24th Series by MATDER

ICMME 2026

International Conference on Mathematics and Mathematics Education

ABSTRACT BOOK

On the Trail of Science from the Seljuks to the Republic: Mathematics in Sivas

SEPTEMBER 10-12, 2026

Sivas Cumhuriyet University Sivas/Türkiye

www.theicmme.org www.matder.org.tr www.cumhuriyet.edu.tr

ABSTRACT BOOK

24th International Conference on Mathematics and Mathematics Education (ICMME-2026)

On the Trail of Science from the Seljuks to the Republic: Mathematics in Sivas

in Memory of Prof. Dr. A. Okay Çelebi

Sivas Cumhuriyet University, Sivas, Türkiye, 10–12 September 2026

© All rights reserved. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the permission of the authors.

All kinds of information in this book is the responsibility of the authors.

Editors

Nurettin DOĞAN
Selçuk University, Türkiye

Yaşar ÇAKMAK
Sivas Cumhuriyet University, Türkiye

Assistant Editors

Tahir FEYZİOĞLU
MATDER, Türkiye

PREFACE

International Conference on Mathematics and Mathematics Education (ICMME-2026) was held on 10-12 September 2026 in Sivas, Türkiye. The conference brought together researchers, educators, and practitioners to exchange ideas, present recent research findings, and discuss future challenges in mathematics and related disciplines. The scientific program included invited plenary lectures and short oral presentations, providing a comprehensive overview of current developments in the field.

MATDER – Mathematicians Association was established in 1995 by mathematicians in Türkiye. Since its foundation, MATDER has organized 14 national and 8 international mathematics conferences. Over the years, these meetings have become major scientific events at both national and international levels. Covering a wide range of topics in pure and applied mathematics, mathematics education, engineering mathematics, and computer science, the ICMME conferences have consistently attracted significant interest from mathematicians and engineers in academia.

The previous ICMME conferences were held in İstanbul (ICMME-2025), Nevşehir (ICMME-2024), Denizli (ICMME-2022), Ankara (ICMME-2021), Konya (ICMME-2019), Ordu (ICMME-2018), Şanlıurfa (ICMME-2017) and Elazığ (ICMME-2016). This year, ICMME-2026 was hosted by Sivas Cumhuriyet University, Sivas, Türkiye, September 10–12, 2026, as an international scientific conference.

The main objective of ICMME-2026 is to contribute to the advancement of mathematical sciences, mathematics education, computer sciences, and their applications, as well as to provide a platform for international collaboration among researchers, educators, mathematicians, statisticians, and interdisciplinary scholars from around the world. Abstracts of all accepted presentations are included in these conference proceedings. In addition, selected and peer-reviewed full papers will be published in the following journals:

- Fractal and Fractional-Special Issue-Advances in Fractal and Fractional Dynamics-With APC
- Fractal and Fractional-Special Issue-Feature Papers for Mathematical Physics Section 2026-With APC
- Symmetry-Special Issue-New Trends on the Mathematical Models and Solitons Arising in Real-World Problems, 3rd Edition-With APC
- Nonlinear Analysis and Computer Simulations
- Turkish Journal of Mathematics and Computer Science (TJMCS)
- International Journal of Maps in Mathematics
- Selcuk University Journal of Engineering Sciences

ICMME-2026 is dedicated to the memory of **Prof. Dr. A. Okay Çelebi**. Prof. Çelebi was the doctoral advisor of Prof. Dr. Ömer AKIN, Honorary President of MATDER, and of several colleagues participating in this conference, and he taught many of us during our undergraduate studies. We remember him with deep respect and gratitude.

The ICMME-2026 conference is jointly organized by MATDER – The Mathematicians' Association and Sivas Cumhuriyet University. We sincerely thank all participants, authors, reviewers, and invited speakers for their valuable contributions to the success of this conference.

On Behalf of The Organizing Committee

Hasan Hüseyin SAYAN

COMMITTEES

HONORARY AND ADVISORY BOARD

Dr. Yılmaz ŞİMŞEK	Governor of Sivas, Türkiye
Ahmet ŞENGÖNÜL	Rector of Sivas Cumhuriyet University, Türkiye
Ömer AKIN	Honorary President of MATDER, Türkiye

ORGANIZING COMMITTEE

Hasan Hüseyin SAYAN	Chairman, MATDER President, Gazi University, Türkiye
Yaşar ÇAKMAK	Chairman, Sivas Cumhuriyet University, Türkiye
Abdulhamit KÜÇÜKASLAN	Vice Chairman, MATDER, Ankara Yıldırım Beyazıt University, Türkiye
Nurettin DOĞAN	Vice Chairman, MATDER, Selçuk University, Türkiye
Mustafa ÖZKAN	MATDER, Gazi University, Türkiye
Ali ÖZTURAN	MATDER, Türkiye
Amiran GOGATISHVILI	Ins. of Math. of The Czech Acad. of Sciences, Czech Republic
Cengiz ÇINAR	MATDER, Gazi University, Türkiye
Ercan ÇELİK	Kırgızistan-Türkiye Manas University, Kyrgyzstan
Fatma AYAZ	MATDER, Gazi University, Türkiye
Gülay KORU	Gazi University, Türkiye
Hacı Mehmet BAŞKONUŞ	Harran University, Türkiye
Hasan BULUT	Fırat University, Türkiye
Hüseyin DEMİR	Samsun Ondokuz Mayıs University, Türkiye
Hüseyin YILDIRIM	Kahramanmaraş Sütçü İmam University, Türkiye
İsmail EKİNCİOĞLU	Istanbul Medeniyet University, Türkiye
Jan ZEMLICKA	Univerzita Karlova, Matematicko-fyzikální fakulta: Praha, Czech Republic
Jehad ALZABUT	Prince Sultan University, Ostim Technical University, Türkiye
Maria Alessandra RAGUSA	Università di Catania, Italy
Matthias HEIL	School of Mathematics, University of Manchester, UK
Mohamed Ibrahim ABBAS	Alexandria University, Egypt
Mohammad Esmael SAMEI	Bu Ali Sina University: Hamedan, Iran
Naim TUĞLU	Gazi University, Türkiye
Nilay AKGÖNÜLLÜ PİRİM	MATDER, Başkent University, Türkiye
Ömer ÖZEN	MATDER, Türkiye
Sorina BARZA	Karlstads Universitet, Sweden
Tahir FEYZİOĞLU	MATDER, Türkiye
Truong Cong QUYNH	The University of Danang: Danang, Vietnam
Tuğba MERT	Sivas Cumhuriyet University, Türkiye
Vagif S. GULIYEV	Baku State University, Azerbaijan, Ahi Evran University, Türkiye

Yasin DEMİRCİ	MATDER, Türkiye
Yusif GASIMOV	Azerbaijan University, Azerbaijan
Zakia HAMMOUCH	Moulay Ismail University, Morocco

LOCAL ORGANIZING COMMITTEE

Ahmet Sinan ÖZKAN	Sivas Cumhuriyet University, Türkiye
Aykut HASBEK	Sivas Cumhuriyet University, Türkiye
Baki KESKİN	Sivas Cumhuriyet University, Türkiye
Hasret DURNA	Sivas Cumhuriyet University, Türkiye
Hatice Kübra PEKMEZ	Sivas Cumhuriyet University, Türkiye
İbrahim ADALAR	Sivas Cumhuriyet University, Türkiye
İdris ZORLUTUNA	Sivas Cumhuriyet University, Türkiye
Mathematics Club Students	Sivas Cumhuriyet University, Türkiye
Mehmet Sefa CİHAN	Sivas Cumhuriyet University, Türkiye
Merve Esra TÜRKAY	Sivas Cumhuriyet University, Türkiye
Mustafa YILDIRIM	Sivas Cumhuriyet University, Türkiye
M. Meral YILDIRIM	Sivas Cumhuriyet University, Türkiye
Nilüfer TOPSAKAL	Sivas Cumhuriyet University, Türkiye
Nuh DURNA	Sivas Cumhuriyet University, Türkiye
Özgür İNCE	Sivas Cumhuriyet University, Türkiye
Öznur GÖLBAŞI	Sivas Cumhuriyet University, Türkiye
Selma GÜLYAZ ÖZYURT	Sivas Cumhuriyet University, Türkiye
Serkan ATMACA	Sivas Cumhuriyet University, Türkiye
Sevim DURAK	Sivas Cumhuriyet University, Türkiye
Şenol BİÇER	Sivas Cumhuriyet University, Türkiye
Tuğba MERT	Sivas Cumhuriyet University, Türkiye
Yalçın GÜLDÜ	Sivas Cumhuriyet University, Türkiye
Yaşar ÇAKMAK	Sivas Cumhuriyet University, Türkiye
Zeliha BEDİR	Sivas Cumhuriyet University, Türkiye
Zeynep ADIGÜZEL	Sivas Cumhuriyet University, Türkiye

SCIENTIFIC COMMITTEE (MATHEMATICS)

Abdulhamit KÜÇÜKASLAN	Ankara Yıldırım Beyazıt University, Türkiye
Adil GÜLER	Marmara University, Türkiye
Ahmet DURAN	İstanbul Technical University, Türkiye
Amiran GOGATISHVILI	Institute of Mathematics of the Czech Academy of Sciences, Czech Republic
Armando CIANCIO	Messina University, Italy
Ayhan ŞERBETÇİ	Ankara University, Türkiye
Ayşe Pelin GÖKGÖZ	Gebze Technical University, Türkiye
Bayram ŞAHİN	Ege University, Türkiye
Cenap ÖZEL	King Abdulaziz University, Saudi Arabia

David CRUZ-URIBE	University of Alabama, USA
Emine CAN	İstanbul Medeniyet University, Türkiye
Emran ALI	Northern University Bangladesh, Bangladesh
Erhan COŞKUN	Karadeniz Technical University, Türkiye
Fouzi HATHOUT	Université de Saïda - Dr. Moulay Tahar, Algeria
Fuad KITTANEH	University of Jordan, Jordan
Hacı Mehmet BAŞKONUŞ	Harran University, Türkiye
Hasan BULUT	Fırat University, Türkiye
Hayriye SUNDU PAMUK	İstanbul Medeniyet University, Türkiye
İsmail EKİNCİOĞLU	Istanbul Medeniyet University, Türkiye
Jan ZEMLICKA	Charles University, Czech Republic
Jean-Christophe BOURIN	University of Franche-Comté, France
Jehad ALZABUT	Prince Sultan University, Saudi Arabia
Kazım İLARSLAN	Kırıkkale University, Türkiye
Kemal AYDIN	Selcuk University, Türkiye
Lars-Erik PERSSON	UiT The Arctic University of Norway, Norway
M. Tamer KOŞAN	Gazi University, Türkiye
Maria Alessandra RAGUSA	Università di Catania, Italy
Martin BOHNER	Missouri University Of Science And Technology, USA
Matthias HEIL	School of Mathematics, University of Manchester, UK
Mustafa ÖZKAN	Gazi University, Türkiye
Naim TUĞLU	Gazi University, Türkiye
Natália BEBIANO	University of Coimbra, Portugal
Natasha SAMKO	Arctic University of Norway, Norway
Nureddin TÜRKAN	İstanbul Medeniyet University, Türkiye
Oktay MUHTAROĞLU	Gaziosmanpaşa University, Türkiye
Rabil AYAZOĞLU	Bayburt University, Türkiye
Ryskul OINAROV	Eurasian National University, Kazakhstan
Shigeru FURUICHI	Mathematical Science, Nihon University, Japan
Siegfried M. RUMP	Hamburg University of Technology, Germany
Sorina BARZA	Karlstads Universitet, Sweden
Murat TOSUN	Sakarya University, Türkiye
Varga KALANTAROV	Koç University, Türkiye
Vagif S. GULIYEV	Baku State University, Azerbaijan, Ahi Evran University, Türkiye
Vincenzo CIANCIO	Messina University, Italy
Yoshihiro SAWANO	Chuo University, Japan
Yusif S. GASIMOV	Azerbaijan University, Azerbaijan
Zakia HAMMOUCH	Moulay Ismail University, Morocco

SCIENTIFIC COMMITTEE (COMPUTER SCIENCE)

Ayhan ERDEM	Gazi University, Türkiye
Ercan Nurcan YILMAZ	Gazi University, Türkiye
Necaattin BARIŞÇI	Gazi University, Türkiye
Yusuf SÖNMEZ	Gazi University, Türkiye
Aydın ÇETİN	Gazi University, Türkiye
Murat YÜCEL	Gazi University, Türkiye
Uğur GÜVENÇ	Düzce University, Türkiye
Hüseyin POLAT	Gazi University, Türkiye
Cemal KOÇAK	Gazi University, Türkiye
Cemal YILMAZ	Gazi University, Türkiye
Hasan Hüseyin SAYAN	Gazi University, Türkiye
Sinan TOKLU	Gazi University, Türkiye
Adem TEKEREK	Gazi University, Türkiye
İbrahim Alper DOĞRU	Gazi University, Türkiye
Tuncay YİĞİT	Süleyman Demirel University, Türkiye
Osman ÖZKARACA	Muğla Sıtkı Koçman University, Türkiye
Gürcan ÇETİN	Muğla Sıtkı Koçman University, Türkiye
Nurettin DOĞAN	Selçuk University, Türkiye
Murat KÖKLÜ	Selçuk University, Türkiye
Metin KAPIDERE	İnönü University, Türkiye
Fatih BAŞÇİFTÇİ	Selçuk University, Türkiye
İlkay ÇINAR	Selçuk University, Türkiye
Adem Alpaslan ALTUN	Selçuk University, Türkiye
Humar KAHRAMANLI ÖRNEK	Selçuk University, Türkiye
Şakir TAŞDEMİR	Sinop University, Türkiye
Hamdi Tolga KAHRAMAN	Karadeniz Technical University, Türkiye
Sabri KOÇER	Necmettin Erbakan University, Türkiye

SCIENTIFIC COMMITTEE (MATHEMATICS EDUCATION)

Ali ERASLAN	Samsun Ondokuz Mayıs University, Türkiye
Ayşe Tuğba ÖNER	İstanbul Medeniyet University, Türkiye
Cemil İNAN	Artuklu University, Türkiye
Cengiz ÇINAR	Gazi University, Türkiye
Çağlar Naci HIDIROĞLU	Pamukkale University, Türkiye
Damla Solmaz GEDİK	İstanbul Medeniyet University, Türkiye
Deniz KAYA	İstanbul Medeniyet University, Türkiye
Çiğdem KILIÇ	İstanbul Medeniyet University, Türkiye
Derya Özlem YAZLIK	İstanbul Medeniyet University, Türkiye
Esra YILDIZ	İstanbul Medeniyet University, Türkiye

Eyüp SEVİMLİ	İstanbul Medeniyet University, Türkiye
Hamza Yaşar OCAK	Marmara University, Türkiye
Hasan ES	Gazi University, Türkiye
Hülya GÜR	Balıkesir University, Türkiye
İsmet AYHAN	Pamukkale University, Türkiye
Melek MASAL	Sakarya University, Türkiye
Nazan MERSİN	İstanbul Medeniyet University, Türkiye
Murat ALTUN	Uludağ University, Türkiye
Murat PEKER	Afyon Kocatepe University, Türkiye
Mustafa DOĞAN	Selçuk University, Türkiye
Mustafa KANDEMİR	Amasya University, Türkiye
Necdet GÜNER	Pamukkale University, Türkiye
Nurullah YAZICI	Karamanoğlu Mehmetbey University, Türkiye
Osman GÜRSOY	Maltepe University, Türkiye
Ömer DERELİ	Necmettin Erbakan University, Türkiye
Ömer Faruk TEMİZER	Fırat University, Türkiye
Özge Yiğitcan NAYIR	Başkent University, Türkiye
Rabia SAVAŞ	İstanbul Medeniyet University, Türkiye
Safure BULUT	Middle East Technical University, Türkiye
Sare ŞENGÜL	Marmara University, Türkiye
Sibel Yeşildere İMRE	Dokuz Eylül University, Türkiye
Süha YILMAZ	Dokuz Eylül University, Türkiye
Şenol DOST	Hacettepe University, Türkiye
Şenol KARTAL	İstanbul Medeniyet University, Türkiye
Şeref MİRASYEDİOĞLU	Başkent University, Türkiye
Tunay BİLGİN	Van Yüzüncü Yıl University, Türkiye
Yasemin KATRANCI	Kocaeli University, Türkiye
Zaki F.A. EL-RAHEM	Alexandria University, Egypt
Zeynep Çiğdem ÖZCAN	İstanbul Medeniyet University, Türkiye

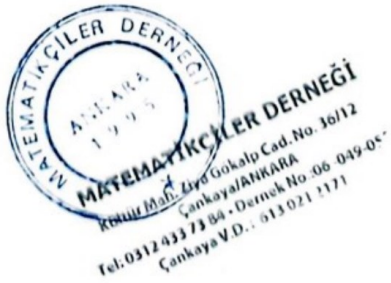
MATEMATİKÇİLER DERNEĞİ
Ziya Gökalp Caddesi No: 36/12 Çankaya –ANKARA, Tel : 433 73 84, bilgi@matder.org.tr
www.matder.org.tr,

24.04.2026

İLGİLİ MAKAMA

Matematikçiler Derneğinin 10-2 Eylül 2026 tarihleri arasında bu yıl 24.sünü Sivas Cumhuriyet Üniversitesi ev sahipliğinde Sivas'da düzenleyeceği Uluslararası Matematik ve Matematik Eğitimi Konferansı (International Conference on Mathematics and Mathematics Education: ICMME 2026) için **Matematikçiler Derneğinin 24.04.2026 tarih ve 7 sayılı Yönetim Kurulu Kararı** ile Kongre Başkanı, Matematikçiler Derneği Yönetim Kurulu Başkanı ve Gazi Üniversitesi Öğr. Üyesi **Prof. Dr. Hasan Hüseyin SAYAN**, Kongre Başkanı ve Sivas Cumhuriyet Üniversitesi Öğr. Üyesi **Prof. Dr. Yaşar ÇAKMAK**, Düzenleme Kurulu Üyeleri; Matematikçiler Derneği Yönetim Kurulu Üyesi ve Selçuk Üniversitesi Öğr. Üyesi **Prof. Dr. Nurettin DOĞAN**, Matematikçiler Derneği Yönetim Kurulu Üyesi ve Gazi Üniversitesi Öğr. Üyesi **Prof. Dr. Mustafa ÖZKAN** ve Matematikçiler Derneği Yönetim Kurulu Üyesi ve Ankara Yıldırım Beyazıt Üniversitesi Öğr. Üyesi **Doç. Dr. Abdülhamit KÜÇÜKASLAN** Akademisyen Temsilcisi olarak görevlendirilmiştir.

