the eigenvalues of the problem

$$-y'' + q(t)y = \lambda y, \;\; y'(0) = y'(1) = 0$$

are given as $\lambda_n = n^2\pi^2,\; n = 0,1,2,...$, then $q \equiv 0$. Since then, his result has been generalized to various versions. In particular, Freiling and Yurko [2] showed that it is sufficient to determine q only the first eigenvalue rather than the whole spectrum. Later, Yurko [3] generalized this result to a wide class of self-adjoint differential operators with arbitrary self-adjoint boundary conditions. These kinds of results are known as Ambarzumyan-type theorems.

In this study, we give an Ambarzumyan-type theorem on a general time scale. Our result is as follows:

Theorem(Ref.[4]): Let $\mathbb{T}$ be a bounded time scale such that $a = inf\mathbb{T}$, $b = sup\mathbb{T}$, and λ_0 be the first eigenvalue of the following boundary value problem:

$$-y^{\Delta\Delta} + q(t)y^{\sigma} = \lambda y^{\sigma}, \;\; y^{\Delta}(a) = y^{\Delta}(\rho(b)) = 0 \; .$$

If $\lambda_0 = \frac{1}{\rho(b)-a}\int_a^{\rho(b)} q(t)\Delta t$, then $q(t) = \lambda_0$ for all $t \in \mathbb{T}^{k^2}$.

Key Words: Time scale, Sturm-Liouville dynamic equation, inverse problem, Ambarzumyan theorem.

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An Investıgatıon of The Relationship Between 9th Grade Students' Performance on Non-Routine Mathematics Problems and Their Mathematics Anxıety

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ABSTRACT

Results of international assessments suggest that Turkish students have difficulties in solving mathematical problems, especially non-routine problems as posed in the PISA test (Organization for Economic Co-operation and Development, [OECD], 2006; Özbay, 2015). As students' attitudes towards mathematics play an important role in their performance in solving problems (Mensah, Kuranchie, & Okyere, 2013), it is important to know more about students' anxiety towards different types of mathematics problems (routine vs. non-routine) in order to facilitate their problem-solving performance.

This study was conducted in the context of an intern teacher's field experience in a private school. The school implemented a different method of assessing students' mathematical understanding by incorporating non-routine problems in the mathematics test in Fall 2017. Non-routine problems referred to problems that students were not familiar with and required higher level thinking skills compared to routine problems. This study investigated the relationship between student performance in routine and non-routine problems and students' mathematics anxiety. Understanding to what extent student performance is affected by mathematics anxiety in solving different types of problems will provide insights for teachers in helping their students build healthy relationship with mathematics and problems solving processes.

Quantitative methods, more specifically, correlational analyses were utilized in order to answer the research question. The participants of this study were 9th grade students from seven different classrooms in a private school who signed the consent forms (N=120). Students' performance in three mathematics examinations were considered and their performance in both 'routine' and 'non-routine problems' were

investigated. In order to determine students' mathematics anxiety levels, students' responses to the Mathematics Anxiety-Apprehension Survey (MASS) developed by Ikegulu (1998) (as cited in Özdemir & Gür, 2011) and adapted to Turkish by Özdemir & Gür (2011) were collected. The survey comprised of 20 Likert-type items. Higher scores indicated higher levels of mathematics anxiety.

The relationship between student mathematics anxiety and mathematics performance was determined by conducting correlation analyses using SPSS. The correlation between student mathematics anxiety and their score in non-routine problems as well as their total mathematics scores were calculated.

The correlation coefficient between student total grades and students' mathematics anxiety was determined as r= -.52* ($p<.01$, two tailed). According to Cohen (1988), this correlation was considered as strongly negative, which indicated a strong negative relationship between students' total exam scores and their mathematics anxiety. The correlation coefficient between students' scores in non-routine problems and students' mathematics anxiety was determined as r= -.35* ($p<.01$, two tailed). According to Cohen (1988), this correlation coefficient signified negative and middle level relationship between two variables.

The results suggested that the level of the relationship between mathematics anxiety and students' total exam scores was stronger compared to the level of the relationship between mathematics anxiety and students' scores in non-routine items. This result may indicate that for this group of students anxiety played a less significant role on student performance in non-routine problems compared to all problems. Possible reasons and implications of the results to student learning and assessment will be discussed in the poster session.

Key Words: Mathematics, Mathematics Anxiety, Non-routine Problems.