Prof. Dr. Hasan Hüseyin SAYAN
Matematikçiler Derneği Yönetim Kurulu Başkanı



CONTENTS

INVITED SPEAKERS **1**
Evolution of Internal Variables in the Hybrid Heat Equation for Anisotropic Viscoelastic Media 2
A Delayed Generalized Lotka–Volterra Model for Threshold Instability and Structural Recognition 3
Geometry of Umbrella Matrices 4
On Some Stability Results of Power-Law Fractal-Fractional Systems 5
Shannon-Whittaker-Kotel'nikov's Theorem Generalized 6
Recent Progresses on Real Analysis and Applications 7
Success in Education and Why is Mathematics? 8
Tracing Mathematical Meaning: A Journey to the Integrated Dynamic Mathematical Meaning Model (BDSM) 9

ABSTRACTS OF ORAL PRESENTATIONS **10**

Algebra and Number Theory ***11***
Generalized Riordan Structures and Sequence Characterizations 12
On Homomorphisms of Nearness Γ-Near Rings 13
On the Generalized Tangent-Fibonacci Polynomials and Related Identities 14
Padovan-Sum Graph 15
q-Leonardo Chebyshev Polynomials 17

Analysis and Theory of Functions ***19***
A Grand Variable Exponent Morrey Space Framework for Sobolev-Type Embeddings, Compactness, and Hölder Regularity in Anisotropic Fractional Variational Problems 20
A Review on the Spectral Analysis of Weighted Mean Matrices 22
Exponential Type Baskakov Operators via Summability Methods 24
Quantum Calderon-Zygmund Singular Integral Operators in the Quantum Lebesgue Spaces 27
The Spectrum of a Generalization of the Cesàro Matrix in Convergent Sequences Space 28
What Is a Complex Partial Differential Equation? 29

Geometry ***30***
Algebraic Structure and Hamilton Matrix Representations of Quaternions over the Gaussian Rational Field 31
Axode Geometry of Persistent Lorentzian Frame Motions 32
Explicit Free Resolutions of Tangent Cones in Affine 4-Space 33
Figures Placed Among Geometric Patterns in Sivas Structures of the Medieval Anatolian Turkish-Islamic Architectural Period 35
Information Geometry of Orthogonal Generalized Doubly Stochastic Matrices 37

Intrinsic Characterization of Persistent Lorentzian Frame Motions in Minkowski Space 38
On the Prolongation of Almost Hsu-Golden Structures to the Second-Order Tangent Bundle 39
Ricci Pseudosymmetry Conditions on Trans Para-Sasakian Manifolds and Ricci Solitons 41
Semi-Orthogonal Matrix Representations of Quaternions in Dual Spaces 43
Spiral Curves and Their Counterparts in Euclidean Three Space 44

Topology 45

Crossed Modules over 2-Groups 46
r-Near Topological Spaces Generated by Means of Minimal Open Sets 47

Applied Mathematics 49

A New Approach to Hermite-Hadamard Inequalities Using Multiplicative Fractional Integrals 50
A Seasonal Jump-Diffusion Model for Extreme Rainfall in the Eastern Black Sea Region: Numerical Schemes and Exceedance Risk 51
A Unified Composite-Weight Framework for Generalized Convexity 53
An Encryption–Decryption Algorithm Using the Fractional Laplace Transform for Composite Special Functions 54
Asymptotic Behavior and Oscillation of Third-Order Semi-Canonical Neutral Differential Equations 56
Certified Fuzzy Output Strips for Information-Diffusion Models 57
Complex Dynamics and Bifurcation Analysis of a Two-Species Model with Harvesting and Allee Effects 58
Conformable Fractional Integrals with Respect to a Function and Chebyshev-Type Inequalities 60
Continuous Dependence on the Viscosity Parameter for the Velocity-Vorticity Model of the g-Navier-Stokes Equations 61
Curvature-Dependent Boundary Conditions at Fluid–Solid Interfaces: An Analysis of Velocity Slip and Heat Transfer 62
Effect of Immigration on the Stability of Leslie-Type Prey-Predator Systems 64
Effects of Surface Charge on DNA Sensing in Biological Nanopores 66
Estimates of the First Eigenvalue of the Schrödinger Equation with a Singularity at Zero 68
Hybrid Orbit Design in Earth-Moon Missions: Patched Conics and CR3BP Manifolds Combined with Differential Correctors 69
If Nature Remembers the Past, Why Should Our Mathematical Models Be Forgetful? 70
Lyapunov-Based Global Stability Analysis of the Endemic Equilibrium in a Reduced SIRD Model for Crimean–Congo Hemorrhagic Fever 71
Mateverse.net: Interactive Educational Web Page Containing Calculus Topics Using MathJax 73

Neural Network-Based Numerical Analysis of Fractional-Order Glucose-Insulin Dynamics 74
On Rich Dynamical Behaviours of a Commensal-Type Symbiosis Model Incorporating Memory Effects 76
On the Numerical Solution of the Diabetes Model with Conformable Derivative 78
Oscillation of Second-Order Damped Differential Equations With a Mixed Neutral Term 79
Proofs of Certain Irrationalities Using Differential Equations 80
Solving Fully FIVP by Using Granular Differentiability Concept 81
Solving the 0-1 Knapsack Problem Using Binary Dhole Optimization Algorithm 82
Stability Analysis of a Discrete-Time Predator–Prey Population Model under the Effects of Immigration and Harvesting 83
Thermal Analysis of a Moving Longitudinally Porous Fin Immersed in a Nanofluid and Exposed to a Magnetic Field: The Pseudo-Spectral Chebyshev Method 85

Discrete Mathematics and Combinatorics 86
Basic Properties of Central Günaltılı Graphs 87
Specific Classes of Directed Graph Groups and Their Structural Properties 88

Probability, Statistics and Data Science 89
Mateverse: A Comprehensive, Visualization-Driven Interactive Ecosystem for Calculus Education 90

Mathematics Education 91
Reflections on Preservice Mathematics Teachers' Interactions with Artificial Intelligence in Problem-Solving 92
Digital Game Design and Implementation for Four-Operation Achievement of Students at Risk of Dyscalculia 94

Computer Sciences 96
An Explainable Deep Learning Approach Based on EfficientNetB3 and Grad-CAM for Enamel Caries Classification in Intraoral Images 97
Disentangling Persistent Excitation and Near-Critical Amplification in Cyber-Threat Indicator Streams: A Mittag-Leffler Hawkes Framework 99
Inverse Analysis of IoT Malware Spread under Partial Observation: A Physics-Informed Neural Network Approach 101
Improving Feature Representation in Underwater Acoustic Target Recognition via Nonlinear PSD Quantization and Custom Colormap Encoding 103
Route and Resource Optimization in UAV-RIS Assisted 6G Wireless Networks: A Comparative Study on Population-Based Metaheuristic Algorithms 105

ABSTRACTS OF ONLINE PRESENTATIONS 107

Analysis and Theory of Functions 108

Computer Sciences 109
Preserving the Completeness of Blockchain-Anchored Digital Evidence under Ledger Failures 110

LIST OF PARTICIPANTS OF ICMME-2026.....112

PARTNERS AND SPONSORS.....115

***Partners*.....115**

***Sponsors*.....115**

INVITED SPEAKERS

Evolution of Internal Variables in the Hybrid Heat Equation for Anisotropic Viscoelastic Media

Prof. Dr. Vincenzo Ciancio
Messina University, Italy

ABSTRACT

The description of viscoelastic media with memory requires a thermodynamic formalism capable of capturing irreversible processes. In this work, a physicalmathematical model for anisotropic viscoelastic materials is developed based on the V. Ciancio—G.A. Kluitenberg theory. The generalized evolution equation is derived, showing its connection with modified Euler-Lagrange equations and the Rayleigh dissipation function. A direct and explicit reduction to the classical Maxwell model for viscoelasticity is demonstrated in the appropriate limit, confirming the consistency of the proposed framework. This work aims to offer a unified theoretical framework, providing a solid foundation for future developments.

A Delayed Generalized Lotka–Volterra Model for Threshold Instability and Structural Recognition

Prof. Dr. Carlo Cattani
University of Tuscia, Italy

ABSTRACT

This paper develops a delayed generalized Lotka{Volterra model for systems in which structural change and recognition of change are not synchronized. The starting point is the idea that a system may accumulate internal transformations while remaining locally stable at the level of interpretation, governance, or perception. Instability becomes effective only when accumulated structural stress crosses a threshold and is recognized after a delay. We introduce a vector of civilizational state variables including systemic complexity, informational entropy, systemic pressure, resilience, and legitimacy. Their interaction is modeled through a delayed generalized Lotka{Volterra system. The delays represent the temporal separation between structural variation and its cognitive, institutional, or social effects. A composite instability functional is then defined, and threshold crossing is interpreted as the transition from latent instability to recognized instability. The paper studies positivity, equilibria, linearization, and local stability of the delayed system. Particular attention is given to delay-induced instability: even when the corresponding non-delayed system is locally stable, sufficiently large delays may destabilize the equilibrium and generate oscillatory instability. The model provides a mathematical framework for delayed recognition, threshold instability, and the emergence of crises in complex adaptive systems. Spectral destabilization and threshold crossing are treated as distinct phenomena: the former is determined locally by characteristic roots, whereas the latter depends on the nonlinear trajectory and on the prescribed instability functional.

Keywords: delayed differential equations; generalized Lotka{Volterra systems; instability thresholds; delayed recognition; civilizational dynamics; resilience; complexity; legitimacy.

MSC 2020: 34K20; 34K60; 37N25; 37C10; 91D10; 91D30

Geometry of Umbrella Matrices

Prof. Dr. Yusuf Yaylı
Ankara University, Türkiye

ABSTRACT

Umbrella matrices are important structures that simultaneously exhibit both transformational and stochastic properties through the row-sum-one condition. This feature preserves the centroid of space, enabling balanced transformations and opening a broad range of applications—from multidimensional geometry and kinematic modeling to inconsistency analysis in decision theory. The presentation reviews the existing research on umbrella matrices and outlines potential directions for future investigations.

On Some Stability Results of Power-Law Fractal-Fractional Systems

Prof. Dr. Zakia Hammouch
ENS Moulay Ismaïl University, Morocco

ABSTRACT

Fractal-fractional operators combine nonlocal memory with nonclassical temporal scaling, but their initial-value formulation depends critically on the adopted integral convention. We study the Caputo-type power-law fractal-fractional derivative and identify a structural admissibility condition for prescribed initial data. From the defining weighted derivative, we derive the corresponding positive Volterra formulation and show that its short-time behaviour is controlled by the exponent . When , a continuous solution cannot generally evolve from an arbitrary non-equilibrium initial state, so an additional compatibility condition is required. For the scalar linear problem, we obtain a multiplicative representation whose symbol belongs to the Bernstein-function framework, yielding an entire generalized relaxation function. This function decays for negative real spectral parameters and grows for positive ones. The analysis extends to finite-dimensional systems, including defective matrices. A model comparison further shows that competing integral conventions, although formally similar, produce different short-time dynamics. The results distinguish initial-value admissibility from spectral stability and show that the integral representation must be verified before applying standard fractional stability criteria. The stability region for general complex spectral parameters remains open.

Shannon-Whittaker-Kotel'nikov's Theorem Generalized

Prof. Dr. Juan Luis García Guirao
Technical University of Cartagena Department of Applied Mathematics and Statistic, Spain

ABSTRACT

The Shannon–Whittaker–Kotel'nikov Sampling Theorem is a fundamental result in signal theory, providing the reconstruction of band-limited signals from uniformly spaced samples. In this talk, we briefly review the historical development of the Sampling Theorem and some of the mathematical results that led to its classical formulation.

We then present a generalized Shannon-type asymptotic recomposition formula for signals that are not necessarily band-limited. We show that Gaussian-type signals satisfy this property and provide two alternative approaches to its proof, based on complex analysis and transformation theory. Finally, we discuss further classes of functions satisfying the proposed recomposition property and the effect of suitable perturbations, extending the classical Shannon framework to a broader family of signals.

Keywords: Sampling Theorem; Shannon–Whittaker–Kotel'nikov Theorem; Asymptotic Recomposition; Cardinal Series.

Recent Progresses on Real Analysis and Applications

Prof. Dr. Maria Alessandra Ragusa
University of Catania, Italy

ABSTRACT

Aim of this talk is to study some progresses of Real Analysis having applications to solutions of partial differential equations and systems. Preparatory to the study of partial differential equations is the action of some integral operators, that we will use extensively. Then, will be depth useful to obtain regularity properties of solutions of elliptic and parabolic equations of second order with discontinuous coefficients, and later of systems. partial differential equations having discontinuous coefficients.

Success in Education and Why is Mathematics?

Prof. Dr. Ömer AKIN
TOBB ETÜ, Türkiye

ABSTRACT

This talk provides a comprehensive synthesis of the educational journey, emphasizing the collaborative roles of students, teachers, and parents (The Golden Wheels). Its purpose is to bridge operational success—through the Academic Action Matrix—with the philosophical understanding of mathematics as the cosmic language of the universe. By addressing modern challenges like technology addiction and defining success through the lens of character and patience (The Human Value Equation), the work outlines a path from individual discipline to societal development. It concludes that true education is not the filling of a vessel, but the cultivation of a mindset capable of deciphering the divine order of nature.

Tracing Mathematical Meaning: A Journey to the Integrated Dynamic Mathematical Meaning Model (BDSM)

Prof. Dr. Sare ŞENGÜL
Marmara University, Atatürk Faculty of Education, Department of Mathematics and Science Education

ABSTRACT

Traditional mathematics teaching and assessment approaches primarily emphasize "result-oriented" thinking; a student's attainment of the correct answer is assumed to be proof of conceptual understanding. However, simply memorizing steps to arrive at the correct result does not reveal the entirety of the mental thought processes and the established meaning connections. This situation highlights the need to shift the object of assessment in mathematics education from "result" to the "thought traces" left in the student's mind. The aim of this study is to introduce the Integrated Dynamic Mathematical Meaning Model (IDMMM), which explains how students connect mathematical structures, coordinate different representations, and transform meaning, in its theoretical and applied dimensions. Applications of the model will be presented at the end of the presentation.

Keywords: Mathematical Meaning-Making, IDMMM, Cognitive Processes, Number-Symbol-Graphic Sense, Dynamic Mathematics Software, Representation Coordination.

ABSTRACTS OF ORAL PRESENTATIONS

Algebra and Number Theory

Generalized Riordan Structures and Sequence Characterizations

Fatma YEŞİL BARAN[1]
1 Amasya University, Department of Computer Engineering, Amasya-Türkiye.
fatma.yesilbaran@amasya.edu.tr

ABSTRACT

The classical Riordan group has established itself as a profoundly powerful algebraic and combinatorial framework within enumerative combinatorics and matrix theory. In its standard definition, a Riordan matrix is uniquely characterized by a pair of analytic generating functions, $g(x)$ and $f(x)$, where the j-th column of the resulting infinite lower-triangular matrix corresponds precisely to the coefficients of the function $g(x)[f(x)]^j$. This elegant structural property provides researchers with robust algebraic tools for solving complex sum equations, inverting infinite sequences, evaluating combinatorial sums, and analyzing the structural properties of special numerical arrays.

In this study, we introduce a novel and significant extension to the traditional Riordan group framework by systematically increasing the number of defining generating functions. By transitioning from a dual-function pair to a multi-function configuration, we establish a generalized Riordan-type matrix structure that captures more intricate combinatorial behaviors. Under this newly developed extended framework, we comprehensively investigate the corresponding row-production sequences characterization, which is universally known to play a crucial role in defining the structural laws, recurrence relations, and construction mechanisms of Riordan-like matrices.

Furthermore, we employ this innovative algebraic approach to construct new matrix representations for several prominent integer sequences that frequently appear in pure and applied mathematics. By exploiting the underlying algebraic and group-theoretic properties of these newly defined multi-function matrices, we derive a series of novel combinatorial identities and algebraic equations that were previously inaccessible through classical methods.

Keywords: Riordan group, Generating functions, Integer sequences.

Acknowledgement: This work was supported by Amasya University Scientific Research Projects (BAP) under project number FMB-BAP 26-0685.

REFERENCES

[1] Shapiro L.W., Getu S., Woan W. J., and Woodson L., The Riordan group, Discrete Appl. Math., 34: 229-239, 1991.

[2] Luzon A., Merlini D., Moron M.A., and Sprugnoli R., Identities induced by Riordan arrays, Linear Algebra and its Applications, 436: 631-647, 2012.

[3] Merlini D., and Verri M.C., Generating trees and proper Riordan arrays, Discrete Mathematics, 218: 167-183, 2000.

On Homomorphisms of Nearness Γ-Near Rings

Mustafa Uçkun [1], Nida Nur Genes [2]
[1] Adıyaman University, Faculty of Arts and Sciences
Department of Mathematics, Adıyaman, Türkiye, muckun@adiyaman.edu.tr
[2] Adıyaman University, Graduate Education Institute
Department of Mathematics, Adıyaman, Türkiye, genes.nidanur44@gmail.com

ABSTRACT

Near set theory is an important mathematical approach developed for the observation, comparison, and classification of objects. This theory provides a framework for identifying similarities among objects and for the mathematical investigation of nearness relations. Based on near set theory, many algebraic concepts have been defined in nearness approximation spaces, and significant results have been obtained within this framework. In this manuscript, various studies on rings, near-rings, Γ-rings, and their generalizations in nearness approximation spaces have been examined.

One of the algebraic concepts introduced into the literature as a result of these studies is the concept of a nearness Γ-near ring. In this paper, the concept of a homomorphism on nearness Γ-near rings is introduced based on the previously defined concept of nearness Γ-near rings. Within the framework of this definition, the fundamental algebraic properties of homomorphisms are investigated, and several results are established. The obtained results are intended to contribute to the theoretical evaluation of homomorphisms on nearness Γ-near rings.

Furthermore, this manuscript aims to contribute to the existing literature on nearness Γ-near rings, support the further development of research in this area, and provide guidance for future studies on other algebraic concepts defined in nearness approximation spaces. In this respect, the present study is expected to serve as a useful reference for future research on nearness Γ-near rings.

Keywords: Nearness approximation space, nearness ring, nearness Γ-near ring, nearness Γ-near homomorphism.

REFERENCES

[1] G. Pilz, Near-Rings: The Theory and Its Applications, North-Holland Publishing Company, Amsterdam, 1983.
[2] N. Nobusawa, On a generalization of the ring theory, Osaka J. Math. 1 (1964), 81-89.
[3] W.E. Barnes, On the Γ-rings of Nobusawa, Pacific J. Math. 18 (1966), 411-422.
[4] Bh. Satyanarayana, A note on Γ-near-rings, Indian J. Math. 41 (1999), 427-433.
[5] J.F. Peters, Near sets: General theory about nearness of objects, Appl. Math. Sci. 1 (2007), 2609-2629.
[6] M.A. Öztürk, M. Uçkun and E. İnan, Near group of weak cosets on nearness approximation spaces, Fund. Inform. 133 (2014), 433-448.
[7] M.A. Öztürk and E. İnan, Nearness rings, Ann. Fuzzy Math. Inform. 17 (2019), 115-131.
[8] B. Çokakoğlu and M. Uçkun, Nearness gamma-near rings, Fund. Contemp. Math. Sci. 3 (2022), 46-56.

On the Generalized Tangent-Fibonacci Polynomials and Related Identities

Semra Kuş
Kırşehir Ahi Evran University, Mucur Vocational High School, Kırşehir, Türkiye, skus@ahievran.edu.tr

ABSTRACT

In recent years, many new families of special polynomials have been introduced within the framework of Fibonomial calculus. Motivated by these developments, in this paper we define the generalized Tangent-Fibonacci numbers and polynomials and investigate their fundamental properties. Several summation formulas and algebraic identities associated with these polynomials are derived. Furthermore, their differential and integral properties are established. We also examine the relationships between the Tangent-Fibonacci polynomials and various families of Fibonomial-based polynomials, obtaining explicit connections and identities among them. The results presented in this study enrich the theory of Fibonomial calculus and provide a new class of special polynomials that may be useful in further theoretical investigations and related applications.

Keywords: Tangent polynomials, Fibonomial calculus, Generating functions

REFERENCES

[1] E. Krot, An Introduction to Finite Fibonomial Calculus, Cent. Eur. J.Math. 2(2004), no. 5, 754-766.

[2] H. Y. Lee, Y. R. Kim, A Study on Tangent-Fibonacchi Polynomials and Numbers, J. Appl. Math. & Informatics Vol. 43(2025), No. 3, pp. 781-794

[3] S. Kuş, E. Erkus-Duman, N. Tuglu, The Appell-Fibonacci Polynomials, Approximation Theory and Special Functions. ATSF 2024. Springer Proceedings in Mathematics & Statistics, vol 503. (2026) Springer, Cham.

[4] S. Kuş, N. Tuglu, and T. Kim, Bernoulli F-polynomials and Fibo–Bernoulli matrices. Adv. Differ. Equ. 2019, 145 (2019).

[5] C. S. Ryoo, A Note on the Tangent Numbers and Polynomials, Adv. Studies Theor. Phys., Vol. 7, 2013, no. 9, 447 – 454.

[6] E. Gulal, N. Tuglu, Apostol-Bernoulli-Fibonacci polynomials, Apostol-Euler-Fibonacci polynomials and their generating functions. Turk. J. Math. Comput. Sci. (2023); 15: 202-210.

Padovan-Sum Graph

Ece Güneş Güngör [1], Elif Özcan [2], Şerife Büyükköse [3]
1 Gazi University Graduate School of Natural and Applied Sciences, Department of Mathematics / 26830501015@gazi.edu.tr
2 Gazi University Graduate School of Natural and Applied Sciences, Department of Mathematics / 26830501014@gazi.edu.tr
3 Gazi University Faculty of Science, Department of Mathematics / sbuyukkose@gazi.edu.tr

ABSTRACT

A graph is an ordered pair (V, E) consisting of a finite, non-empty set

$V = \{1,2, \dots, n\}$ whose elements are called vertices, and a finite set E whose elements are called edges; it is denoted by $G = (V, E)$ or simply by G .Here, E is defined as $E = \{ij: i, j \in V\}$. A graph can always be drawn by means of its vertices and the edges connecting them. This drawing is not unique.

In any graph G, for vertices $i, j \in V$, if ij is an edge, the vertices i and j are said to be adjacent and this is denoted by $i \sim j$. In graph theory, the number of edges incident to the i -th vertex is called the degree of the i -th vertex and is denoted by $d(i)$. For a simple graph G with n vertices, the adjacency matrix of G is given by $A(G) = (a_{ij})$, where the entries of this matrix are defined as follows:

$$a_{ij} = \begin{cases} 1 & ; \quad if\ i \sim j \\ 0 & ; \quad otherwise \end{cases}$$

The Padovan sequence $\{P_n\}$ is an integer sequence defined by the initial values $P_0 = 1$, $P_1 = 1$, $P_2 = 1$ and the recurrence relation:

$$P_{n+3} = P_{n+1} + P_n \quad , \text{for } n \geq 0$$

The first few terms of the Padovan sequence are:

1,1,1,2,2,3,4,5,7,9,12,16,21,28,…

For any $n \geq 1$, the Padovan-sum graph $G_n = (V, E)$ is a simple graph defined on the vertex set $V = \{1,2, \dots, n\}$ with the edge set:

$$E = \{ij: i, j \in V, i \neq j,\ and\ i + j\ is\ a\ Padovan\ number\}$$

In this study, the properties of Padovan-sum graphs, such as vertex degree and adjacency, will be investigated in connection with Padovan numbers.

Keywords: Graph, Padovan Numbers, Weighted Graph, Padovan-Sum Graph

REFERENCES

[1] D. Taşcı, Ş. Büyükköse, G. Ö. Kızılırmak, E. Sevgi, The Bounds For The Largest Eigenvalues of Fibonacci-Sum and Lucas-Sum Graphs, TWMS J. Appl. Engi. Math. V.12, N.1, 2022, pp. 367-371

[2] Gross, JL, Yellen, J., & Anderson, M. (2018). Graph Theory and Applications, Chapman and Hall/CRC.

[3] Arman, A., Gunderson, D. S., Li, P. C., (2017), Properties of the Fibonacci-sum graph, arXiv:1710.10303v1[math.CO] 27 Oct 2017 https://arxiv.org/abs/1710.10303.

[4] Michel Waldschmidt, Linear recurrences:an introduction, ICSFA-2018, November 22-24, 2018

q-Leonardo Chebyshev Polynomials

Naim TUGLU [1], Gizem ŞİMŞEK [2]
1 Gazi University Department of Mathematics Teknikokullar 06500 Ankara Türkiye
naimtuglu@gazi.edu.tr
2 Gazi University Graduate School of Natural and Applied Sciences, 06500 Teknikokullar, Ankara, Türkiye
gizemberksan24@gmail.com

ABSTRACT

Chebyshev polynomial of the first kind, defined by for $n \geq 2$,

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

with the initial conditions $T_0(x) = 1$, $T_1(x) = x$. Chebyshev polynomials of the second kind defined by same recurrence relation

$$U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x)$$

but different initial conditions $U_0(x) = 1$ and $U_1(x) = 2x$. The Chebyshev polynomials are studied in a variety of ways by different researchers see [2-5, 7,8].

Leonardo Pisano numbers are defined by recurrence relation

$Le_n = Le_{n-1} + Le_{n-2} + 1 \qquad n \geq 2$

with the initial conditions $Le_0 = Le_1 = 1$ in [1]. There is a relation between the Leonardo numbers and the Fibonacci numbers for $n \geq 0$ such that $Le_n = 2F_{n+1} - 1$.

In this presentation, we introduced the q-analogues of Leonardo Chebyshev polynomials called the q-Leonardo Chebyshev polynomials of first and second kind. We obtained explicit formulas of these polynomials, generating functions, some summations properties, determinantal representations of these polynomials. We show that there are connections between the q-Leonardo Chebyshev polynomials and q-Chebyshev polynomials

Keywords: Chebyshev polynomials, Leonardo Chebyshev polynomials, q-Calculus.

REFERENCES

[1] Y. Alp and E.G. Koçer, Some properties of Leonardo numbers, Konuralp J. Math. 9 (2021), 183-189.

[2] M. Atakihiyeva and N. Atakishyev, On discrete q –extensions of Chebyshev polynomials, Commun. Math. Anal. 14 (2013), 1-12.

[3] A. T. Benjamin and D. Walton, Counting on Chebyshev polynomials. Math. Mag. 82 (2009), 117-126.

[4] E. Ercan, M. Çetin Firengiz, and N. Tuglu, Some results on q –analogue of the Chebyshev polynomials, Filomat, 10(1) (2015), 3599-3607

[5] J. C. Mason D. C. Handscomb, Chebyshev Polynomials, Chapman & Hall/CRC Press, Boca Raton, FL, 2003.

[6] N. Tuglu, S. S. Ekinci and M.Çetin Keskin, q –Leonardo polynomials. Notes on Number Theory Discrete Math. 32(2) (2026), 281–299.

[7] T. J. Rivlin, Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory; John Wiley and Sons: New York, NY, USA, 1990.

[8] F. Taşdemir and V.S. Durusoy, Gaussian Chebyshev polynomials and their properties. Symmetry 17 (2025), 1040.

Analysis and Theory of Functions

A Grand Variable Exponent Morrey Space Framework for Sobolev-Type Embeddings, Compactness, and Hölder Regularity in Anisotropic Fractional Variational Problems

Merve Esra TÜRKAY[1]
1 Sivas Cumhuriyet University, Sivas, Türkiye, mesra@cumhuriyet.edu.tr

ABSTRACT

This work introduces a unified analytical framework, based on grand variable exponent Morrey (GVEMS) spaces, for anisotropic nonlocal fractional variational problems with variable-exponent growth on bounded Lipschitz domains. A nonlocal energy space is defined through a Gagliardo-type double integral governed by a variable exponent and a fractional order, and it is shown, by means of a dyadic annulus decomposition combined with a fractional maximal-function argument, that every function in this energy space admits an admissible Hajłasz-type gradient controlled in the grand variable exponent Morrey norm. This energy-to-Hajłasz reduction is obtained as a proved result rather than assumed as a hypothesis, and it identifies precisely which structural properties of the nonlocal kernel guarantee its validity. Building on this reduction, together with known boundedness properties of the Hardy–Littlewood maximal operator, Calderón–Zygmund singular integrals and Riesz-type potentials on grand variable exponent Morrey spaces, and Morrey–Campanato identifications in the variable exponent setting, four main results are established: a continuous embedding into a variable exponent Hölder space, obtained through an explicit chaining estimate along dyadic radii; a compact embedding into subcritical variable exponent Lebesgue spaces, via the Arzelà–Ascoli theorem; a Poincaré-type oscillation estimate on balls, derived directly from the Hajłasz pointwise inequality; and a Campanato-type Hölder regularity corollary. A pointwise Hölder-type estimate for Riesz potentials is also proved, together with a discussion, via Green-function comparability results for anisotropic nonlocal operators, of when the potential-representation hypothesis needed for a sharper refinement of the embedding actually holds. As a variational application, it is shown that for anisotropic fractional energy functionals that are coercive on the energy space, with subcritical lower-order perturbations of variable-exponent growth, the compact embedding yields compactness of the lower-order term on bounded energy sets, which is precisely the structural ingredient required by the direct method and by critical-point theory. The failure of compactness at the critical exponent threshold, and the additional structure needed to recover it there, is also addressed. Altogether, the results extend the classical fractional Sobolev, Morrey and Campanato theories to a genuinely anisotropic, variable-exponent, grand-space setting relevant to nonlocal variational problems with nonstandard growth arising in the calculus of variations and nonlinear analysis.

Keywords: Grand variable exponent Morrey spaces, Hajłasz–Sobolev spaces, fractional Sobolev embedding, Riesz potentials, anisotropic nonlocal variational problems.

REFERENCES

[1] P. Hajłasz, Sobolev spaces on an arbitrary metric space, Potential Anal. 5 (1996), 403–415.

[2] L. Diening, P. Harjulehto, P. Hästö, M. Růžička, Lebesgue and Sobolev Spaces with Variable Exponents, Lecture Notes in Mathematics, Vol. 2017, Springer-Verlag, Heidelberg, 2011.

[3] T. Iwaniec and C. Sbordone, On the integrability of the Jacobian under minimal hypotheses, Arch. Ration. Mech. Anal. 119 (1992), 129–143.

[4] D.R. Adams, A note on Riesz potentials, Duke Math. J. 42 (1975), 765–778.

[5] T. Ohno and T. Shimomura, Sobolev embeddings for Riesz potentials of functions in grand Morrey spaces of variable exponents over non-doubling measure spaces, Czechoslovak Math. J. 64(139) (2014), 209–228.

[6] N. C.EDDINE, M. A.RAGUSA,, The Compact Embeddings and the Concentration-Compactness Principles for Anisotropic Variable Exponent Sobolev Spaces and , *The Journal of Geometric Analysis* 35(11) (2025).

A Review on the Spectral Analysis of Weighted Mean Matrices

Zeynep ADIGÜZEL [1,*], **Nuh DURNA** [1,**]
1 Sivas Cumhuriyet University, Faculty of Science, Department of Mathematics, 58140, Sivas,
* zeynepadiguzel@cumhuriyet.edu.tr
** ndurna@cumhuriyet.edu.tr

ABSTRACT

Summability theory is an important branch of mathematics that provides meaningful limits for sequences and series that are not convergent in the classical sense. It has found wide applications in various fields, including Fourier series, functional analysis, and operator theory, by extending the classical notion of convergence. The summability method introduced by Cesàro is one of the most fundamental regular summability methods and has served as the basis for many generalized matrix transformations [1].

The matrix representation of summability methods has made it possible to investigate the action of infinite matrices on sequence spaces. In this context, weighted mean matrices emerged as a natural generalization of Cesàro matrices. By selecting appropriate weight sequences, various averaging methods can be obtained, allowing the classical Cesàro method to be studied within a broader framework. The theory of regular matrix transformations developed by Silverman, Toeplitz, and Hardy constitutes one of the major milestones in establishing the mathematical foundations of weighted mean matrices [1,2].

In operator theory, the spectrum of a matrix is a fundamental concept that characterizes the algebraic and analytical properties of the corresponding operator. Determining the spectrum provides significant information about the invertibility, stability, and overall behavior of the operator. Therefore, investigating the spectral properties of weighted mean matrices is of considerable importance in both theoretical and applied mathematics [5].

This paper presents a review of the spectral analysis of weighted mean matrices by summarizing the main results available in the literature. First, the definition and basic properties of weighted mean matrices are introduced, followed by an overview of the concepts of the spectrum, point spectrum, continuous spectrum, and residual spectrum. The spectral behavior of these matrices on the classical sequence spaces c_0, c, and ℓ_p is then discussed. Furthermore, the principal analytical techniques employed in spectral investigations and the major results reported in the literature are comparatively evaluated [3,4].

In conclusion, weighted mean matrices occupy a significant position in both the development of summability theory and the analysis of infinite-dimensional operators. Determining their spectra contributes not only to the theoretical understanding of operator theory but also to solving various problems arising in functional analysis and related fields. Future studies involving different weight sequences and new sequence spaces are expected to provide further valuable contributions to the literature [2,5,6].

Keywords: Weighted mean matrix, spectrum, fine spectrum.

REFERENCES

[1] Hardy, G. H. (1949). Divergent Series. Oxford University Press.

[2] Wilansky, A. (1984). Summability through Functional Analysis. North-Holland.

[3] Altay, B., & Başar, F. (2007).On the fine spectrum of the generalized difference operator over the sequence spaces c0 and c. International Journal of Mathematics and Mathematical Sciences.

[4] Mursaleen, M., & Noman, A. K. (2010). On generalized weighted mean difference sequence spaces. Computers & Mathematics with Applications, 59, 2726–2733.

[5] Goldberg, S. (1966). Unbounded Linear Operators. McGraw-Hill.

[6] Rhoades, B. E. (2003). Summability methods and their applications. Rocky Mountain Journal of Mathematics, 33(2), 497–528.

Exponential Type Baskakov Operators via Summability Methods

Seyma Çelik [1], Dilek Söylemez Özden [2]
1 Selcuk University, Faculty of Science, Department of Mathematics, Konya, Türkiye
e-mail: seymacelik216@gmail.com
2 Selcuk University, Faculty of Science, Department of Mathematics, Konya, Türkiye
e-mail: dsozden@gmail.com

ABSTRACT

In this presentation, we investigate approximation properties of exponential Baskakov operators via power series statistical convergence. We also study Baskakov and King type Baskakov operators in weighted space. Furthermore, we obtain rates of convergence in weighted function spaces, and compute quantitative estimates by means of the weighted modulus of continuity.

Yılmaz et al. **[6]** introduced modified Baskakov operators preserving $e^{2\mu x}$, for any $\mu \geq 0, n \in \mathbb{N}$ $x \in [0,\infty)$.

$$K_{n,\beta}(f;x) = \sum_{k=0}^{\infty} f\left(\frac{k}{n}\right)\binom{n+k-1}{k}\frac{\left(\beta_n(x)\right)^k}{\left(1+\beta_n(x)\right)^{n+k}}, \quad n \in \mathbb{N}$$

where $\beta_n(x) = \frac{e^{-\frac{2ax}{n}}-1}{1-e^{\frac{2a}{n}}}$. Moreover, the following equalities hold

$$K_{n,\beta}(1,x) = 1,$$

$$K_{n,\beta}(e^{-t},x) = \left(\frac{e^{-\frac{2ax}{n}}-1}{1-e^{\frac{2a}{n}}}\left(1-e^{-\frac{1}{n}}\right)+1\right)^{-n},$$

$$K_{n,\beta}(e^{-2t},x) = \left(\frac{e^{-\frac{2ax}{n}}-1}{1-e^{\frac{2a}{n}}}\left(1-e^{-\frac{2}{n}}\right)+1\right)^{-n}.$$

We recall the necessary background on power series statistical convergence, as these concepts will form the core of our results. Ünver and Orhan (**[5]**) introduced the notion of P-statistical convergence, where P is a regular power series method explained in (**[2], [5]**). Let $E \subset \mathbb{N}_0$. If

$$\delta_P\ (E) \coloneqq \lim_{t\to R^-} \frac{1}{p(t)}\sum_{n\in E}^{\infty} p_n t^n$$

exists, then $\delta_P(E)$ is called the power series density of E.

Let $x = (x_n)$ be a real sequence. Then x is said to be P-statistically convergent to L if, for every $\varepsilon > 0$,

$$\lim_{t\to R^-}\frac{1}{p(t)}\sum_{|x_n-L|\geq\varepsilon}p_n t^n=0,$$

that is,

$$\delta_P(\{n\in\mathbb{N}_0:|\,x_n-L\,|\geq\varepsilon\})=0,$$

for every $\varepsilon>0$. In this case we write

$$st_P-\lim x=L.$$

Now, we recall the weighted spaces B_ρ and C_ρ as

$$B_\rho=\{f:\mathbb{R}\to\mathbb{R}:|\,f(x)\,|\leq M_f\rho(x)\text{ on }\mathbb{R}\text{ for some }M_f>0\},$$

and

$$C_\rho=\{f:\mathbb{R}\to\mathbb{R}:f\in C_\rho\text{ and }f\text{ is continuous on }\mathbb{R}\},$$

Here ρ is a weight function satisfy standard assumption (see; **[3]**) and they are equipped with the norm

$$\|\,f\,\|_\rho=\sup_{x\in\mathbb{R}}\left\{\frac{|\,f(x)\,|}{\rho(x)}|\,\forall x\in\mathbb{R}\right\}.$$

Now let $\rho_1$and ρ_2 be two weight functions satisfying

$$\lim_{|x|\to\infty}\frac{\rho_1(x)}{\rho_2(x)}=0. \qquad (1)$$

Then $C_{\rho_1}\subset C_{\rho_2}$, $B_{\rho_1}\subset B_{\rho_2}$. Assume that

$$\rho(x)=1+(\varphi(x))^2,$$

and that $\rho_2$satisfies (1), where $\varphi(x)$ is a continuous increasing function on $\mathbb{R}$.

Keywords: Power series statistical convergence, weighted Korovkin-type theorem, rate of convergence.

REFERENCES

[1] Atlıhan ÖG, Yurdakadim T, Taş E. A new approach to the approximation by positive linear operators in weighted spaces. *Ukrainian Mathematical Journal*. 2023; 74:1649–1657.

[2] Boos *J.* Classical and Modern Methods in Summability. *Oxford: Oxford University Press; 2000.*

[3] Gadjiev AD. The convergence problem for a sequence of positive linear operators on unbounded sets and theorems analogous to that of P. P. Korovkin. *Doklady Akademii Nauk SSSR*. 1974; 218:1000–1003

[4] Söylemez D. Weighted Korovkin-Type Theorem and Some Properties of Szász--Mirakjan Operators Preserving Exponential Functions via Power Series Statistical Convergence. *Journal of Function Spaces*. 2026;2026(1)

[5] Ünver M, Orhan C. Statistical convergence with respect to power series methods and applications to approximation theory. Numerical Functional Analysis and Optimization. 2019;40(5):535–547.

[6] Yılmaz, Ö. G., Gupta, V., Aral, A., On Baskakov operators preserving the exponential function, Journal of Numerical Analysis and Approximation Theory, 46(2) (2017), 150--161.

Quantum Calderon-Zygmund Singular Integral Operators in the Quantum Lebesgue Spaces

Abdulhamit Küçükaslan [1]
1 Department of Mathematics, Ankara Yıldırım Beyazıt University, Ankara, Türkiye

ABSTRACT

The study of quantum calculus, or q-calculus, has evolved significantly since the foundational works of F.H. Jackson in the early 20th century, emerging as a vital framework that bridges discrete analysis and classical Lebesgue theory. This work investigates the structural and boundedness properties of integral operators within the setting of quantum Lebesgue spaces when $0 < q < 1$, and focus on the boundedness of quantum analogues of classical operators of harmonic analysis as quantum Calderon-Zygmund singular integral operator in the quantum Lebesgue spaces defined via the Jackson q-measure. By employing modern techniques from quantum harmonic analysis, we establish the weak-type (1, 1) boundedness and the strong-type (p, p) boundedness for $1 < p < \infty$ by using Marcinkiewicz interpolation theorems.

Furthermore, this work integrates historical perspectives with contemporary advancements, referencing academic developments up to now to demonstrate the ongoing relevance of q-integral inequalities in modern functional analysis. Our results extend classical operator theory to the q-environment, providing a rigorous basis for further applications in quantum dynamical systems and partial q-difference equations.

Keywords: q-integral, q-Lebesgue spaces, q-Calderon-Zygmund singular integral operator.

REFERENCES

[1] F. H. Jackson, On q-functions and a certain difference operator, Trans. Roy. Soc. Edinb. 46, 64–72 (1908).
[2] F. H. Jackson, On q-definite integrals, Quart. J. Pure Appl. Math. 41, 193–203 (1910).

The Spectrum of a Generalization of the Cesàro Matrix in Convergent Sequences Space

Rabia KILIÇ [1]
1 Niğde Ömer Halisdemir University, Faculty of Science, Department of Mathematics, 51240, Niğde, rabiakilic@ohu.edu.tr

ABSTRACT

Spectral analysis has become an active and continuously developing research area within functional analysis and applied mathematics. In particular, the spectral properties of various classes of matrix operators acting on different sequence spaces have attracted considerable attention in the literature. Among the classical summability methods, the Cesàro matrix occupies a central position, and a variety of its generalizations have subsequently been proposed and extensively investigated. For instance, Durna and Türkay [2] studied the spectrum of the q-Cesàro matrix, while Sinha *et al.* [3] investigated the spectral properties of the (p,q)-Cesàro matrix.

The convergent sequence space c consists of all convergent sequences and is equipped with the norm $\|x\|_c = \sup_n |x_n|$. The dual space of c is norm isomorphic to the sequence space ℓ_1, consisting of all absolutely summable sequences.

Let X and Y be Banach spaces, let $T \in B(X,Y)$. A complex number λ is called a regular value of T if the operator $(\lambda I - T)^{-1}$ exists such that is bounded and has a dense domain. The set of all such λ, denoted by $\rho(T,X)$ is called the resolvent set of T. Furthermore $\sigma(T,X) = \mathbb{C}\backslash\rho(T,X)$ set is spectrum. The elements of the spectrum are called spectral values. The spectrum has many sub-decompositions. However, three discrete subsets are commonly studied: continuous spectrum, residual spectrum, and point spectrum.

In this study, we investigated the spectrum of a generalized Cesàro matrix introduced by Şengönül and Başar [4] on the sequence space c.

Keywords: Generalized Cesàro matrix, spectrum, fine spectrum.

REFERENCES

[1] Goldberg, S., Unbounded Linear Operators, McGraw Hill, New York, 1966.

[2] N. Durna and M.E. Türkay, The spectrum of q- Cesáro matrices on c and its various spectral decomposition for $0 < q < 1$, Operators and Matrices 15(3) (2021), 795–813.

[3] P. Sinha, M. Sen, B. C. Tripathy, The Spectrum and Fine Spectrum of (p, q)-Cesáro Matrices over the sequence space $\ell_s (0 < s < \infty)$, Complex Analysis and Operator Theory, 20(4) (2026), 107.

[4] M. Şengönül and F. Başar, Some new Cesáro sequence spaces of non-absolute type which include, Soochow Journal of Mathematics, 31(1) (2005), 107-119.

What Is a Complex Partial Differential Equation?

Pelin Ayşe Gökgöz
Cyber Security Vocational School, Gebze Technical University, 41400 Gebze, Kocaeli, Turkey,
pelingokgoz@gtu.edu.tr

ABSTRACT

Complex partial differential equations are an important part of complex analysis. They bring together differential operators, integral representation formulas, and boundary value problems in planar domains. This talk will introduce complex partial differential equations, starting with the Cauchy-Riemann operator. Then it will move on to higher-order complex equations, such as polyanalytic and polyharmonic equations. The main goal is to explain the ideas of complex partial differential equations in a simple way.

Special attention will be given to differential operators and classical boundary value problems, such as the Schwarz, Dirichlet, Robin, and Riquier problems. Complex partial differential equations and these problems show how we can use methods and Green functions together to find solutions and understand when solutions are possible. We will also discuss the importance of planar domains, like the unit disk and ring domains, where we can use explicit formulas and integral representation formulas in a very useful way.

This talk aims to explain why complex partial differential equations are important by looking at the reasons behind them, their history, and some examples. This way, we can see why complex partial differential equations are significant and how they keep inspiring research in analysis.

This talk is dedicated to the memory of Professor Ahmet Okay Çelebi, whom we remember with gratitude and respect.

Keywords: Complex partial differential equations, boundary value problems, planar domains.

REFERENCES

[1] Ü. Aksoy, H. Begehr, A. Çelebi et al. Complex Partial Differential Equations. J Math Sci 287, 867-883 (2025).

[2] H.G.W. Begehr, Complex Analytic Methods for Partial Differential Equations: An introductory text, World Scientific, Singapore, 1994.

Geometry

Algebraic Structure and Hamilton Matrix Representations of Quaternions over the Gaussian Rational Field

Gülay KORU YÜCEKAYA[1]
[1]Gazi University, Gazi Education Faculty, Primary Mathematics Education Department, Teknikokullar, 06500, Ankara-Türkiye; gkoru@gazi.edu.tr

ABSTRACT

The foundations of quaternion theory were laid by Hamilton in 1843 with the introduction of the quaternion number system. Hamilton introduced an associative but noncommutative multiplication rule and thereby established a powerful algebraic framework for representing rotations in three-dimensional Euclidean space. In particular, real quaternions have become an important tool for describing and investigating rotational motions. With the development of quaternion theory, various quaternion systems defined over different number fields and algebraic structures have emerged and have been studied through algebraic, geometric, and matrix-based approaches.