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Characterization of Physics and Mathematics of Quantum Entanglement Via Ion-Photon System

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ABSTRACT

Quantum entanglement is a quantum mechanical phenomenon in which the quantum states of two or more objects. For example, it is possible to prepare two particles in a single quantum state such that when one is observed to be spin-up, the other one will always be observed to be spin-down and vice versa. This work have investigated quantum entanglement between a three- level trapped ion and two laser beams. The trapped ion is initially in its excited-raman-ground (e-r-g) of superposition states and the laser beams is in the coherent state.

We present a comprehensive analysis of the pattern of information entropy arising in the time evolution of ion-photon system. Describing the quantum entanglement is shown negativity and concurrence with respect to Schmidt coefficients. Quantum measurements may be defined in ion-photon system with different configurations of the Lamb-Dicke parameter (LDP). In the initial unentangled state, typical family of the Lamb-Dicke parameter can be obtained qudit states in a trapped ion. Finally, determination of physics and mathematics of quantum entanglement is illustrated with respect to reduced density matrix. It is seen that the mathematics of quantum entanglement is important in quantum information theory.

Key Words: Quantum entanglement, density matrix, quantum measure.

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Examining Classroom Experiences and Perspectives of Middle School Mathematics Teachers On The Use of Tablets

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ABSTRACT

The use of tablets in education is becoming important and popular in many respects, especially in mathematics classrooms (Ditzler, Hong and Strudler, 2016). Previous studies documented a variety of benefits of using tablets in mathematics lessons including representations of graphs, easy interaction with peers, and use of archive notes (Fister & McCarthy, 2008). Tablet use in Turkey is a rather new phenomenon, which was initiated by the "Fatih Project" implemented by the Ministry of National Education, 2016). Studies on tablet use in the context of Turkey mainly focused on the attitudes of the teachers and students towards the use of tablets using surveys (Akbay & Küçük, 2013; Batur, Gülveren & Balcı, 2013; Çetinkaya and Keser, 2014). There were only few studies in the Turkish context that focused on the experiences of teachers and students who used tablets effectively during lessons (Sarıtepeci & Durak, 2016). There is a need for understanding changes that tablet use initiates in teacher's lesson planning and in the interaction between teachers and students, especially in the context of Turkey where this type of research is scarce. This study investigated classroom experiences and perspectives of middle school mathematics teachers on the use of tablets in a private school where tablet use is adapted by the whole school.

This study utilized qualitative methods. The private school was selected as a case because teachers implemented it in their classrooms consistently. The main data source for this study consisted of interviews with eight mathematics teachers and their academic coordinator, which were transcribed and coded into different themes using content analysis. Interviews provided insights on teachers' impressions and experiences, their thoughts and future plans for use of the tablet, strategies

related to tablet use and objectives, and their evaluations of influence on student learning.

The initial results of data analysis revealed a majority of teachers believed tablets were necessary as a 21st century requirement and reduction in paper use. Teachers viewed using tablets as a tool for professional development and reported support from the school administration. An important theme across interviews was being able to give fast and effective feedback by using tablets. Some teachers focused on subject specific themes related to mathematics including dynamic technology structure and providing easier access to real life problems. Teachers in general considered tablets were beneficial for students since they provided individual learning opportunities and increase in student motivation.

The results suggested that adapting technology as the whole school, getting professional feedback about planning lessons with tablets, technological support and professional development helped teachers integrate tablet use in their classrooms effectively which is different than previous research conducted in the context of Turkey. Implications on teachers' use of tablets in mathematics classroom for practice and research will be shared during the session.

Key Words: Tablet use in mathematics, technology use of math teachers, classroom experiences

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Exploring Personal Theories of a Prospective Middle School Mathematics Teacher

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ABSTRACT

Fieldwork experiences during teacher education provide opportunities to help prospective teachers understand the relationship between theoretical knowledge at the university and teaching practices. One useful way to facilitate understanding between theories of teaching and learning, and teaching practices as observed is writing reflective journals during fieldwork activities (Körkkö, Kyrö-Ämmälä & Turunen, 2016). While reflective journals can be in many different forms, in general they are intended to reveal personal understandings of a prospective teacher in the context of different types of school activities. Analysing reflective journals is common in teacher education programs because this type of research reveals growth of the professional skills of the prospective teachers during teacher education programs and more specifically, field work.

The aim of this study was to determine the nature and frequency of personal theories developed by a prospective middle school mathematics teacher by investigating her reflective journals during fieldwork. Developing personal theories about observations during field experience is important as it may be a sign that prospective teacher is able to integrate theoretical knowledge at the university with practical knowledge at the school site (Maaranen & Stenberg, 2017). In this study we use a definition of personal theory as proposed by Elbaz (1983): "teacher's complex set of understandings, which are actively used to shape and direct the work of teaching" (Elbaz, 1983, p. 3). Nature of personal theories may change during the field experience. Exploring this change over time may be used to understand professional growth of a prospective teacher which can help teacher educators better facilitate prospective teacher learning.

This study used qualitative methods to understand personal theories of a prospective mathematics teacher through three semesters during fieldwork. The

prospective teacher was selected as a case because she was one of the first graduates of a teacher education program in a private university which focused heavily on fieldwork in the context of Turkey (Özcan, 2013). The main data source of this study consisted of about 40 journal documents written by the prospective teacher. The authors conducted data analysis by reading each journal and determined personal theories according to the definition by Elbaz (1983). The researchers agreed on all the personal theories. After determining the personal theories, each personal theory was coded by using content analysis which led to different themes. The initial results of data analysis revealed the following themes: students' learning, students' affects, students' interests, classroom management and being a teacher. A majority of the theories were about students while classroom management was the topic which was least mentioned. Additionally, the number of the personal theories observed in journals decreased after the first semester. Examples of personal theories and changes in the content as well as implications of the results on teacher learning and teacher education will also be shared during the poster session.

Key Words: Prospective mathematics teacher, reflective journals, personal theories.

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Generalized Derivations of Hyperlattices

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ABSTRACT

Firstly, Marty introduced the notion of hyperstructure in [6] at 8th Congress of Scandinavian Mathematicians. Normally, the composition of two elements is an element in classical algebraic structures, but the composition of two elements is a set in algebraic hyperstructures. After this study, many authors studied this subject. The many concepts in pure and applied mathematics were applied to hyperstructures [2, 4]. In [5], Konstantinidou and Mittas introduced hyperlattices and superlattices. In particular some interesting results of the theory of hyperlattices studied by Rasouli and Davvaz in [7, 8].

Let L be a nonempty set and $\vee: L \times L \to P^*(L)$ be a hyperoperation, where $P(L)$ is a power set of L and $P^*(L) = P(L) - \emptyset$ and $\wedge: L \times L \to L$ be an operation. Then $(L,\vee,\wedge)$ is a hyperlattice if

(1) for all $a \in L, a \in a \vee a$, $a \wedge a = a$;

(2) for all $a, b \in L, a \vee b = b \vee a,\ \ a \wedge b = b \wedge a$;

(3) for all $a, b, c \in L, (a \vee b) \vee c = a \vee (b \vee c);\ \ (a \wedge b) \wedge c = a \wedge (b \wedge c)$;

(4) for all $a, b \in L, a \in [a \wedge (a \vee b)] \cap [a \vee (a \wedge b)]$;

(5) if $a \in a \vee b$ for all $a, b \in L,$ then $a \wedge b = b$.

Derivations in rings and near-rings have been studied by many mathematicians in several ways. Bresar [3] introduced the generalized derivation in rings and many mathematicians studied on this concept. N. O. Alshehri applied the notion of generalized derivation in ring theory to lattices [1]. Now, we define the notion of derivation on hyperlattice. In this study, we aim to generalize some results given in [1] to generalized derivations of hyperlattices. In this way, we define generalized derivation on hyperlattice and give an example.

A mapping $D: L \to L$ is called a generalized derivation on hyperlattice L if there exists a derivation $d: L \to L$ such that

I. $D(x \vee y) \subseteq D(x) \vee D(y)$

II. $D(x \wedge y) \in (D(x) \wedge y) \vee (x \wedge d(y))$

for all $x, y \in L$. The pair (L, D) is said to be a differential hyperlattice or is said to be hyperlattice with generalized derivation.The map D is called strong generalized derivation if $D(x \vee y) = D(x) \vee D(y)$ and satisfies the condition II. Then the pair (L, D) is called a strongly differential hyperlattice.

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Key Words: Generalized derivations, hyperlattice.