Investigating the structural properties and matrix representations of quaternions and their generalizations over different fields is important both for algebraic theory and for potential applications in geometry, robotics, spatial kinematics, computer graphics, image processing, and signal processing.

In this study, quaternions defined over the Gaussian rational field $\mathbb{Q}(i)$ are considered algebraically. Their fundamental algebraic properties are investigated, and addition, scalar multiplication, and quaternion multiplication are defined together with the basic relations associated with these operations. In addition, Hamilton operators corresponding to these quaternions are constructed and their matrix representations are obtained. The relationships between quaternion multiplication and the corresponding Hamilton matrices are determined, allowing quaternionic operations to be expressed through matrix operations and linear transformations. Thus, the relationship between the structural properties of quaternion algebra over $\mathbb{Q}(i)$ and its Hamilton matrix representations is clarified, providing a systematic matrix-based representation.

Keywords: Gaussian rational field, quaternion algebra, Hamilton matrices.

REFERENCES

[1] W. R. Hamilton, Elements of Quaternions, Longmans, Green and Co., London, 1866.
[2] H. H. Hacısalihoğlu, Motion Geometry and Quaternions Theory, Gazi University Publications, Ankara, 1983.
[3] M. Özdemir, Quaternions and Geometry, Golden Point Publishing, İzmir, 2020.
[4] M. Bekar and Y. Yaylı, Involutions of Complexified Quaternions and Split Quaternions, Advances in Applied Clifford Algebras 23(2) (2013), 283–299.
[5] I. Stewart and D. O. Tall, Algebraic Number Theory, Chapman and Hall, 1979.

Axode Geometry of Persistent Lorentzian Frame Motions

Derya Kahveci [1] , Yusuf Yaylı [2]
1 Ankara University Faculty of Science, Department of Mathematics, 06100, Ankara, Turkey, dkahveci@ankara.edu.tr
2 Ankara University Faculty of Science, Department of Mathematics, 06100, Ankara, Turkey, yayli@science.ankara.edu.tr

ABSTRACT

The geometry of fixed and moving axodes provides one of the fundamental geometric tools for interpreting one-parameter rigid-body motions. In Lorentzian kinematics, the causal structure of Minkowski space enriches the geometry of frame motions and leads to distinctive features that differ significantly from their Euclidean counterparts. Consequently, the investigation of axodes offers valuable insight into the geometric nature of persistent Lorentzian motions.

This presentation focuses on the geometric aspects of axodes generated by persistent Lorentzian frame motions associated with regular curves in three-dimensional Minkowski space. Rather than emphasizing algebraic formulations, attention is devoted to the geometric interpretation of persistence through the interaction between moving frames and the corresponding fixed and moving axodes. The persistence condition is discussed together with its influence on the geometry of the associated kinematic surfaces, highlighting the role of constant-pitch motions in Lorentzian geometry.

Illustrative examples are presented to demonstrate the geometric behavior of the obtained constructions and to clarify the relationship between frame motions and their associated axodes. The presented results show that axode geometry provides a natural and effective framework for understanding the differential geometric structure of persistent Lorentzian motions and further strengthens the connection between geometric kinematics and Lorentzian differential geometry.

Keywords: Axodes, persistent motions, Minkowski space, Lorentzian kinematics, moving frames.

REFERENCES

[1] O. Bottema and B. Roth, Theoretical Kinematics, North-Holland Press, New York, 1979.

[2] D. Kahveci and Y. Yaylı, Geometric kinematics of persistent rigid motions in three-dimensional Minkowski space, Mech. Mach. Theory 167 (2022), 104535.

Explicit Free Resolutions of Tangent Cones in Affine 4-Space

Özgür İnce
Department of Mathematics, Faculty of Science, Sivas Cumhuriyet University, 58050 Sivas, Türkiye,
ozgince@cumhuriyet.edu.tr

ABSTRACT

This paper is devoted to the study of minimal free resolutions of tangent cones associated with complete intersection monomial curves in affine 4-space. By applying the gluing technique for numerical semigroups, we characterize specific families of these curves where the tangent cone retains the complete intersection property. Our analysis focuses particularly on curves defined by 4-generated numerical semigroups, constructed by gluing two semigroups generated by two elements. Furthermore, we present explicit computations of their minimal free resolutions under specific conditions.

In the singularity theory of monomial curves, the tangent cone plays a pivotal role, and its minimal free resolution yields essential insights into the local characteristics of the curve's singularity. While a complete intersection curve in affine 4-space is characterized by a Betti sequence of (1,3,3,1), its tangent cone does not necessarily inherit the complete intersection property. To address this challenge, we employ the gluing method—specifically nice gluing and certain non-nice gluing conditions—to isolate families of curves where the tangent cone remains a complete intersection, thereby preserving the (1,3,3,1) resolution. We explicitly derive the matrix representations for these resolutions and confirm the exactness of the corresponding complexes via the Buchsbaum-Eisenbud criterion. The computational aspects of this study were executed using the numericalsgps package in GAP, alongside Macaulay2 and Singular.

Keywords: Complete intersection, Free resolution, Gluing, Monomial curve, Tangent cone.

REFERENCES

[1] Arslan, F. (1999). Cohen-Macaulayness of tangent cones. Proceedings of the American Mathematical Society, 128, 2243–2251.

[2] Arslan, F., & İnce, Ö. (2025). Complete intersection monomial curves and free resolutions. Communications in Algebra, 1–10.

[3] Arslan, F., Mete, P., & Şahin, M. (2009). Gluing and Hilbert functions of monomial curves. Proceedings of the American Mathematical Society, 137(7), 2225–2232.

[4] Barucci, V., Fröberg, R., & Şahin, M. (2014). On free resolutions of some semigroup rings. Journal of Pure and Applied Algebra, 218, 1107–1116.

[5] Bresinsky, H. (1975). On prime ideals with generic zero. Proc. Amer. Math. Soc., 47(2) 329–332.

[6] Buchsbaum, D. A., & Eisenbud, D. (1973). What makes a complex exact? Journal of Algebra, 25, 259–268.

[7] Decker, W., Greuel, G.-M., Pfister, G., & Schönemann, H. (2022). SINGULAR 4-3-0 — A computer algebra system for polynomial computations [Computer software]. https://www.singular.uni-kl.de

[8] Delorme, C. (1976). Sous-monoïdes d'intersection complète de N. Annales Scientifiques de l'École Normale Supérieure, 9(1), 145–154.

[9] Gimenez, P., & Srinivasan, H. (2019). The structure of the minimal free resolution of semigroup rings obtained by gluing. Journal of Pure and Applied Algebra, 223, 1411–1426.

[10] Grayson, D. R., & Stillman, M. E. (n.d.). Macaulay2, a software system for research in algebraic geometry [Computer software]. http://www.math.uiuc.edu/Macaulay2/

[11] Herzog, J. (1970). Generators and relations of abelian semigroup rings. Manuscripta Mathematica, 3, 175–193.

[12] Herzog, J. (1975). When is a regular sequence super regular? Nagoya Mathematical Journal, 83, 183–195.

[13] Herzog, J., Rossi, M. E., & Valla, G. (1986). On the depth of the symmetric algebra. Transactions of the American Mathematical Society, 296, 577–606.

[14] İnce, Ö. (2023). Free Resolutions of Tangent Cones of Complete Intersection Monomial Curves [Doctoral dissertation, Middle East Technical University]. https://hdl.handle.net/11511/103041

[15] Mete, P. (2021). On the Betti numbers of the tangent cones for Gorenstein monomial curves. Turkish Journal of Mathematics, 45(6), 2603–2613.

[16] Mete, P., & Zengin, E. (2019). Minimal free resolutions of the tangent cones of Gorenstein monomial curves. Turkish Journal of Mathematics, 43, 2782–2793.

[17] Robbiano, L., & Valla, G. (1980). On the equations defining tangent cones. Mathematical Proceedings of the Cambridge Philosophical Society, 88(2), 281–297.

[18] Stamate, D. I. (2018). Betti numbers for numerical semigroup rings. In Multigraded Algebra and Applications (NSA 24, pp. 133–157). Springer International Publishing.

[19] Şahin, M., & Stella, L. G. (2019). Gluing semigroups and strongly indispensable free resolutions. International Journal of Algebra and Computation, 29(2), 263–278.

[20] Şahin, M., & Şahin, N. (2019). Betti numbers for certain Cohen-Macaulay tangent cones. Bulletin of the Australian Mathematical Society, 99(1), 68–77.

[21] Şahin, N. (2023). Free resolutions for the tangent cones of some homogeneous pseudo symmetric monomial curves. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 72(1), 129–136.

[22] The GAP Group. (2022). GAP – Groups, Algorithms, and Programming, Version 4.12.2 [Computer software]. https://www.gap-system.org

[23] Vu, T. (2014). Periodicity of Betti numbers of monomial curves. Journal of Algebra, 418, 66–90.

Figures Placed Among Geometric Patterns in Sivas Structures of the Medieval Anatolian Turkish-Islamic Architectural Period

Fatmagül Saklavcı
Doctor. Kayseri University, Fsaklavci@gmail.com, 05052529561, Doi: 0000-0001-9274-5111

ABSTRACT

In our study, an evaluation has been made of the figures incorporated into the geometric ornamentation of Sivas structures from the Anatolian Seljuk period. In Anatolia, which has become a multicultural and multinational, a bridge between Asia and Europe since the 11th century, decoration and building elements with many different origins took place in the Anatolian Seljuk architecture. The vegetal and geometric patterned strips, inscription and muqarnas strips and moldings, ornamental strips and siege arches, gülbezek and hovels designed especially for the portal frame, which are the most striking parts of the buildings, are the most important examples of Seljuk stonework. Although the number of geometric ornament strips surrounding the three sides of the portals in the buildings differ, they have the same symbolic meanings. Ornamental areas consisting of geometric shapes are usually designed with a grift or plain composition and mostly star forms. In some buildings, various figures are placed inside the ornamental elements such as stars and rosettes, which are among these ornaments. Among these structures, carved figures appear amidst the geometric ornamentation on the facades and monumental portals of the Divriği Great Mosque, the Izzettin Keykavus Hospital, and the Gök Medrese. These figures, alongside geometric designs, carry various symbolic meanings. In medieval Anatolia—particularly during the Seljuk era—celestial elements such as the moon, sun, planets, and stars; figures like fish, dragons, lions, deer, and birds; and forms such as the wheel of fortune, flowers, and stars set within rosettes were all used to symbolize concepts of protection and power. Motifs such as the eagle, moon, sun, planets, and symbols of light were among the most frequently employed icons by the Anatolian Seljuks. Star forms, meanwhile, symbolize the sultan as God's representative on earth. In architectural ornamentation, forms representing infinity—such as circles, spheres, nail-head motifs, and rosettes—serve as miniature symbols of the "celestial dome," conveying religious and cosmological meanings associated with God, the heavens, and the universe. These cosmic designs can be interpreted as an expression of the desire to protect structures from evil and to show reverence for sacred spaces. While it is possible that these forms symbolize zodiac signs and planets, it is equally likely that they represent the sultan.

Keywords: Geometric ornaments, Medieval Anatolian Seljuk period, Sivas, figure.

REFERENCES

[1] O. Aslanapa, Türk Sanatı, İstanbul, 1984.
[2] D. Kuban, Anadolu Türk Mimarisinin Kaynak ve Sorunları, İstanbul,1964.
[3] S. Ögel, Taçkapılar, Anadolu Selçukluları ve Beylikler Dönemi Uygarlığı, Kültür Bakanlığı Yayınları, Ankara,2006.
[4] Ç. Ö. Tunç, Anadolu Selçuklu Dönemi Taçkapilari Süsleme Şeritlerinde Tezyinat, Pamukkale Üniversitesi İlahiyat Fakültesi Dergisi Mart/, 3/ 5, (2016), 114-131.

[5] Ö. Alp, Orta Asya'dan Anadolu'ya Kültürel Sembollere Giriş. Ankara: Eflatun Yayınevi, 2009.

[6] S. Ögel, Anadolu'nun Selçuklu Çehresi. İstanbul: Akbank Yay., 1994.

[7] T. Yazar and Doğan N. Şaman Doğan, Anadolu Selçuklu ve Beylikler Dönemi Mimari Süslemesinde Küre, Küre ve Koni Kesiti/Kabara, Rozet. Hacettepe Üniversitesi Türkiyat Araştırmaları Dergisi. (2013),19: 221-245.

[8] E. Esin, Kün-ay, 7. Türk Tarih Kongresi, TTK Basımevi, Ankara, 1972.

[9] B. Karamağaralı, İçiçe Daire Motiflerinin Mahiyeti Hakkında, Sanat Tarihinde İkonografik Araştırmalar Güner İnal'a Armağan. 249-270. Ankara, 1993.

[10] Durukan Aynur. Ak Han'ın Süsleme Programı, Sanat Tarihinde İkonografik Araştırmalar Güner İnal'a Armağan. Ankara, (1993), 143-161.

[11] B. Ögel, Türk Mitolojisi I. Ankara: TTK, 2010.

[12] Öney Gönül (1992). *Anadolu Selçuklu Mimari Süslemesi ve El Sanatları*. Ankara: Türkiye İş Bankası Kültür Yay.

Information Geometry of Orthogonal Generalized Doubly Stochastic Matrices

Mert Çarboğa [1], Yusuf Yaylı [2]
1 Department of Statistics, Yüksek İhtisas University,Ankara,Türkiye
Email: mert.carboga@yuksekihtisas.edu.tr
2 Department of Mathematics, Faculty of Science, Ankara University, Ankara, Türkiye
E-mail: yayli@science.ankara.edu.tr

ABSTRACT

This study investigates the orthostochastic images of orthogonal generalized doubly stochastic matrices obtained through the Hadamard-square transformation. First, the component with determinant (+1), which is identified with the umbrella Lie group, is considered, and a general algebraic identity explicitly describing the composition of the Cayley and Hadamard-square transformations is derived. It is also shown that the pullback of the Fisher metric on the orthostochastic image is a constant multiple of the Frobenius metric on the source space. As an application of this information-geometric structure, the doubly stochastic process generated by the Hadamard-square image of Brownian motion on the umbrella Lie group is examined. While the component with determinant (-1) is evaluated comparatively, it is shown that, in the three-dimensional case, the Fisher metric is constant and the resulting orthostochastic images are always invertible.

Keywords: Orthogonal generalized doubly stochastic matrix, Umbrella matrix, Cayley transformation, orthostochastic matrix, Fisher metric, Brownian motion.

REFERENCES

[1] M. Çarboğa and Y. Yaylı, Geometry of Circulant Umbrella Matrices, *The American Mathematical Monthly* (2026). https://doi.org/10.1080/00029890.2026.2673808

[2] G. Oderda, A. Smoktunowicz and R. Kozera, Efficient numerical algorithms for constructing orthogonal generalized doubly stochastic matrices, *Applied Numerical Mathematics* 142 (2019), 16–27. https://doi.org/10.1016/j.apnum.2019.02.008

[3] G. Montúfar, J. Rauh and N. Ay, On the Fisher metric of conditional probability polytopes, *Entropy* 16(6) (2014), 3207–3233. https://doi.org/10.3390/e16063207

Intrinsic Characterization of Persistent Lorentzian Frame Motions in Minkowski Space

Derya Kahveci [1] , Yusuf Yaylı [2]
1 Ankara University Faculty of Science, Department of Mathematics, 06100, Ankara, Turkey, dkahveci@ankara.edu.tr
2 Ankara University Faculty of Science, Department of Mathematics, 06100, Ankara, Turkey, yayli@science.ankara.edu.tr

ABSTRACT

Persistent frame motions constitute a distinguished class of one-parameter Lorentzian motions whose instantaneous twists preserve a constant pitch during the evolution of the motion. Such motions naturally arise in the geometric study of moving frames and play an important role in the kinematic interpretation of curves in Minkowski space. While previous investigations have mainly focused on the algebraic characterization of persistence through curvature and torsion functions, the intrinsic geometric structure underlying this property has received comparatively less attention.

In this presentation, we investigate persistent Lorentzian frame motions from an intrinsic differential geometric perspective. Considering Lorentzian moving frames along regular curves in three-dimensional Minkowski space, we examine the geometric quantities governing the persistence condition and discuss how these invariants characterize the corresponding frame motions. The emphasis is placed on the intrinsic interpretation of persistence rather than on the complete classification of special curves. In particular, the presentation illustrates how differential geometric invariants naturally describe the behavior of persistent motions and establish geometric relations between different moving frame constructions.

Several representative examples are discussed to illustrate the proposed framework and to emphasize the geometric significance of the obtained characterization. The presented approach provides a unified viewpoint for understanding persistent Lorentzian frame motions through intrinsic geometry and contributes to the interaction between differential geometry and geometric kinematics.

Keywords: Persistent motions, Lorentzian geometry, moving frames, intrinsic invariants.

REFERENCES

[1] J.M. Selig and M. Carricato, Persistent rigid-body motions and Study's Ribaucour problem, J. Geom. 108 (1) (2017), 149-169.

[2] D. Kahveci and Y. Yaylı, Persistent Lorentzian Rigid Motions Generated by Slant Helices in Minkowski 3-Space, Mathematics 14 (13) (2026), 2415.

On the Prolongation of Almost Hsu-Golden Structures to the Second-Order Tangent Bundle

Neslihan Yıldırım [1] , **Mustafa ÖZKAN** [2]
1 Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye, neslihanyildirim@gazi.edu.tr
2 Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye, ozkanm@gazi.edu.tr

ABSTRACT

Additional structures play an important role in the geometry of differentiable manifolds. Structures on a differentiable manifold induced by a tensor field of type (1,1) have been studied by many researchers.

Definition ([4]). Let M be C^∞ a differentiable manifold. A golden structure on M is defined as a tensor field ϕ of type $(1,1)$ and of class C^∞, where $\phi^2 - \phi - \mathrm{I} = 0$ is satisfied.

Definition ([6]). Let M be a C^∞ differentiable manifold and F be a tensor field of type $(1,1)$. The tensor field F is said to be a Hsu structure on M if it satisfies the condition

$$\mathrm{F}^2 = \alpha^{\mathrm{r}} I,$$

where $\alpha \in \mathrm{C} \setminus \{0\}$, $r \in Z$ and I is the identity tensor.

Definition ([1]). Let M be a differentiable manifold and J be a $(1,1)$ tensor field that satisfies the equation below,

$$J^2 = J - \frac{1 - 5\alpha^r}{4} I,$$

where $\alpha \in \mathrm{C} \setminus \{0\}$, $r \in Z$. Here, J is called an almost Hsu-golden structure on M, and the pair (M, J) is an almost Hsu-golden manifold.

In this study, we investigate the second-order lift of an almost Hsu-golden structure J to the second-order tangent bundle $T_2(M)$ of a differentiable manifold M. We examine the conditions under which the lifted structure is integrable and analyse its parallelism with respect to certain natural connections.

Keywords: Hsu structure, almost Hsu-golden structure, second-order tangent bundle, prolongations, lift.

REFERENCES

[1] B. Ay and M. Özkan, Almost Hsu-golden structure on manifolds, Turk. J. Math. Comput. Sci. 17(1) (2025), 264–274.
[2] L. Bisht, On Hsu-structure manifold, Nijenhuis tensor, Int. J. Adv. Res. Technol. 2 (2013), 87–96.
[3] M. Crasmareanu and C. E. Hretcanu, Golden differential geometry, Chaos Solitons Fractals, 38 (2008), 1229-1238.
[4] U. De, A. Gezer and C. Karaman, Results concerning semi-symmetric metric Fconnections on the Hsu-B manifolds, Commun. Korean Math. Soc. 38 (2023), 837-–846.
[5] C.J. Hsu, Note on the integrability of a certain structure on differentiable manifold, Tohoku Math. J. Second Ser., 12 (1960), 349-360.

[6] C. J. Hsu, On some structures which are similar to the quaternion structure, Tohoku Math. J. Second Ser., 12 (1960), 403-428.

[7] M. Özkan and F. Yılmaz, Prolongations of golden structures to tangent bundles of order r, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. 65(1) (2016), 35–47.

[8] K. Yano, S. Ishihara, Tangent and Cotangent Bundles, Marcel Dekker Inc., New York, 1973.

Ricci Pseudosymmetry Conditions on Trans Para-Sasakian Manifolds and Ricci Solitons

Mehmet Atçeken[1] , **Tuğba Mert**[2]
1 Aksaray University, Faculty of Science and Letters, Department of Mathematics
mehmetatceken@aksaray.edu.tr
2 Cumhuriyet University, Faculty of Science, Department of Mathematics
tmert@cumhuriyet.edu.tr

ABSTRACT

The aim of this study is to investigate the geometric consequences of Ricci pseudosymmetry conditions imposed on trans-para-Sasakian manifolds and to provide a detailed analysis of the resulting curvature properties. In this context, the interaction between the Ricci tensor, the curvature tensor, and the underlying trans-para-Sasakian structure is examined under appropriate pseudosymmetry conditions. By making use of the fundamental identities characterizing trans-para-Sasakian manifolds, several geometric relations associated with these curvature restrictions are derived and analyzed. Particular attention is devoted to determining how the imposed Ricci pseudosymmetry conditions influence the structural and curvature behavior of the manifold. Furthermore, the study investigates the position of trans-para-Sasakian manifolds within the broader framework of almost paracontact metric geometry by identifying the classes of manifolds that are generalized by the trans-para-Sasakian structure and the particular cases to which this structure reduces under suitable restrictions on its defining functions. In this way, connections between trans-para-Sasakian manifolds and several important subclasses of paracontact metric manifolds are clarified. The results obtained contribute to a better understanding of the role of Ricci pseudosymmetry in the classification and characterization of trans-para-Sasakian manifolds and provide a framework for further investigations concerning curvature restrictions in paracontact geometry.

Keywords: Trans Para-Sasakian Manifold, Para-Cosymplectic Manifolds, α-Para-Sasakian Manifolds.

REFERENCES

[1] S. Kaneyuki and M. Konzai, Paracomplex structure and affine symmetric spaces, Tokyo J. Math. 8 (1985), 301–308.

[2] S. Zamkovoy, Canonical connection on paracontact manifolds, Ann. Glob. Anal. Geom. 36 (2009), 37–60.

[3] J.C. Marrero and D. Chinea, On trans-Sasakian manifolds, Ann. Mat. Pura Appl. 162(4) (1992), 77–86.

[4] S. Zamkovoy, On the geometry of trans-para-Sasakian manifolds, Filomat 33(18) (2019), 6015–6024.

[5] M.D. Siddiqi, Characteristics of lightlike hypersurfaces of trans-para-Sasakian manifolds, Int. J. Maps Math. 3(2) (2020), 109–128.

[6] I. Ansari, S.A. Khan, S. Rizvi and M.A. Qayyoom, Submanifolds in semi-Riemannian manifolds with Golden structure, Palestine J. Math. 13(4) (2024), 993–1004.

[7] M. Atçeken, T. Mert, P. Uygun and M.S. Stanković, New insights on pseudoparallel submanifold in trans-Sasakian manifolds, Filomat 39(8) (2025), 2577–2586.

[8] T. Mert and M. Atçeken, A note on pseudoparallel submanifolds of Lorentzian para-Kenmotsu manifolds, Filomat 37(15) (2023), 5095–5107.

[9] T. Mert, M. Atçeken and P. Uygun, On invariant submanifolds of Lorentz Sasakian space forms, J. Math. Ext. 17(2) (2023), 1–17.

[10] M.D. Upadhyay and K.K. Dube, Almost contact hyperbolic (f,g,η,ξ)(f,g,\eta,\xi)-structure, Acta Math. Acad. Sci. Hungar. 28 (1976), 13–15.

[11] M. Crasmareanu and C.E. Hretcanu, Golden differential geometry, Chaos Solitons Fractals 38 (2008), 1124–1146.

[12] C.E. Hretcanu, Submanifolds in Riemannian manifold with Golden structure, Workshop on Finsler Geometry and its Applications, Hungary, 2007.

[13] R. Deszcz and W. Grycak, On manifolds satisfying some curvature conditions, Colloq. Math. 57(1) (1989), 89–92.

Semi-Orthogonal Matrix Representations of Quaternions in Dual Spaces

Gülsüm Yüca [1], Yusuf Yaylı [2]
1 Department of Mathematics, Aksaray University, Aksaray, Turkey, gulsumbicer@gmail.com
2 Department of Mathematics, Ankara University, Ankara, Turkey, yayli@science.ankara.edu.tr

ABSTRACT

This paper studies the mathematical link between quaternions and dual transformations. We focus on 3D and 4D rotations. We want to show these rotations in Lorentzian space. Rotational motions are very important in geometry and kinematics. We offer a new way to express these motions. We use unit real quaternions for this purpose. This method helps us find new rotation matrices in Lorentzian space.

First, we use Euclidean and Lorentzian spaces. We take a rotation matrix made from a quaternion. We prove that this matrix keeps the same axis invariant in both Euclidean and Lorentzian spaces. We transform orthogonal matrices into semi-orthogonal matrices. We do this by using dual transformations.

Next, we study a one-parameter motion. We define this motion using a quaternion curve in 4D space. We write down the semi-orthogonal matrix step by step. We also provide practical examples. We draw pictures of these examples. This makes the math easier to see and understand. For instance, we build special surfaces from circle curves.

Finally, we expand our ideas to dual spaces. Dual spaces are very useful in robotics and physics. We use dual unit quaternions to model rigid body motions. We investigate screw motions in dual Lorentzian space. We calculate the screw axes for these motions. Then, we show that the matrices keep the same screw axis invariant.

Keywords: Dual quaternion, dual transformation, kinematics, Lorentzian space, rotation matrix.

REFERENCES

[1] R. Dohi, Y. Maeda, M. Mori and H. Yoshida, A dual transformation between SO(n + 1) and SO(n, 1) and its geometric applications, Linear Algebra Appl. 432 (2010), 770-776.

[2] G. Yüca and Y. Yaylı, A dual transformation between SO(3) and SO(2, 1) and its geometric applications, Proc Natl Acad Sci, India, Sect A Phys Sci. 88 (2018), 267-273.

[3] R. López, Differential geometry of curves and surfaces in Lorentz-Minkowski space, Int Electron J Geomet. 7 (2014), 44-107.

Spiral Curves and Their Counterparts in Euclidean Three Space

Fouzi HATHOUT
Department of mathematics, Faculty of mathematics, informatic and telecommunication, University of Saida, 20000 Saida, Algeria
Email: f.hathout@gmail.com & fouzi.hathout@univ-saida.dz

ABSTRACT

Let α be a regular curve in Euclidean 3-space E^3. Considering the f-position vector $\alpha_f=\int f d\alpha$, we define a new class of special curves in E^3, called slant spiral curves. These curves are characterized by the property that their f-position vector $\alpha_{f\equiv cst}$ makes a constant angle with the unit normal vector of α. We begin by providing a brief overview of spiral curves, including their characterization in terms of curvature and torsion conditions, as well as their classification. Next, we provide the characterization, necessary and sufficient conditions, and classification of slant spiral curves in terms of curvature and torsion. Finally, we conclude our presentation by presenting illustrative examples.

Keywords: f-position vector, helix, slant helix, spiral, slant spiral, B-spiral.

REFERENCES

[1] R. C. Archibald, The Logarithmic Spiral, Amer. Math. Monthly 25, 189-193,(1918).
[2] Y. Aminov, Differential Geometry and Topology of Curves, CRC Press, Boca Raton, (2000)
[3] B. Y. Chen, When does the position vector of a space curve always lie in its rectifying plane?, Amer. Math. Monthly, 110, 147-152, (2003).
[4] A. Gray, Logarithmic Spirals, Mod. Diff. Geo. of Curves and Surfaces with Mathematica, 2nd ed. Boca Raton, FL: CRC Press, 40-42, (1997).
[5] F. Hathout, A new class of curves generalizing helix and rectifying curves, Int. J. of Geo. 11(4), 65-74, (2022).
[6] J. D. Lawrence, A Catalog of Special Plane Curves. New York: Dover, 184-186, (1972).
[7] T. Ikawa, On Some Curves in Riemannian Geometry, Soochow J. Math., 7, 37-44,(1980).
[8] D. A. Singer, Curves whose curvature depends on distance from the origin, this Monthly, 106, 835-841,(1999).

Topology

Crossed Modules over 2-Groups

Sedat TEMEL [1]
1 Recep Tayyip Erdogan University, Rize, TÜRKİYE, sedat.temel@erdogan.edu.tr

ABSTRACT

Crossed modules have long been recognized as one of the fundamental algebraic models for homotopy 2-types and have established strong connections between algebra, topology, and category theory. In recent years, the development of higher-dimensional algebra has motivated the extension of classical crossed module theory to more sophisticated categorical settings. In this context, crossed modules over 2-groups provide a natural higher-dimensional framework that connects abstract categorical structures with manageable algebraic models.

In this work, we investigate crossed modules over 2-groups, where both the base and the total object are 2-groups (group-groupoids) satisfying the corresponding compatibility conditions. These structures extend the classical notion of crossed modules over groups while preserving the rich interplay between algebraic and categorical properties. Our primary objective is to clarify the relationships between crossed modules over 2-groups and other higher-dimensional categorical models.

The first main result establishes that the category of crossed modules over 2-groups is equivalent to the category of cat¹-2-groups, viewed as group objects in the category of cat¹-groupoids. The second main result proves an equivalence between crossed modules over 2-groups and the category of group-2-groupoids. These equivalences provide alternative but mathematically equivalent descriptions of the same higher-dimensional structures, allowing problems arising in one category to be transferred and studied within another, often more convenient, categorical framework.

Keywords: Crossed module, 2-group, groupoid.

REFERENCES

[1] Temel, Sedat. "Further remarks on group-2-groupoids." *Applied general topology* 22.1 (2021): 31-46.

[2] Temel, Sedat. "The theory of cat^1-2-groups among higher categorical models." *AIMS Mathematics* 11.3 (2026): 6141-6161.

r-Near Topological Spaces Generated by Means of Minimal Open Sets

Berrin Nur Abbasoğlu, Serkan Atmaca

Berrin Nur Abbaoğlu, b.nur.a4244@gmail.com, Topology Division, Department of Mathematics, Graduate School of Natural and Applied Sciences, Sivas Cumhuriyet University, Sivas, Türkiye

Serkan Atmaca, seatmaca@cumhuriyet.edu.tr, Topology Division, Department of Mathematics, Graduate School of Natural and Applied Sciences, Sivas Cumhuriyet University, Sivas, Türkiye

ABSTRACT

Near set theory, introduced by Peters in 2002 as a generalization of rough set theory, focuses on the similarity between objects based on their feature descriptions. Unlike classical set theory, it examines objects through functional descriptions, where proximity is directly proportional to the number of shared features.

Near approximation spaces constitute the fundamental building blocks of this theory. They are defined by object feature functions and the indiscernibility relations formed via these functions. While these spaces operate on a logic analogous to the lower and upper approximations in rough set theory, they offer a much more flexible and comprehensive perspective for classifying objects. The lower approximation distinctly defines the boundaries of elements that definitely belong to a set, whereas the upper approximation provides a broader framework encompassing all elements that may be related to that set. This enables more inclusive and reliable inferences to be drawn from uncertain data.

The topological dimension of near set theory, namely r-near topological spaces, was first defined by Atmaca in 2020. Subsequently, in 2021, Atmaca and Zorlutuna introduced the concept of r-near continuous functions to the literature, and in 2024, Sarıkaya and Atmaca introduced the concepts of r-near bases and r-near subspaces, further solidifying the theoretical groundwork of this structure. These developments constitute significant steps toward addressing the concept of nearness within a topological framework.

In the present study, it is aimed to systematically construct near approximation spaces by utilizing minimal open sets from any given topological space. The characteristic functions of these minimal open sets are formulated as object feature functions, thereby enabling the derivation of r-near structures from classical topological data. This approach establishes a solid and meaningful relationship between classical topology and near set theory; the resulting r-near topological structures allow for a reinterpretation of the concept of nearness over topological spaces, thus presenting a distinctive mathematical framework that deepens the interaction between these two fields.

Keywords: Near set theory, r-near topological spaces, minimal open sets.

REFERENCES

[1] S. Atmaca, r-Near topologies on nearness approximation spaces, J. Intell. Fuzzy Systems 39 (2020), 5885-5891.

[2] S. Atmaca and I. Zorlutuna, Continuity on r-near topological spaces, J. Intell. Fuzzy Systems 41 (2021), 3629-3633.

[3] S. Sarıkaya and S. Atmaca, Some notes on r-near topological spaces, [Journal Name, Volume, Pages] (2024).

[4] Z. Pawlak and J.F. Peters, How near, Systemy Wspomagania Decyzji I, 57 (2002).

[5] J.F. Peters, Near sets. Special theory about nearness of objects, Fund. Inform. 75 (2007), 407-433.

[6] B.N. Abbasoğlu, On separation axioms of r-near topological spaces, Master's Thesis, Sivas Cumhuriyet University, Institute of Natural Sciences, 2025.

Applied Mathematics

A New Approach to Hermite-Hadamard Inequalities Using Multiplicative Fractional Integrals

Umut Baş[1,*], Huseyin Budak[2], Hüseyin Yildirim[3],
1,3 Department of Mathematics, Faculty of Science, Kahramanmaraş Sütçü İmam University, Kahramanmaraş 46100, Türkiye.
2 Department of Mathematics, Faculty of Science and Arts, Kocaeli University, Kocaeli 41001, Türkiye.
E-mail: umutbas9661@gmail.com
E-mail: hsyn.budak@gmail.com
E-mail: hyildir@ksu.edu.tr

ABSTRACT

Abstract. In this article, a novel bridge is established between multiplicative calculus and fractional calculus. Various properties of fractional integral operators defined with respect to another function in the context of multiplicative analysis are rigorously proven. Subsequently, these multiplicative fractional integrals are adapted to the Hermite-Hadamard inequality, leading to a new inequality. Furthermore, a trapezoid-type version of the Hermite-Hadamard inequality is established for the multiplicative fractional integrals with respect to another function, and these results are also supported by examples and graphical representations.

Keywords: Multiplicative Calculus; Non-Newtonian Calculus; Fractional Integrals; Hermite-Hadamard Inequalities.

REFERENCES

[1] Bashirov, A. E., Kurpınar, E. M., & O¨ zyapıcı, A. (2008). Multiplicative calculus and its applications. Journal of mathematical analysis and applications, 337(1), 36-48. https://doi.org/10.1016/j.jmaa.2007.03.081

[2] Georgiev, S., & Zennir, K. (2022). Multiplicative Differential Calculus (1st ed.). Chapman and Hall/CRC. https://doi.org/10.1201/9781003299080

[3] Butt, S. I., Khan, D., Breaz, V. D., Cot I., & Mohsin, B. B. (2025). Fractional integral inequalities for strongly convex functions via multiplicative calculus with applications. Boundary Value Problems, 2025(1), 102. https://doi.org/10.1186/s13661-025-02061-3

[4] Zhou, Z., & Du, T. (2024). Analytical properties and related inequalities derived from multiplicative Hadamard k-fractional integrals. Chaos, Solitons & Fractals, 189, 115715. https://doi.org/10.1016/j.chaos.2024.115715

A Seasonal Jump-Diffusion Model for Extreme Rainfall in the Eastern Black Sea Region: Numerical Schemes and Exceedance Risk

Tuğçem Partal[1], **Gülşen Orucova Büyüköz**[2]
1 Department of Computer Engineering, Recep Tayyip Erdoğan University, Rize, Türkiye, tugcem.partal@erdogan.edu.tr
2 Department of Mathematics and Computer Sciences, Necmettin Erbakan University, Konya, Türkiye, gorucova@erbakan.edu.tr

ABSTRACT

The Eastern Black Sea region of Türkiye receives the highest annual precipitation in the country, and short-duration extreme rainfall events in the region frequently trigger destructive flash floods, as exemplified by the July 2021 disaster in Rize. Existing studies of regional precipitation rely predominantly on static statistical tools such as trend tests and extreme value distributions, whereas continuous-time stochastic models that jointly describe ordinary rainfall variability and abrupt extreme events remain scarce. Ornstein–Uhlenbeck type rainfall models without jumps, compound Poisson descriptions of daily rainfall, and SDE-based hydrological risk assessment have been proposed, and, most recently, mean-reverting jump-diffusion processes have been suggested for rainfall dynamics, but without seasonal structure or a systematic study of numerical discretization. In this work, we propose a seasonal mean-reverting jump-diffusion model for daily rainfall intensity in which the drift term relaxes the process toward a deterministic seasonal level obtained from day-of-year averages, a Brownian component captures ordinary day-to-day variability, and a compound Poisson component with exponentially distributed jump sizes, in the spirit of the jump-diffusion framework of Kou, represents extreme events. The jump intensity and the jump size distribution are calibrated from threshold exceedances in the data, and the diffusion parameters are estimated from long-term daily station records by a moment-based procedure based on the discrete autoregressive representation of the mean-reverting component. The numerical solution of the model is investigated through jump-adapted stochastic schemes: the Euler–Maruyama and Milstein methods are implemented on the calibrated model, and their strong convergence behavior and numerical stability are compared for decreasing step sizes. Threshold exceedance probabilities and return periods of extreme rainfall are then estimated by Monte Carlo simulation and compared with those produced by the corresponding pure diffusion model without jumps. Numerical results show that omitting the jump component systematically underestimates the probabilities of extreme events and that seasonality is essential for reproducing the observed rainfall regime. The comparison of schemes further indicates the extent to which higher-order discretization improves accuracy in the presence of jumps.

Keywords: stochastic differential equations, jump-diffusion process, extreme rainfall, Euler–Maruyama scheme, Milstein scheme, exceedance probability

REFERENCES

[1] E. Platen and N. Bruti-Liberati, Numerical Solution of Stochastic Differential Equations with Jumps in Finance, Springer-Verlag, Berlin, 2010.

[2] D.J. Higham and P.E. Kloeden, Numerical methods for nonlinear stochastic differential equations with jumps, Numer. Math. 101 (2005), 101-119.

[3] S.G. Kou, A jump-diffusion model for option pricing, Manag. Sci. 48 (2002), 1086-1101.

[4] J. GhoshDastider, D. Pal and P.K. Mishra, Controlling the rain fall statistics using mean-reverting jump diffusion model, arXiv:2604.08338 (2026).

[5] E.Y. Setyorini and E.R.M. Putri, Rainfall modelling in East Java using a modified Ornstein-Uhlenbeck model, Jurnal Matematika UNAND 14 (2025), 46-61.

[6] N.C. Dzupire, P. Ngare and L. Odongo, A Poisson-Gamma model for zero inflated rainfall data, J. Probab. Stat. 2018 (2018), 1012647.

[7] K. Unami, F.K. Abagale, M. Yangyuoru, A.H.M.B. Alam and G. Kranjac-Berisavljevic, A stochastic differential equation model for assessing drought and flood risks, Stoch. Environ. Res. Risk Assess. 24 (2010), 725-733.

[8] E. Eris and N. Agiralioglu, Homogeneity and trend analysis of hydrometeorological data of the Eastern Black Sea Region, Turkey, J. Water Resour. Prot. 4 (2012), 99-105.

A Unified Composite-Weight Framework for Generalized Convexity

Ali Harun Daldallı[1,2], Abdullah Akkurt[1], Hüseyin Yıldırım[1]
[1] Department of Mathematics, Faculty of Science, Kahramanmaras Sutcu Imam University, 46100 Kahramanmaras, Türkiye; aakkurt@ksu.edu.tr; hyildir@ksu.edu.tr
[2] Department of Mathematics, Faculty of Science and Arts, Hatay Mustafa Kemal University, 31060 Hatay, Türkiye; aliharun.daldalli@mku.edu.tr

ABSTRACT

This seminar presents a reusable composite-weight framework for organizing broad families of generalized convexity, including classes related to h-convexity [1]. An admissible convex generator φ on [0,1] is first transformed into a canonical normalized weight and is then coupled with a continuous modulating weight ψ. Their product determines a composite weighted convexity class through a single two-endpoint inequality. The decomposition separates the geometric shape carried by the normalized generator from the global modulation supplied by ψ, so different weighted classes can be constructed and compared without rebuilding the underlying method.

$$\varphi\#(t) = [\varphi(t) - \varphi(0)]/[\varphi(1) - \varphi(0)], \qquad W_{\varphi,\psi}(t) = \varphi\#(t)\psi(t).$$

The seminar focuses on this architecture rather than on a complete catalogue of subclasses. Under natural admissibility assumptions, the framework provides endpoint normalization, positivity and pointwise bounds, closure under nonnegative linear combinations, inclusion relations induced by weight ordering, and a direct comparison with classical convexity. Its integral behaviour is encoded by three moments obtained from W, (1−t)W, and tW.

$$I_W = \int_0^1 W(t)\,dt, \qquad \mu_1 = \int_0^1 (1-t)W(t)\,dt, \qquad \mu_2 = \int_0^1 tW(t)\,dt.$$

Using the Dragomir-Agarwal identity [2] together with Hölder and Hölder-İşcan refinements [3], these constants generate a common template for Hermite-Hadamard and derivative-based integral estimates. Particular choices of φ and ψ recover familiar weighted settings or produce new families, such as exponential-trigonometric variants [4]. The main point is that once the composite weight and its three moments are known, the analytical mechanism remains unchanged. This offers a compact platform for constructing, comparing, and extending generalized convexity models.

Keywords: generalized convexity, composite weight, h-convexity, Hermite-Hadamard inequality, weighted convexity, exponential-trigonometric convexity.

REFERENCES

[1] S. Varošanec, On h-convexity, Journal of Mathematical Analysis and Applications 326 (2007), 303-311.

[2] S. S. Dragomir and R. P. Agarwal, Two inequalities for differentiable mappings and applications to special means of real numbers and to trapezoidal formula, Applied Mathematics Letters 11 (1998), 91-95.

[3] İ. İşcan, New refinements for integral and sum forms of Hölder inequality, Journal of Inequalities and Applications 2019 (2019), Article 304.

[4] M. Kadakal, İ. İşcan, P. Agarwal and M. Jleli, Exponential trigonometric convex functions and Hermite-Hadamard type inequalities, Mathematica Slovaca 71 (2021), 43-56.