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Investigating a Middle School Mathematics Teacher's Responses to Pivotal Teaching Moments

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ABSTRACT

Attending to students' needs and acting accordingly is essential in building student-centered mathematics instruction. In exploring teachers' decision making in the moment of teaching, pivotal teaching moments (PTMs) (Stockero & Zoest, 2013) were defined as instances of teachers' changing or improving lesson plans according to students' thinking when unexpected student misconceptions, mathematical questions or comments come up. Although how teachers respond to PTMs shapes student learning, little is known about how to act on PTMs productively in different contexts. The purpose of this study is to investigate a teacher's instructional decisions and actions in response to PTMs.

This paper documents a case study of a teacher who was purposefully selected because of her orchestration of mathematical discussions in a meaningful way even though she only had 2 years of experience working in a private school. One of the authors observed and reflected on the teacher's instructional practices during one semester. The data sources of this study included semi-structured interviews with the teacher before and after conducting observations of four lessons from two different classrooms as well as planning meetings, during when the teacher collaborated with colleagues and developed lesson plans. Data analysis utilized several frameworks from the literature in understanding teachers' instructional decisions and actions in responding to student thinking and specifically focused on the PTMs (Cengiz, Kline, & Grant, 2011; Stockero & Zoest, 2013). Analysis of the lesson plans, observations and interviews revealed several PTMs. There were only a few unexpected episodes which may be due to the fact that the lesson plans were detailed and were developed by a group of teachers including experienced teachers. Analysis of transcripts of unexpected moments in teaching by using instructional actions framework (Cengiz et

al., 2011) revealed that teacher instructional actions included supporting and extending student understanding actions. More specifically recording and acknowledging student thinking and inviting students to evaluate a claim, challenging/providing counter arguments to student claims were observed as the teacher responded to changes in the lesson flow. We discuss unexpected answers, comments and questions in relation to teacher and student learning by considering both pre-and post-lesson interviews with the teacher.

Considering students' engagement and answers in the discussion, it may be argued that teacher was able to create a rich discussion environment by demonstrating such instructional actions. Additionally, teacher interviews revealed that the teacher was able to use the PTMs in planning for the next lessons which demonstrated teacher learning through experience. Results of this study have implications for understanding teacher decision making for the purpose of developing meaningful mathematical discussions during the moment of teaching, which is found difficult for even experienced teachers. In the poster session teacher responses to different types of pivotal teaching moments will be discussed by exploring differences and similarities in instructional actions and implications for teacher practices will be shared.

Key Words: Teacher planning, middle school mathematics, instructional actions

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Some Fixed Point Results For Contractive Type Mappings in B-Metric Spaces

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ABSTRACT

There has been a numerous generalization of metric spaces. One such a well known generalization is b-metric space defined by Czerwik [1]. After that many authors have obtained some fixed point theorems in b-metric spaces. Hussain et al. [2] introduced the notion of wt-distance on b-metric spaces, which is a b-metric version of w-distance of Kada et al. [4] and they obtained some fixed point theorems in a partially ordered b-metric space by using wt-distance. Then, Mohanta [5] proved some fixed point theorems by using the wt-distance on a b-metric spaces. Saadati et al. [7] obtained some fixed point theorems for classes of contractive type multi-valued operators via wt-distances in the setting of a complete b-metric space. In 2012, Samet et al. [6] introduced the concepts of α-ψ-contractive and α-admissible mappings. Then, many authors investigated some fixed point results by using this idea. Karapinar et al. [3] extended the results of Samet et al to the concept of b-metric space and they investigated Ulam-Hyers stability results for fixed point theorems by using α-ψ-contractive mapping of type-(b) in the sense of b-metric spaces. In this paper, we first prove some fixed point theorems by using wt-distance on complete b-metric spaces and we extend the results of Karapinar et al. [3]. Also, we introduce the notion of wt_0-distance and we obtain some coupled fixed point theorems via wt_0-distance on b-metric spaces

Key Words: Fixed point, b-metric, wt-distance.

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ABSTRACTS OF WORKSHOP

Student-Centered Mathematics Instruction-Workshop

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ABSTRACT

The aim of this workshop is to underline different instructional models, to discuss the main parts of them, sharing sample instructional design process and details of considerations in each step, and to make participants prepare draft outline for student-centered mathematics unit.

The main parts of the workshop will be fundamental concepts in instructional design, instructional design model, sample instructional design, preparation of sample instructional unit plans, sharing of the draft plans and feedback for development.

The workshop is open for all interested participants, especially practitioners in schools and educational institutions.

Key Words: Instructional design, mathematics, students' centered mathematics teaching

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