An Encryption–Decryption Algorithm Using the Fractional Laplace Transform for Composite Special Functions

Hüseyin Demir [1], Hatice Muti [2], Hünkar Acar[3]
[1]Faculty of Computer and Information Sciences, Department of Software Engineering, Samsun University, 55080, Samsun, Turkey, e-mail: huseyin.demir@samsun.edu.tr
[2]Department of Aviation Management, College of Civil Aviation, Samsun University, 55080, Samsun, Turkey, e-mail: hatice.muti@samsun.edu.tr
[3]Faculty of Computer and Information Sciences, Department of Software Engineering, Samsun University, 55080, Samsun, Turkey, e-mail: hunkaracar7@gmail.com

ABSTRACT

The rapid growth of secure digital communication has encouraged the development of encryption schemes based on advanced integral transforms and special functions. In this study, a new encryption and decryption framework is proposed using the fractional Laplace transform (FLT) applied to a class of functions composed of Bessel functions, exponential functions. The proposed approach leverages the additional degree of freedom provided by the fractional order of the transformation; this serves as an encryption key while preserving the reversibility necessary for correct decryption. The analytical properties of the fractional Laplace transform, including Bessel functions, exponential terms, have been examined for structured composite functions, and explicit transformation formulas suitable for cryptographic applications have been obtained. The encryption process is carried out by transforming the original signal into the fractional Laplace domain, while decryption is performed through the corresponding inverse fractional Laplace transform using the correct fractional order. The dependence of the reconstructed signal on the fractional parameter shows high precision; this increases the security of the proposed method against unauthorized recovery. Numerical examples demonstrate the accuracy, stability, and computational efficiency of the algorithm. The results, when compared with previous literature studies, show that the combination of fractional Laplace transform with special function-based signal representations provides a flexible mathematical framework for secure information transmission and suggests new directions for transform-based cryptographic systems. In addition, the study will focus on cryptanalysis.

Keywords: Fractional Laplace transform; Fractional calculus; Bessel functions; Integral transforms; Special functions.

REFERENCES

[1] Andrews, L. C. (1998). Special Functions of Mathematics for Engineers (2nd ed.). SPIE Press.

[2] Belgacem, F. B. M., & Karaballi, A. A. (2006). "Sumudu Transform Fundamental Properties Investigations and Applications." International Journal of Applied Mathematics and Stochastic Analysis, 2006, Article ID 91083.

[3] Debnath, L., & Bhatta, D. (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press.

[4] Demir, H., Muti, H., & Acar, H. (2025). Applying Laplace transforms to cryptography using Bessel and exponential functions and performing detailed time and memory analysis. Gümüşhane Üniversitesi Fen Bilimleri Dergisi, 15(4), 1244-1256. https://doi.org/10.17714/gumusfenbil.1780179

[5] Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). Theory and Applications of Fractional Differential Equations. Elsevier.

[6] Oldham, K. B., & Spanier, J. (1974). The Fractional Calculus. Academic Press.

[7] Podlubny, I. (1999). Fractional Differential Equations. Academic Press.

[8] Watson, G. N. (1995). A Treatise on the Theory of Bessel Functions (2nd ed., reprint). Cambridge University Press.

Asymptotic Behavior and Oscillation of Third-Order Semi-Canonical Neutral Differential Equations

Orhan Özdemir[1], Ercan Tunç[2]
1 Department of Mathematics, Faculty of Arts and Sciences, Tokat Gaziosmanpaşa University, 60240, Tokat, Türkiye. email: orhanozdemir37@yahoo.com
2 Department of Mathematics, Faculty of Arts and Sciences, Tokat Gaziosmanpaşa University, 60240, Tokat, Türkiye. email: ercantunc72@yahoo.com

ABSTRACT

The qualitative analysis of third-order neutral differential equations has attracted considerable attention in recent years because of the theoretical importance of these equations and their wide range of applications in physics and engineering. However, most of the existing results are established under the canonical assumption, whereas the semicanonical case has received much less attention. In this presentation, we consider a class of third-order semicanonical neutral differential equations and introduce sufficient conditions for the asymptotic behavior and oscillation of the solutions. The main tools employed are transforming the considered equation into an equivalent canonical form and utilizing Riccati-type substitutions to establish the desired results.

The canonicalization procedure simplifies the analysis of the considered semicanonical equation and allows us to apply techniques developed for canonical equations. The presented results extend several existing oscillation criteria and complement the available results for third-order neutral differential equations. Several examples are presented to demonstrate the applicability and significance of the proposed criteria.

Keywords: Asymptotic behavior, oscillation, neutral differential equations.

REFERENCES

[1] K. Saranya, V. Piramanantham, E. Thandapani and E. Tunç, Oscillation criteria for third-order semi-canonical differential equations with unbounded neutral coefficients, Stud. Univ. Babeş-Bolyai Math. 69(1) (2024), 115–125.

[2] E. Thandapani, B. Göktürk, O. Özdemir and E. Tunç, Oscillatory behavior of semi-canonical nonlinear neutral differential equations of third-order via comparison principles, Qual. Theory Dyn. Syst. 22 (2023), Article No. 30.

Certified Fuzzy Output Strips for Information-Diffusion Models

Hazm Talab [1], Ömer Akın [2]
1 Department of Computer Engineering, Faculty of Engineering, TOBB University of Economics and Technology, Söğütözü Caddesi No:43, 06560 Ankara, Türkiye; htalab@etu.edu.tr
2 Department of Mathematics, TOBB University of Economics and Technology, Söğütözü Caddesi No:43, 06560 Ankara, Türkiye; omerakin@etu.edu.tr)

ABSTRACT

Fuzzy parameters are useful when behavioral rates cannot be measured precisely. A common practice is to evaluate a nonlinear model only at the lower and upper endpoints of each fuzzy parameter and present the result as an output strip. This may be misleading because an extreme output can occur at an interior parameter value. Building on fuzzy models of unverified-information diffusion [1] and recent alpha-cut algorithms for fuzzy initial-value problems [2], we develop a certification approach for a compact SEIR-type information-diffusion model. Transmission, pre-dissemination correction, and intervention rates are represented by triangular fuzzy numbers. At each alpha level, they form a cuboid of ordinary parameter values. We determine when selected corners give the exact output range, following vertex-based propagation [3]. The four state variables are normalized population fractions, following a standard SEIR convention [4]. Monotonicity proofs yield exact alpha-cut strips for the diffusion takeoff threshold and eventual additional reach. A model-specific conserved quantity, or first integral [5], gives a closed-form strip for a peak dissemination pressure and a rigorous upper bound on the largest fraction of active disseminators. In 120 randomized supercritical settings, simulated peaks attain 94%–100% of this bound. However, exact peak size remains numerically supported rather than fully certified, while peak time cannot be certified from endpoints because near-threshold cases may evolve much more slowly. The study therefore separates outputs into certified, empirically supported, and endpoint-non-certified categories. This distinction helps researchers report mathematically justified uncertainty statements rather than visual ranges alone.

Keywords: fuzzy systems, information diffusion, alpha-cuts, certified strips.

REFERENCES

[1] S. Hosseini and A. Zandvakili, "The SEIRS-C model of information diffusion based on rumour spreading with fuzzy logic in social networks," Int. J. Comput. Math. 99(9) (2022), 1918–1940.

[2] N. Doğan, S. Bayeğ, R. Mert and Ö. Akın, "Singularly perturbed fuzzy initial value problems," Expert Syst. Appl. 223 (2023), 119860.

[3] W. Dong and H. C. Shah, "Vertex method for computing functions of fuzzy variables," Fuzzy Sets Syst. 24(1) (1987), 65–78.

[4] K. Heng and C. L. Althaus, "The approximately universal shapes of epidemic curves in the Susceptible-Exposed-Infectious-Recovered (SEIR) model," Sci. Rep. 10 (2020), 19365.

[5] D. Prodanov, "Comments on some analytical and numerical aspects of the SIR model," Appl. Math. Model. 95 (2021), 236–243.

Complex Dynamics and Bifurcation Analysis of a Two-Species Model with Harvesting and Allee Effects

Deniz ELMACI [1], Figen KANGALGİL [2], Seval IŞIK [3]
1 Bergama Vocational School, Dokuz Eylül University, İzmir, 35700, Turkey, deniz.elmaci@deu.edu.tr
2 Bergama Vocational School, Dokuz Eylül University, İzmir, 35700, Turkey, figen.kangalgil@deu.edu.tr
3 Department of Mathematics and Science Education, Faculty of Education, Sivas Cumhuriyet University, Sivas, 58140, Turkey
skaracan@cumhuriyet.edu.tr

ABSTRACT

Prey – predator models, the first and simplest model of which was formulated by Lotka-Volterra, constitute one of the most important classes of mathematical models for describing interactions between biological populations and have been extensively studied to understand their complex dynamical behaviours. Owing to their broad applicability, numerous modifications have been introduced over the years to enhance their biological realism. Among these, the incorporation of harvesting effects has attracted considerable attention because of its significant influence on population dynamics and species persistence. Including the Allee effect, the decline in the per capita growth rate when a population's density increases below a critical threshold, is among the modifications made to the model to increase its realism.

This study investigates the dynamics of the Allee and harvest effects on a discrete – time model. We first establish the conditions for existence and the local asymptotic stability criteria for the co-existence fixed points. By employing center manifold theory and bifurcation analysis, we show that the model undergoes codimension-one bifurcations. Furthermore, bifurcation diagrams and phase portraits are presented to illustrate the qualitative changes in the system dynamics. Numerical simulations are performed to validate the theoretical findings and to reveal the rich dynamical behaviour of the model.

Keywords: Prey – predator model, stability, bifurcation analysis.

REFERENCES

[1] A. J. Lotka, Elements of Physical Biology, Baltimore: Williams and Willkins, 1925.
[2] V. Volterra, Variazioni e fluttazionidel numero di indivui in species animali conviventi Memorie Accademia Nazionaledei Lincei, 2:31-313, 1926.
[3] W. C. Allee, Animal aggretions: a study in general sociology. Chicago: University of Chicago Press; 1931.
[4] Y. A. Kuznetsov, Elements of Applied Bifurcation Theory, Springer-Verlag, New York, NY, USA, 4th edition, 2023.
[5] F. Kangalgil, Neimark--Sacker bifurcation and stability analysis of a discrete-time prey-predator model with Allee effect in prey, Advances in Difference Equations, 2019(1), 1-12 2019.
[6] F. Kangalgil, N. Topsakal, N. Öztürk, Analyzing bifurcation, stability, and chaos control for a discrete-time prey-predator model with Allee effect. Turkish Journal of Mathematics, 46(6), 2047-2068, 2022.

[7] D. Elmacı, F. Kangalgil, Complex dynamics of a discrete prey–predator model exposing to harvesting and Allee effect on the prey species with chaos control. International Journal of Bifurcation and Chaos, 34(09), 2450114, 2024.

[8] S. Işık, F. Kangalgil, M. J. Uddin, S. M. S. Rana, An investigation of the discrete-time model subject to immigration, harvesting, and Allee effect. International Journal of Bifurcation and Chaos, 35(02), 2550022, 2025

Conformable Fractional Integrals with Respect to a Function and Chebyshev-Type Inequalities

Sümeyye ERMEYDAN ÇİRİŞ[1], Hüseyin YILDIRIM[2]
1 Department of Mathematics, Faculty of Science, University of Kahramanmaraş Sütçü Imam, 46100, Kahramanmaraş, Turkey, sumeyye_ermeydan@hotmail.com
2 Department of Mathematics, Faculty of Science, University of Kahramanmaraş Sütçü Imam, 46100, Kahramanmaraş, Turkey, hyildir@ksu.edu.tr.

ABSTRACT

In this paper, we introduce a new class of conformable fractional integrals defined with respect to an increasing function $h(x)$. The proposed operators provide a unified framework that includes several classical fractional integrals as special cases, such as the Riemann–Liouville, Hadamard, and Katugampola integrals, obtained through suitable choices of the function $h(x)$ and the associated parameters.The proposed approach provides a flexible framework for generating various fractional integral operators within a unified setting. In addition, we study some basic properties of these operators and derive several Chebyshev-type integral inequalities. Our results generalize and extend many existing inequalities in fractional calculus, offering a broader framework for the investigation of integral inequalities related to fractional operators.

Keywords: Fractional calculus; conformable fractional integrals; Chebyshev inequality; generalized operators; integral inequalities.

REFERENCES

[1] U.N. Katugampola, New approach to generalized fractional integral. Appl. Math. Comput., 218, (2011) 860-865.
[2] U.N. Katugampola, A new approach to generalized fractional derivatives. Bull. Math. Anal. Appl., 6, (2014), 1-15.
[3] T. Abdeljawad, On conformable fractional calculus. J.Comput. Appl. Math., 279, (2015) 57-66.
[4] F. Jarad, E. Uğurlu, T. Abdeljawad, D. Baleanu, On a new class of fractional operators. Advances in Difference Equations, 247, (2017).
[5] H. Yıldırım, Z. Kırtay, ., Ostrowski Inequality for Generalized Fractional Integral and Related Inequalities. Malaya Journal of Mathematics, (2)3, (2014), 322-329.

Continuous Dependence on the Viscosity Parameter for the Velocity-Vorticity Model of the g-Navier-Stokes Equations

Özge Kazar[1]
1 Department of Electronics and Automation, Vocational School of Technical Sciences, Hitit University, Çorum 19000, Türkiye. E-mail: ozgekazar@gmail.com

ABSTRACT

The velocity-vorticity formulation has been extensively studied for the Navier-Stokes equations because of its advantages in the mathematical and numerical analysis of fluid flows. Motivated by these advantages, we introduced a velocity-vorticity formulation for the g-Navier-Stokes equations with periodic boundary conditions in our previous work [1]. In that work, we established the existence and uniqueness of weak and strong solutions. In this study, we investigate the continuous dependence of strong solutions on the viscosity parameter for this model.

Continuous dependence and structural stability have been studied for various fluid models. Çelebi et al. investigated the continuous dependence of solutions on the Forchheimer coefficient for the convective Brinkman-Forchheimer equations [2]. They also studied the continuous dependence of solutions on the coefficients of the Brinkman-Forchheimer equations [3]. Özlük and Kaya established the structural stability of the g-Bénard problem with respect to the viscosity parameter [4]. Kaya studied the structural stability of the Bénard problem with Voigt regularization by proving the continuous dependence of solutions on the regularization parameters [5]. Motivated by these results, we investigate the continuous dependence of strong solutions on the viscosity parameter for the velocity-vorticity model of the g-Navier-Stokes equations.

In this study, we establish suitable estimates for the difference of two strong solutions corresponding to the equations with different viscosity coefficients. Consequently, the velocity-vorticity model of the g-Navier-Stokes equations is structurally stable with respect to the viscosity parameter.

Keywords: g-Navier-Stokes equations, velocity-vorticity formulation, structural stability, continuous dependence, viscosity parameter.

REFERENCES

[1] Ö. Kazar and M. Kaya, On the weak and strong solutions of the velocity-vorticity model of the g-Navier-Stokes equations, Turkish Journal of Mathematics, 47(6), (2023), 1694-1714.

[2] A. O. Çelebi, V. K. Kalantarov, and D. Uğurlu, Continuous dependence for the convective Brinkman-Forchheimer equations. Applicable Analysis, 84(9), (2005), 877-888.

[3] A. O. Çelebi, V. K. Kalantarov, and D. Uğurlu, On continuous dependence on coefficients of the Brinkman-Forchheimer equations. Applied Mathematics Letters, 19(8), (2006), 801-807.

[4] M. Özlük and M. Kaya, On the strong solutions and the structural stability of the g-Bénard problem. Numerical Functional Analysis and Optimization, 39(4), (2018), 383-397.

[5] M. Kaya, Structural stability for the Benard problem with Voight regularization. Gazi University Journal of Science, 31(3), (2018), 890-896.

Curvature-Dependent Boundary Conditions at Fluid–Solid Interfaces: An Analysis of Velocity Slip and Heat Transfer

Semran Ipek[1], Kiril St. Shterev[2], Stefan K. Stefanov[2], Ali Dinler[3]
[1]Istanbul Medeniyet University, Department of Physics Engineering, Uskudar Istanbul, Türkiye
[2]Bulgarian Academy of Sciences, Institute of Mechanics, Academic Georgi Bonchev Street, Sofia 1113, Bulgaria
[3]Istanbul Medeniyet University, Department of Mathematics, Uskudar Istanbul, Türkiye

ABSTRACT

Boundary conditions at fluid–solid interfaces are commonly formulated for locally planar surfaces; however, micro- and nanoscale systems frequently contain curved boundaries whose local radius of curvature can become comparable to molecular transport scales. For momentum transfer, kinetic simulations of curved gas flows show that convex and concave geometries have opposing effects on velocity slip: increasing curvature enhances slip at convex boundaries but suppresses it at concave boundaries [1]. The associated near-wall non-equilibrium behavior can be characterized through the Knudsen layer and the velocity defect, defined as the deviation between the kinetic solution and the corresponding continuum prediction. A relationship between wall shear stress and velocity defect provides a basis for curvature-sensitive slip corrections [2]. Curvature-dependent slip formulae subsequently reproduce Direct Simulation Monte Carlo data over a range of accommodation coefficients and surface curvatures and clarify the physical mechanism underlying velocity inversion in cylindrical Couette flow [3].

A closely related geometric effect is observed in thermal transport. At a convex fluid–solid interface, increasing curvature produces a significant reduction in interfacial heat transfer and a larger temperature jump. Molecular-scale analysis links this behavior to curvature-induced changes in the Kapitza length and thermal accommodation coefficient in both gas and liquid phases. For gases, convex curvature also increases the penetration depth of non-equilibrium effects and expands the Knudsen layer adjacent to the surface [4].

Taken together, these results indicate that surface curvature should be treated as an explicit parameter in boundary conditions for microscale transport rather than as a purely geometric correction to planar models. The resulting viewpoint links geometry, non-equilibrium boundary layers, velocity slip, velocity defect, and thermal resistance within a common mathematical framework, providing a basis for improved continuum descriptions of momentum and heat transfer in curved micro- and nanoscale systems.

The research leading to these results has been funded under 2502 joint program between the Scientific and Technological Research Council of Turkey (TUBITAK) and the Bulgarian Academy of Sciences (BAS) with a grant number of 119F279.

Keywords: Micro/Nanofludics, Surface curvature, boundary slip, Knudsen layer, velocity defect, interfacial heat transfer.

REFERENCES

[1] A. Dinler, R.W. Barber, S.K. Stefanov and D.R. Emerson, Curvature dependence of heat transfer at a fluid-solid interface, Physical Review E 98 (2018), 033104.

[2] A. Dinler, R.W. Barber, S.K. Stefanov, D.R. Emerson and K. Orucoglu, Role of surface shape on boundary slip and velocity defect, Physical Review E 86 (2012), 016314.

[3] A. Dinler, R.W. Barber, S.K. Stefanov, D.R. Emerson and K. Orucoglu, On the degree of boundary slip over nonplanar surfaces, Microfluidics and Nanofluidics 15 (2013), 807–816.

[4] A. Dinler, Heat transfer at the convex fluid-solid interface, Journal of Applied Physics 121 (2017), 124302.

Effect of Immigration on the Stability of Leslie-Type Prey-Predator Systems

Deniz ELMACI [1] , **Figen KANGALGİL** [2]
1 Bergama Vocational School, Dokuz Eylül University, İzmir, 35700, Turkey, deniz.elmaci@deu.edu.tr
2 Bergama Vocational School, Dokuz Eylül University, İzmir, 35700, Turkey, figen.kangalgil@deu.edu.tr

ABSTRACT

Mathematical models have long served as indispensable tools for understanding the mechanisms governing complex biological and ecological systems. Discrete-time models are well suited for describing populations with non-overlapping generations and have attracted considerable attention owing to their ability to exhibit rich dynamical behaviours that are not always observed in continuous-time formulations. Such models provide an effective model for investigating population persistence, species coexistence, and the emergence of complex dynamics in ecological interactions.

Prey-predator models constitute one of the most important classes of ecological models and have been extensively employed to analyse interactions among species sharing the same environment. The population dynamics of these systems are influenced by both intrinsic biological characteristics and extrinsic environmental factors. Among the latter, immigration plays a crucial role in shaping population distributions, altering species interactions, and affecting long-term ecological outcomes.

In this study, the dynamical effects of immigration are investigated within a Leslie-Gower prey-predator model. The continuous-time system is discretized using the Forward Euler method to obtain its discrete-time counterpart. The existence of co-existence fixed points is established, and their local stability properties are analysed through analytical techniques and topological classification. Furthermore, numerical investigations are carried out using Maple and MATLAB to illustrate the theoretical results and explore the model's dynamical behaviour under different parameter regimes. The obtained results demonstrate how immigration influences the stability of the coexistence state and contributes to the emergence of various dynamical patterns in the discrete system. These findings provide a deeper understanding of the role of immigration in prey-predator interactions and offer useful insights into the long-term behaviour of ecological populations.

Keywords: Leslie-Gower prey – predator model, immigration, bifurcation analysis.

REFERENCES

[1] A. J. Lotka, Elements of Physical Biology, Baltimore: Williams and Willkins, 1925.
[2] V. Volterra, Variazioni e fluttazionidel numero di indivui in species animali conviventi Memorie Accademia Nazionaledei Lincei, 2:31-313, 1926.
[3] L. J. Allen, An Introduction to Mathematical Biology Pearson Prentice Hall. Upper Saddle River, New Jersey, 2007.
[4] Y. A. Kuznetsov, Elements of Applied Bifurcation Theory, Springer-Verlag, New York, NY, USA, 4th edition, 2023.

[5] F. Kangalgil, & S. Işik, Effect of immigration in a predator-prey system: Stability, bifurcation and chaos. AIMS Math, 7(8), 14354-14375, 2022.

[6] C. Köme, & Y. Yazlik, Stability, bifurcation analysis and chaos control in a discrete predator–prey system incorporating prey immigration. Journal of Applied Mathematics and Computing, 70(5), 5213-5247, 2024.

[7] N. Zhang, F. Chen, Q. Su, & T. Wu, Dynamic Behaviors of a Harvesting Leslie-Gower Predator-Prey Model. Discrete Dynamics in Nature and Society, 2011(1), 473949, 2011.

[8] S. Işık, F. Kangalgil, M. J. Uddin, S. M. S. Rana, An investigation of the discrete-time model subject to immigration, harvesting, and Allee effect. International Journal of Bifurcation and Chaos, 35(02), 2550022, 2025

Effects of Surface Charge on DNA Sensing in Biological Nanopores

Durdane Yilmaz[1], Kaan Kececi[2], Ali Dinler[3]
[1]Beykoz University, Department of Engineering, Istanbul, Türkiye
[2]Istanbul Medeniyet University, Department of Chemistry, Istanbul, Türkiye
[3]Istanbul Medeniyet University, Department of Mathematics, Istanbul, Türkiye

ABSTRACT

Nanopores are single-molecule sensing platforms whose performance depends on the coupling between pore geometry, surface charge, and electrokinetic transport [1]. Although surface-charge engineering through mutation is widely used to enhance sensing performance, its quantitative effect across different nanopore architectures has not been examined within a unified mathematical framework [2]. In addition, previous studies suggest the inclusion of geometry-sensitive hydrodynamic considerations attributed to the boundary, when describing flow fields in complex nanopore structures [3,4]. In this study, we present a continuum electrokinetic model, based on the coupled Poisson–Nernst–Planck–Navier–Stokes (PNP–NS) system of partial differential equations, to quantify how surface charge density (SCD) governs ionic current blockade in four biological nanopores on translocation of DNA molecules [5].

Our numerical solutions reveal a threshold surface charge regime, common to all four geometries, in which the normalized blockade ratio ($\Delta I/I_0$) increases monotonically with surface charge density only up to a pore-dependent critical value, beyond which the electrostatic contribution to blockade weakens. AeL consistently yields the highest normalized blockade among the four pores, followed by MspA, CsgG, and α-HL, and this ranking is invariant across all three particle configurations. Comparing uniform and locally applied surface charge shows that, within the parameter range studied, the magnitude of the charge density influences the blockade signal more strongly than its spatial location, while folded ssDNA consistently produces the deepest current blockade among the analytes considered. At an intermediate applied potential of 180 mV, AeL produces approximately 1.5-fold higher normalized blockade than MspA and 1.5- to 6-fold higher blockade than α-HL, depending on the translocation scenario, while α-HL displays a distinct threshold response relative to the other three pores, underscoring the pore-dependent nature of the surface-charge–blockade relationship.

These results establish a quantitative, PDE-based framework linking surface charge density, pore geometry, and analyte configuration to ionic current modulation, offering a mathematically grounded design principle for optimizing mutation-based biological nanopore sensors. By showing that blockade performance is governed jointly by a pore-specific charge threshold and by particle geometry rather than by surface charge magnitude alone, the framework provides practical guidance for selecting and engineering nanopore-analyte combinations in resistive-pulse sensing applications, including single-molecule DNA sequencing and protein characterization.

Keywords: Resistive Pulse Sensing, Finite Element Method, Biological Nanopores, DNA Sequencing, Surface Charge Density

REFERENCES

[1] K. Kececi and A. Dinler, Understanding the characteristics of small DNA detection and effect of electrolyte on translocation performance in nanopore sensors, ChemNanoMat 12 (2026), e202500701.

[2] D. Yilmaz, D. Kaya, K. Kececi and A. Dinler, Role of nanopore geometry in particle resolution by resistive-pulse sensing, ChemistrySelect 6 (2021), 59–67.

[3] A. Dinler, R.W. Barber, D.R. Emerson, S.K. Stefanov and K. Orucoglu, Role of surface shape on boundary slip and velocity defect, Physical Review E—Statistical, Nonlinear, and Soft Matter Physics 86 (2012), 016314.

[4] A. Dinler, R.W. Barber, D.R. Emerson, S.K. Stefanov and K. Orucoglu, On the degree of boundary slip over nonplanar surfaces, Microfluidics and Nanofluidics 15 (2013), 807–816.

[5] A. Dinler, K. Kececi, D. Kaya and D. Yılmaz, Investigating the effect of charged components on translocation of DNA molecules in track-etched nanopores, IEEE Sensors Journal 20 (2020), 4041–4046.

Estimates of the First Eigenvalue of the Schrödinger Equation with a Singularity at Zero

İlyas Haşimoğlu
Karabük University, Faculty of Business, Department of Business Administration, Turkey
i.hasimoglu@karabuk.edu.tr

ABSTRACT

In this study, we investigate estimates of the first eigenvalue of the following Schrödinger equation with Drichlet conditions

$$u'' - \frac{\nu^2 - \frac{1}{4}}{x^2} u - Q(x) + \lambda u = 0, \ x \in (0,1), \quad (1)$$

$$u(0) = u(1) = 0 \quad (2)$$

where the function $Q(x)$ is a real locally integrable function on the interval (0,1) for which the integral condition

$$\int_0^1 x^\alpha (1-x)^\beta (Q(x))^\gamma \, dx = 1, \qquad \alpha, \beta, \gamma \in R \text{ and } \gamma \neq 0 \qquad (3)$$

is satisfied. The set of such functions Q is denoted by $T_{\alpha,\beta,\gamma}$. Some estimates of $m_{\alpha,\beta,\gamma} = \inf_{Q \in T_{\alpha,\beta,\gamma}} \lambda_1(Q)$ were obtained for different values of the parameters of the integral condition (3).

1) If $\gamma > 0$, then $m_{\alpha,\beta,\gamma} = j_{\nu,1}^2$
2) If $\gamma < 0$ and $\alpha, \beta > 0$, then $m_{\alpha,\beta,\gamma} \leq 4\pi^2\nu^2 + 1$
3) If $\gamma < 0$ and $2\gamma - 1 < \alpha < 0 \leq \beta$, then $m_{\alpha,\beta,\gamma} \leq 4\pi^2((\alpha - 2\gamma + 1)^{\frac{1}{\gamma}} + \nu^2)$
4) If $\gamma < 0$ and $2\gamma - 1 < \beta < 0 \leq \alpha$, then $m_{\alpha,\beta,\gamma} \leq 4\pi^2((\beta - 2\gamma + 1)^{\frac{1}{\gamma}} + \nu^2)$

Previously, in [1], the regularized trace of equations (1) - (2) was investigated

Keywords: Differential equations, Eigenvalues, Schrödinger equation, Drichlet conditions

REFERENCES

[1] HASIMOGLU ILYAS (2012). Regularized trace of Sturm Liouville equation with singularity on a bounded segment. Commun.Fac.Sci.Univ.Ank., 61(1), 1-9., Doi: 10.3970/cmes.2012.085.367

Hybrid Orbit Design in Earth-Moon Missions: Patched Conics and CR3BP Manifolds Combined with Differential Correctors

Beliz Özcan [1], Abdulhamit Küçükaslan [2]
1 Ankara Yıldırım Beyazıt University, 06010 Ankara, Turkiye, andac0685@gmail.com
2 Department of Mathematics, Ankara Yıldırım Beyazıt University, 06010 Ankara, Turkiye, a.kucukaslan@aybu.edu.tr

ABSTRACT

In this study, traditional high-energy methods and low-energy routes based on phase space topology in Earth-to-Moon orbital transfers are comparatively analyzed, and a hybrid orbital architecture combining the advantages of these two different dynamics is presented. In the first stage, the patched-conics and Hohmann transfer approach, which are considered standard in the literature, are modeled. It is shown through simulations that this classical method provides an extremely fast mission duration by minimizing the time of flight (such as 3-5 days); however, it requires a massive braking propellant (V), especially in the lunar sphere of influence (LOI), to insert the spacecraft into orbit. In the second stage, the study transitions to the Circular Restricted Three-Body Problem (CR3BP) framework, which harbors the chaotic structure of the system, and analyzes the invariant manifolds around the Lagrange points (L1, L2). It is determined that these manifold-based gravitational highways (Low-Energy Transfers) provide highly significant savings by bringing the propellant cost theoretically close to zero, but create a severe disadvantage in terms of time by increasing the mission duration to the order of months.

As the main contribution of the study, a new targeting mechanism is developed in order to overcome this fundamental time-propellant trade-off (TOF vs. V) in astrodynamics. This hybrid architecture, which enables the spacecraft to depart from Earth with a traditional, powerful, and fast burn and smoothly attach to the L1 stable manifold instead of going directly to the Moon, is formulated as a deterministic Boundary Value Problem (BVP). In the generation of the hybrid trajectory, instead of evolutionary optimization algorithms that perform random searches, the State Transition Matrix (STM) and Newton-Raphson based Differential Correctors are used. As a result, the acceleration and speed advantage provided by the Hohmann route and the fuel efficiency offered by the manifold dynamics are blended in the phase space with purely mathematical precision.

Keywords: Three body problem, Hohmann transfer, manifold.

REFERENCES

[1] V. Szebehely, Theory of Orbits: The Restricted Problem of Three Bodies. Academic Press, 1967.

[2] H. D. Curtis, (2020). Orbital Mechanics for Engineering Students (4th ed.) Butterworth-Heinemann.

[3] Lo, M. W., & Ross, S. D. (1998). Low energy interplanetary transfers using invariant manifolds of L1, L2 and halo orbits. AAS/AIAA Space Flight Mechanics Meeting.

If Nature Remembers the Past, Why Should Our Mathematical Models Be Forgetful?

Nilay AKGÖNÜLLÜ PİRİM [1]
1 Baskent University, Kahramankazan Vocational School Kahramankazan 06980, Ankara, Turkey, napirim@baskent.edu.tr

ABSTRACT

Fractional-order differential equations have emerged as powerful tools for modeling real-world phenomena due to their inherent ability to capture memory and hereditary properties. In contrast to classical integer-order models, fractional systems provide a more realistic framework for describing biological interactions, particularly in ecological dynamics such as prey–predator systems.

In this study, a fractional-order prey–predator model is considered to better represent the memory-dependent behavior observed in natural ecosystems. The system is formulated using Caputo fractional derivatives, which are widely preferred for their compatibility with initial conditions in physical problems. To obtain approximate solutions of the proposed system, the Hermite Collocation Method is employed due to its efficiency and high accuracy in handling nonlinear fractional differential equations.

This study highlights the importance of incorporating memory effects into mathematical models and confirms that fractional-order approaches, combined with efficient numerical techniques, offer significant advantages over classical models in representing complex biological systems.

Keywords: Fractional-order differential equations, Prey–predator model,Hermite collocation methods, Memory effect

REFERENCES

[1] I. Podlubny, (1999). Fractional Differential Equations. Academic Press.
[2] K. Diethelm, (2010). The Analysis of Fractional Differential Equations. Springer.
[3] Miller, K. S., & Ross, B. (1993). An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley.
[4] N. A. Pirim, F. Ayaz, Hermite collocation method for fractional order differential equations. An International Journal of Optimization and Control 8 (2), 228-236
[5] N. A. Pirim, F.Ayaz, A new technique for solving fractional order systems: Hermite collocation method,Applied Mathematics, 7(18), 2307, 2016.

Lyapunov-Based Global Stability Analysis of the Endemic Equilibrium in a Reduced SIRD Model for Crimean–Congo Hemorrhagic Fever

Yusuf Gürsoy [1], Nilüfer Topsakal [2], Figen Kangalgil [3]
1 Department of Mathematics, Institute of Science,Sivas Cumhuriyet, Sivas, Türkiye , yusufgursoy@cumhuriyet.edu.tr
2 Department of Mathematics, Faculty of Science, Sivas Cumhuriyet, Sivas, Türkiye, ntopsakal@cumhuriyet.edu.tr
3 Bergama Vocational School ,Dokuz Eylul, Izmir, Türkiye, figen.kangalgil@deu.edu.tr

ABSTRACT

Crimean–Congo Hemorrhagic Fever (CCHF) is a significant tick-borne viral disease that poses a considerable threat to public health, particularly in endemic regions. Mathematical epidemic models provide a robust framework for comprehending disease transmission mechanisms and predicting their long-term epidemiological behaviour. The present study investigates the global asymptotic stability of the endemic equilibrium of a reduced $SIRD$ epidemic model, which describes the transmission dynamics of CCHF. The reduced model is obtained through dimensional reduction of the original $SIRD$ system while preserving its epidemiological characteristics. In accordance with the previously determined endemic equilibrium and the basic reproduction number, a suitable Lyapunov function is constructed for the purpose of analysing the qualitative behaviour of the system. The time derivative of the proposed Lyapunov function is derived by employing equilibrium identities in conjunction with suitable algebraic manipulations. It is demonstrated that this derivative is negative definite in the biologically feasible region for all values of R_0.

Consequently, according to Lyapunov's direct method, the endemic equilibrium is proved to be globally asymptotically stable. The obtained result guarantees that all trajectories initiated in the feasible region converge to the endemic equilibrium, indicating the long-term persistence of the disease under endemic conditions. The proposed analysis is intended to complement the existing qualitative results for the considered model by establishing the global behaviour of the endemic equilibrium. Furthermore, it provides a rigorous mathematical framework for studying the long-term dynamics of Crimean–Congo Hemorrhagic Fever.

Keywords: Crimean–Congo Hemorrhagic Fever Endemic Equilibrium, Lyapunov function, global asymptotic stability.

This work was supported by the Scientific Research Projects Coordination Unit of Sivas Cumhuriyet University under Doctoral Thesis Project No. F-2025-735.

REFERENCES

[1] Ö. Ergönül, Crimean-Congo haemorrhagic fever, Lancet Infect. Dis. 6 (2006), 203–214.

[2] T. Gunes, A. Engin, O. Poyraz, N. Elaldi and S. Kaya, Crimean-Congo hemorrhagic fever virus in ticks collected from humans, livestock, and the environment in Turkey, Vector-Borne Zoonotic Dis. 9 (2009), 653–661.

[3] H. Mohammadi, M.K.A. Kaabar, J. Alzabut, A.G.M. Selvam and S. Rezapour, A complete model of Crimean-Congo hemorrhagic fever (CCHF) transmission cycle with nonlocal fractional derivative, J. Funct. Spaces (2021), Article ID 1273405.

[4] H. Alnafisah and M.A. Sohaly, A comprehensive mathematical framework for epidemic dynamics: Stability, stochastic effects, and vaccination strategies based on the SIR model, J. Low Freq. Noise Vib. Act. Control (2026), Advance online publication. doi:10.1177/14613484251382403.

[5] Y. Gürsoy, N. Topsakal and F. Kangalgil, The Mathematical Modelling of Crimean Congo Haemorrhagic Fever Disease: An Approach Based on SIRD Model, Erzincan Univ. J. Sci. Technol. 19 (2026), 357–378. doi:10.18185/erzifbed.1727278.

[6] H.K. Khalil, Nonlinear Systems, 3rd ed., Prentice Hall, Upper Saddle River, NJ, 2002.

[7] M. Martcheva, An Introduction to Mathematical Epidemiology, Springer, New York, 2015.

[8] T. Burtur, A. Zewdie and T. Ayele, Mathematical modeling and analysis for the dynamics of Crimean-Congo hemorrhagic fever in Ethiopia, 2025.

Mateverse.net: Interactive Educational Web Page Containing Calculus Topics Using MathJax

Cenk Alp Arslan, Onur Alp Bahadır, Cüneyt Çevik
Department of Mathematics, Faculty of Science, Gazi University, Ankara, Turkey
e-mail: calp.arslan1@gazi.edu.tr, oalp.bahadir@gazi.edu.tr, ccevik@gazi.edu.tr

ABSTRACT

The foundation of mathematical knowledge and skills holds significant importance, particularly in university-level academic studies. Calculus stands out as a critical discipline that develops mathematical thinking and problem-solving abilities. However, it is often perceived by students as an abstract and complex field. This project aims to contribute to teaching calculus in a more accessible and effective manner by developing an interactive educational web page using MathJax technology. The primary objective is to present fundamental calculus topics, such as limits, derivatives, integrals, and series, interactively and visually. MathJax, a library that effectively displays mathematical notations on web pages, enhances the accessibility and sharing of mathematical content on the web (Cervone, 2012). It will be utilized to provide high-quality and comprehensible presentations of mathematical expressions and graphs. The web platform will offer short explanations, reinforcing examples, applied problems, and step-by-step dynamic solutions to help students effectively learn both theory and practice. By making mathematical expressions interactive, MathJax provides students with the opportunity to develop their problem-solving skills and visualize complex topics in real time (Bray & Tangney, 2017). Furthermore, interactive graphs and visualization tools will be employed to facilitate the visual representation of functions, limits, and derivatives, reducing student misconceptions and confusion. Previous studies emphasize that interactive learning tools have significant positive effects on student achievement in educational settings (Hillmayr & Ziernwald, 2020). This study is designed using a mixed-methods research approach that incorporates both qualitative and quantitative data. The development phase involves a comprehensive literature review, content creation, and web development using HTML, CSS, and JavaScript. Ultimately, this project aims to increase students' mathematical competencies and support their academic success by making calculus topics more understandable and interactive through the integration of modern educational technologies.

Keywords: MathJax, calculus, interactive education, mathematical notations.

REFERENCES

[1] D. Cervone, MathJax: a platform for mathematics on the Web, Notices of the AMS 59 (2012), 312-316.

[2] A. Bray and B. Tangney, Technology usage in mathematics education research – A systematic review of recent trends, Computers & Education 114 (2017), 255-273.

[3] D. Hillmayr, L. Ziernwald, F. Reinhold, S. I. Hofer, and K. M. Reiss, The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis, Computers & Education 153 (2020), 103897.

Neural Network-Based Numerical Analysis of Fractional-Order Glucose-Insulin Dynamics

Hasan Bulut [1], Şeyma Firdevs Hızal [1] and Funda Türk[2]
1 Department of Mathematics, Faculty of Science, Fırat University, Elazığ, Türkiye;
hbulut@firat.edu.tr; sfkorkmaz@firat.edu.tr
2 Department of Mathematics, Faculty of Science, Bartın University, Bartın, Türkiye;
fturk@bartin.edu.tr

ABSTRACT

Glucose–insulin regulation is a complex physiological process that requires the simultaneous consideration of nonlinear interactions among pancreatic β-cells and various hormonal and metabolic components. Although classical integer-order differential models provide an important framework for describing these interactions, they may be limited in representing history-dependent behavior and memory effects inherent in biological systems. In this study, a fractional-order glucose–insulin model incorporating the interactions among β-cells, insulin, glucose, and growth hormone is considered to investigate the underlying system Dynamics numerically. The Caputo fractional derivative is employed to account for memory effects within the model.

To obtain approximate solutions of the nonlinear fractional system, a computational approach based on a Bayesian regularization neural network is employed. The designed network consists of a single hidden layer with 25 neurons and a log-sigmoid activation function, and the model solutions are evaluated for different fractional-order values. The accuracy of the proposed approach is assessed by comparing the neural network predictions with the reference solutions and by examining several error and performance measures. The numerical results demonstrate that the proposed neural network approach can accurately approximate the dynamic behavior of the model variables and provide stable solutions for different fractional orders. Furthermore, variations in the fractional-order parameter noticeably affect both the system's numerical behavior and solution accuracy. These findings indicate that Bayesian regularization neural networks provide an effective and reliable computational framework for the numerical analysis of fractional-order glucose-insulin systems involving memory effects.

Keywords: Fractional-order glucose–insulin model, Caputo derivative, Bayesian regularization neural network, numerical approximation

REFERENCES

[1] M. Farman, A. Ahmad, A. Zehra, K.S. Nisar, E. Hincal and A. Akgul, Analysis and controllability of diabetes model for experimental data by using fractional operator, *Mathematics and Computers in Simulation* 218 (2024), 133–148.

[2] K.S. Nisar and M. Farman, Investigation of fractional order model for glucose-insulin monitoring with PID and controllability, *Scientific Reports* 15(1) (2025), 8128.

[3] M. Alhazmi, Comparing the numerical solution of fractional glucose–insulin systems using generalized Euler method in sense of Caputo, Caputo–Fabrizio and Atangana–Baleanu, *Symmetry* 16(7) (2024), 919.

[4] B. Souayeh et al., A stochastic Bayesian regularization approach for the fractional food chain supply system with Allee effects, *Fractal and Fractional* 6(10) (2022), 553.

On Rich Dynamical Behaviours of a Commensal-Type Symbiosis Model Incorporating Memory Effects

Seval IŞIK [1]
1 Sivas Cumhuriyet University, Department of Mathematics and Science Education, Faculty of Education, 58140, Sivas, Turkey, skaracan@cumhuriyet.edu.tr

ABSTRACT

Populations, as dynamic components of one another's environments, coexist in complex interactions that provide advantages or disadvantages for the species involved. Symbiosis in ecological models is a relationship representing the coexistence and survival strategies of different biological species. Among the various types of symbiotic interactions, commensalism attracts attention as a unidirectional relationship in which one species benefits significantly from the association, while the other remains completely unaffected. Thus, mathematically, modelling these relationships provides an in-depth understanding of populations in which at least one species interacts.

Although mathematical models offer profound insights into these relationships, the time formulation used to represent them is important, as continuous-time models may overlook the discrete nature of many ecological systems, leaving a gap in our understanding of how such populations behave across discrete time steps. To address this gap, discretising from continuous-time models to discrete-time models yields a more realistic representation of species with seasonal breeding, non-overlapping generations, and sudden environmental transitions.

The present study explores the qualitative behaviour of a discrete-time commensalism model with a memory effect, derived by discretising its continuous-time counterpart to capture richer, more complex dynamics. Then, the existence and local asymptotic stability of the model's fixed points are analysed. By choosing a bifurcation parameter, it is shown that the model undergoes a bifurcation. Consequently, the coexistence fixed point loses stability, leading to complex dynamical behaviour. Lastly, comprehensive numerical examples are presented to support the analytical results and provide deeper insights into the long-term dynamics of commensal populations.

Keywords: Commensalism model, stability analysis, bifurcation theory.

REFERENCES

[1] U. Sommer, Freshwater and Marine Ecology (Springer), 2023.
[2] A. A. Elsadany and A. E Matouk, Dynamical behaviors of fractional-order Lotka-Volterra predator-prey model and its discretization, J. Appl. Math. Comput. 49, (2015), 269-283.
[3] Y.A. Kuznetsov, Elements of Applied Bifurcation Theory, 4th edition (Springer-Verlag, NY, USA), 2023.
[4] D. Elmacı and F. Kangalgil, Complex dynamics of a discrete prey-predator model exposing to harvesting and Allee effect on the prey species with chaos control, Int. J. Bifurcation and Chaos 34, (2024), 2450114.

[5] S. Kartal, Caputo and conformable fractional order guava model for biological pest control: discretization, stability and bifurcation, Journal of Computational and Nonlinear Dynamics, 18(12), (2023), 121002.

On the Numerical Solution of the Diabetes Model with Conformable Derivative

Şüheda NERSE[1], İnci ÇİLİNGİR SÜNGÜ[2]
1 Ondokuz Mayıs University, Graduate School of Education, Department of Mathematics, Turkiye, shdnrs@gmail.com
2 Ondokuz Mayıs University, Faculty of Education, Department of Education of Mathematics and Sciences, Turkiye, icilingir@omu.edu.tr

ABSTRACT

Nowadays, diabetes is regarded not only as a health problem but also as a mathematical problem. Numerous mathematical models have been developed to better track the progression of the disease. In this study, the PEDC (Healthy individuals– Prediabetes individuals – Diabetics without complications – Diabetics with complications) mathematical model, which explains the progression of diabetes, was examined and extended using the definition of the conformal derivative. The system of equations with conformal derivative definition was numerically solved using the fourth-order Runge-Kutta (RK4) and Heun methods. Time-dependent numerical solutions for the populations of Healthy (P), Prediabetes (E), Diabetics without Complications (D), and Diabetics with Complications (C) were calculated using the model parameters and initial conditions. The solutions obtained for various fractional orders were presented graphically, and numerical results were obtained that allow for a comparison of model behaviour across different fractional orders. The model's fundamental dynamic structure was examined and its equilibrium behaviour evaluated by comparing it with the analytical solution. In addition, the results obtained from the two numerical methods at different fractional orders were presented in tabular form, compared and interpreted. This study demonstrates the applicability of the RK4 and Heun methods in the numerical solution of diabetes equations containing conformable fractional derivatives and provides a computational framework for comparing solutions obtained at various fractional orders.

Keywords: Diabetes model, Conformable derivative, Runge Kutta Method, Heun Method.

REFERENCES

[1] Boutayeb, W., Lamlili, M. E., Boutayeb, A., & Derouich, M. (2016, April). The dynamics of a population of healthy people, pre-diabetics and diabetics with and without complications with optimal control. In Proceedings of the Mediterranean Conference on Information & Communication Technologies 2015: MedCT 2015 Volume 1 (pp. 463-471). Cham: Springer International Publishing.

[2] Boutayeb, A., & Chetouani, A. (2006). A critical review of mathematical models and data used in diabetology. Biomedical engineering online, 5(1), 43.

[3] Vanitha, R., & Porchelvi, R. (2017). A linear population model for diabetes mellitus. Bulletin of Pure and Applied Sciences-Mathematics and Statistics, 36(2), 311-315.

Oscillation of Second-Order Damped Differential Equations With a Mixed Neutral Term

Ümmügülsüm Dal[1], Orhan Özdemir[2]
1 Graduate Education Institute, Tokat Gaziosmanpaşa University,
60240, Tokat, Türkiye. email: ummugulsumdal@yahoo.com
2 Department of Mathematics, Faculty of Arts and Sciences, Tokat Gaziosmanpaşa University,
60240, Tokat, Türkiye. email: orhanozdemir37@yahoo.com

ABSTRACT

Oscillation theory has long been an active area of research due to its wide range of applications in mathematics, physics, engineering, and the applied sciences. In recent years, increasing attention has been devoted to neutral differential equations because they provide more realistic mathematical models for processes involving after-effects and memory. The presence of damping terms further enriches the mathematical structure of these equations, while also introducing significant analytical challenges. The analysis is further complicated by the simultaneous presence of delayed and advanced arguments in the neutral term and unbounded neutral coefficients.

This presentation investigates the oscillatory behavior of a class of second-order damped differential equations with mixed neutral terms. The main objective is to establish new sufficient conditions guaranteeing that every solution of the considered equations is oscillatory. An oscillation theorem is established by comparing the considered equation with first-order functional differential equations. The applicability of the obtained results is illustrated by an example.

Keywords: Oscillation, neutral differential equations, damping term.

REFERENCES

[1] J.R. Graef, O. Özdemir, A. Kaymaz and E. Tunç, Oscillation of damped second-order linear mixed neutral differential equations, Monatsh. Math. 194 (2021), 85–104.

[2] S. Şahin, Novel oscillation conditions for second-order damped differential equations with bounded and unbounded neutral terms, J. Math. 2025 (2025), Article ID 4504087, 8 pages.

[3] T. Kusano and M. Naito, Comparison theorems for functional differential equations with deviating arguments, J. Math. Soc. Japan 33 (1981), 509–533.

Proofs of Certain Irrationalities Using Differential Equations

İbrahim Adalar
Department of Mathematics, Faculty of Science, Sivas Cumhuriyet University, 58140 Sivas, Türkiye,
iadalar@cumhuriyet.edu.tr

ABSTRACT

In particular, the idea presented in [2] of combining the theory of linear differential equations with constant coefficients with the problem of the irrationality of certain numbers is quite intriguing. Thanks to this connection, proofs of the irrationality of the values of the basic trigonometric functions at nonzero points and of the well-known constants can be provided by a single theorem, without resorting to the results of transcendental theory. In this study, we take this relationship a step further and extend it. To this end, we consider a special Cauchy-Euler differential equation that can be transformed into a differential equation with constant coefficients and whose characteristic equation has complex roots. For a broader class of numbers known to be irrational, we present an alternative proof of their irrationality.

Keywords: Irrationality.

REFERENCES

[1] Havil, J. The Irrationals: A Story of the Numbers You Can't Count On. Princeton University Press. 2023.

[2] Murty, M. R., Murty, V. K. (1977). Irrational numbers arising from certain differential equations. Canadian Mathematical Bulletin, 20(1), 117-120.

[3] Niven, I. Irrational numbers, The Carus Mathematical Monographs, no. 11. The Mathematical Association of America. 1956.

Solving Fully FIVP by Using Granular Differentiability Concept

Tahir Ceylan [1], Ömer Akın [2]
1 Department of Mathematics, Sinop University, Sinop, Türkiye, tceylan@sinop.edu.tr
2 Department of Mathematics, TOBB Economics and Technology University, Ankara, 06560, Türkiye, omerakin@etu.edu.tr

ABSTRACT

In this study, we consider the fuzzy initial value problem of the second order fuzzy differential equations by employing the concepts of horizontal membership function and granular differentiability. Here we examine problems with fuzzy coefficients, fuzzy initial values and fuzzy forcing functions. We compare the granular differentiability approach with different solution methods. We have done this by getting help from Akın et al. (2013). Finally we present some examples describing the proposed method.

Keywords: Keyword one, keyword two, keyword three.

REFERENCES

[1] M. Mazandarani, N. Pariz and A.V. Kamyad, Granular differentiability of fuzzy-number-valued functions, IEEE Trans. Fuzzy Syst. 26 (1) (2018).

[2] Ö. Akın, T. Khaniyev, Ö. Oruç and I.B. Türkşen, An algorithm for the solution of second order fuzzy initial value problems, Expert Systems with Applications 40 (3), (2013), 953-957.

[3] A. Çelikyilmaz, and I.B. Türkşen. (2009). Modeling Uncertainty with Fuzzy Logic: With Recent Theory and Applications, Springer (2009).

[4] H. Yang and Y. Wu, The BVP of a class of second order linear fuzzy differential equations is solved by Green function method under the concept of granular differentiability. Comp. Appl. Math. 43, 293 (2024).

[5] M. Misukoshi, Y. Chalco-Cano, H. Román-Flores, and R.C. Bassanezi, Fuzzy differential equations and the extension principle, Information Sciences, 177, (2007), 3627–3635.

Solving the 0-1 Knapsack Problem Using Binary Dhole Optimization Algorithm

Gülşen Orucova Büyüköz [1], Tuğçem Partal [2]
1 Department of Mathematics and Computer Science, Necmettin Erbakan University, Konya, Turkey, gorucova@erbakan.edu.tr
2 Department of Computer Engineering, Faculty of Engineering, Recep Tayyip Erdoğan University, Rize, Türkiye, tugcem.partal@erdogan.edu.tr

ABSTRACT

The Dhole Optimization Algorithm (DOA) is a recently proposed meta-heuristic inspired by the cooperative hunting behaviour and vocal coordination of dhole packs, and has demonstrated competitive performance in continuous evaluation functions and real-world engineering problems. However, DOA was originally designed for continuous search spaces and cannot be directly applied to binary combinatorial optimization problems. This paper proposes a Binary Dhole Optimization Algorithm (BDOA) that transforms DOA's continuous position-updating mechanism into a binary search space using transfer functions. S-shaped and V-shaped families of transfer functions have been utilised to convert continuous solutions into binary solutions. To address solutions that violate the knapsack capacity constraint, a repair operator has been integrated which removes the items with the lowest profit-to-weight ratio until feasibility is restored. Furthermore, an improvement mechanism has been incorporated into the repair process: for feasible solutions with remaining unused capacity, additional items are greedily added based on their profit-to-weight ratios; thus, the total profit is further increased without violating the capacity constraint. This repair and optimization strategy not only guarantees the feasibility of the solution but also enhances solution quality throughout the search process. The proposed BDOA was evaluated on a series of standard 0-1 knapsack problem instances of varying sizes, and the performance of different transfer function variants was compared in terms of solution quality and computation time. Furthermore, the proposed algorithm has been compared with various algorithms in the literature. The experimental results have demonstrated the competitiveness and effectiveness of the proposed approach in solving binary optimization problems.

Keywords: Dhole optimization algorithm, binary optimization, 0-1 knapsack problem, metaheuristic algorithms, combinatorial optimization.

REFERENCES

[1] B. O. Mohammed, H. S. Aghdasi, and P. Salehpour, Dhole optimization algorithm: a new metaheuristic algorithm for solving optimization problems, Cluster Comput. 28 (2025), 430.
[2] G. B. Dantzig, Discrete-variable extremum problems, Oper. Res. 5 (1957), 266-288.
[3] R. P. Rooderkerk and H. J. van Heerde, Robust optimization of the 0-1 knapsack problem: Balancing risk and return in assortment optimization, Eur. J. Oper. Res. 250 (2016), 842-854

Stability Analysis of a Discrete-Time Predator–Prey Population Model under the Effects of Immigration and Harvesting

Emre Baklacı [1], Nilüfer Topsakal [2], Figen Kangalgil [3]
1 Sivas Cumhuriyet University, Institute of Science, Department of Mathematics, Sivas, Türkiye, emrebaklaci0@gmail.com
2 Sivas Cumhuriyet University, Faculty of Sciences, Department of Mathematics, Sivas, Türkiye, ntopsakal@cumhuriyet.edu.tr
3 Dokuz Eylül University, Bergama Vocational School, Department of Management and Organization, İzmir, Türkiye, figen.kangalgil@deu.edu.tr

ABSTRACT

Population dynamics is one of the major research areas that investigates the temporal evolution of biological systems, and mathematical models describing predator–prey interactions play a significant role in understanding ecological processes. In real-world ecosystems, populations are influenced not only by interspecific interactions but also by various external factors such as immigration, environmental changes, and harvesting activities. Therefore, incorporating immigration and harvesting mechanisms into classical predator–prey models enables a more realistic representation of ecological systems.

In this study, a discrete-time predator–prey population model incorporating the effects of immigration and harvesting is considered, and its dynamical behaviour is investigated in detail. First, the existence of biologically meaningful equilibrium points of the model is examined, and the mathematical conditions required for their existence are established. Subsequently, local stability analysis is carried out for each equilibrium point by means of the Jacobian matrix, and the topological classification of the equilibrium points is performed using the corresponding eigenvalues.

Furthermore, the effects of key parameters, including the immigration rate and harvesting intensity, on the dynamics of the system are analysed, and the influence of variations in these parameters on the stability of the populations is investigated. To verify the theoretical findings, various numerical simulations are performed using the Maple software package. Phase-plane diagrams and trajectory plots are generated for different parameter values, demonstrating a strong agreement between the theoretical analyses and the numerical results.

The obtained results indicate that immigration and harvesting have a decisive influence on the dynamical behaviour of predator–prey populations. The findings of this study contribute to a better understanding of the dynamics of discrete-time ecological models and provide valuable insights for the sustainable management of natural resources and the conservation of biological populations.

Keywords: Stability analysis, predator–prey population model, immigration, harvesting.

REFERENCES

[1] Elaydi, S. N. (2005). An introduction to difference equations (3rd ed.). Springer.

[2] Elaydi, S. (2007). Discrete chaos: With applications in science and engineering (2nd ed.). Chapman & Hall/CRC.

[3] Işık, S. and Kangalgil, F. (2022). On the analysis of stability, bifurcation and chaos control of discrete time predator prey model with Alle effect on predator, Hacettepe J. of Math. 51(2), 404-420.

[4] Kangalgil, F. (2019). Kesik zamanlı Allee etkili bir av–avcı modelinin kararlılığı ve flip çatallanması. Cumhuriyet Science Journal, 40(2), 356–366.

[5] Kangalgil, F., Topsakal, N., Öztürk, N. (2022). Analyzing bifurcation,stability and chaos control for a discrete time prey-predator model with Alle effect. Turkish J. Math, 46(6),2047–2068.

[6] Kangalgil, F., Topsakal, N., Kuzucu, Ö. (2025). Local dynamic analysis ofthe covid-19 mathematical model based on a discrete time prey-predator population model. Turkish J. Math, 49(1),139–156

[7] Kangalgil, F., Uddin, M.D., Rana, S. (2025). An investigation of the discrete-time model subject to immigrationi harvesting, and Alle effect. Int.J.Bifurcation and Chaos in Ap.Sci and En.35(2),

[8] Kılıç, H., Topsakal, N., Kangalgil, F. (2020). Stability analysis of a discrete time prey-predator population model with immigration. Cumhuriyet Science Journal, 41(4), 884–900.

Thermal Analysis of a Moving Longitudinally Porous Fin Immersed in a Nanofluid and Exposed to a Magnetic Field: The Pseudo-Spectral Chebyshev Method

Hüseyin DEMİR [1], İnci ÇİLİNGİR SÜNGÜ[2] , İbrahim KELEŞ[3]
1 Samsun University, Faculty of Engineering and Natural Sciences, Department of Software Engineering, Turkiye, huseyin.demir@samsun.edu.tr
2 Ondokuz Mayıs University, Faculty of Education, Department of Education of Mathematics and Sciences, Turkiye, icilingir@omu.edu.tr
3 Samsun University, Faculty of Engineering and Natural Sciences, Department of Mechanical Engineering, Turkiye, ibrahim.keles@samsun.edu.tr

ABSTRACT

In this study, heat transfer and temperature distribution in a longitudinal fin with a porous structure, moving within a nanofluid medium and subjected to an external magnetic field (MHD), have been theoretically investigated. The energy equation describing the physical problem has been formulated to account for conduction, convection, additional energy losses arising from the porous medium, and the MHD effects caused by the magnetic field. The differential equation was rewritten using appropriate dimensionless variables, and the problem was expressed in terms of the Peclet number (Pe), the magnetic field parameter (M^2), the porosity parameter (S_p) and the convection parameter (N_c). The Pseudo-Spectral Chebyshev Method was chosen to solve the dimensionless equation due to its high accuracy and rapid convergence. Temperature profiles, base heat flux and blade efficiency (η_{fin}) were calculated using spectral derivative matrices generated with Chebyshev–Gauss–Lobatto nodal points. The effects of the Peclet number (Pe), the magnetic field parameter (M^2), the porosity parameter (S_p) and the convection parameter (N_c) on temperature profiles, base heat flux and fin efficiency (η_{fin}) were analysed in detail. The findings indicate that as the Peclet number and magnetic field strength increase, the temperature along the fin drops rapidly, and that fin motion (Pe) plays a dominant enhancing role in heat transfer and efficiency.

Keywords: Moving fin, Porous medium, Nanofluid, MHD, Chebyshev pseudo-spectral method, Fin efficiency.

REFERENCES

[1] S. Kiwan and M. A. Al-Nimr, Using porous fins for heat transfer enhancement. Journal of Heat Transfer, (2001), 123(4), 790–795.

[2] S. U. Choi, Enhancing Thermal Conductivity of Fluids with Nanoparticles, Proceedings of the ASME 1995 International Mechanical Engineering Congress and Exposition. Developments and Applications of Non-Newtonian Flows. San Francisco, California, USA. November 12–17, 1995. pp. 99-105. ASME. https://doi.org/10.1115/IMECE1995-0926

[3] L. N. Trefethen, *Spectral methods in MATLAB*. Society for industrial and applied mathematics, 2000.

Discrete Mathematics and Combinatorics

Basic Properties of Central Günaltılı Graphs

İbrahim GÜNALTILI
Eskişehir Osmangazi University,
Department of Mathematics and Computer Sciences,
igunalti@ogu.edu.tr ORCID:0000-0002-4346-0758

ABSTRACT

A günaltılı graph is a connected graph G=(V , E) property that any two distinct vertices have exactly one common path or none all in G. A central günaltılı graph is a günaltılı graph such that for amy three vertices u, v aad w there is exactly one vertex x that lies simultenously on a shortest (u,v)-path, a shortest (v,w)-path and a shortest (w,u)-path. !t is proved that a central günaltılı graph can be obtained from a one-vertex graph by an expansion procedure. From this characterization some nice properties are derived. The concept of central günaltılı graph is firstly introduced in this article. It was shown that there is a close relation between central günaltılı graph and some at first sight fairly distinct mathematical structures. One of these is a special class of Heliy hypergraphs. This class consists o{ the hypergraphs wiith vertex.-set V and edge-set E C P(V) such that A ϵE iff V\AϵE and É⊂E∩É=ϕ than∃A,BϵÉ: A∩B=ϕ.

In the seque: a structural characterization of central günaltılı graph is given and some nice properties are derived.

Keywords: Günaltılı graph, path, hypergraph

REFERENCES

[1] A.S. Asratian, T.M.J. Denley and R. Hoggkvist, Bipartite graphs and their applications, Cambridge Uni. Press,1998, United Kingdom.

[2] İ. Günaltılı, A. Ulukan, and Ş. Olgun, Some Properties Of Fınıte {0,1}-Graphs , Konualp Journal Of Mathematics,1(1), 2013, 34-39.

[3] İ. Günaltılı, Classification Of Some {0,1}-Semigraphs, İnternation Journal of Innovative Research in Computer Science & Technology,4(1),2016,10-12.

Specific Classes of Directed Graph Groups and Their Structural Properties

Fatih Polat [1], **Mehmet Sefa Cihan**[2]
1 Institute of Natural Science, Sivas Cumhuriyet University, Sivas, Türkiye, fatihpolatt2001@gmail.com
2 Department of Mathematics, Science Faculty, Sivas Cumhuriyet University, Sivas, Türkiye, msefacihan@cumhuriyet.edu.tr

ABSTRACT

This study investigates selected classes of directed graphs associated with group presentations and aims to identify and classify these graph classes as a basis for the subsequent algebraic study of the corresponding groups. Digraph groups arise naturally from group presentations in which each defining relator involves two generators, with the corresponding directed graph encoding the generators and relations of the presentation [1,2]. In previous work, Cihan identified and classified families of directed graphs satisfying (|V(\Gamma)|=|A(\Gamma)|-1), establishing 35 possible cases [3]. Further analysis of leaf-free cases provided additional structural information about these graph classes and their associated groups [4].

Building on these results, the present study focuses on determining particular classes of directed graphs within the considered framework and establishing their defining graph-theoretic characteristics. The selected classes are identified through structural properties of the underlying directed graphs, with the aim of providing a clear and systematic classification that can be used in subsequent work. Rather than focusing directly on the algebraic properties of the associated groups, the main purpose of this study is to establish the necessary graph-theoretic framework for the next stage of the research, in which the corresponding groups will be investigated in terms of their generating sets, defining relations, rank, commutativity, and finiteness. Thus, the results provide a foundation for examining the correspondence between the structural properties of directed graphs and the algebraic properties of the associated groups, and for extending the analysis to broader classes of directed graph groups [3,4].

Keywords: Digraph, Group, Finite Cyclic.

REFERENCES

[1] M. S. Cihan and G. Williams, Finite groups defined by presentations in which each defining relator involves exactly two generators, J. Pure Appl. Algebra 228 (2024), 107499.
[2] M. S. Cihan and G. Williams, Strong digraph groups, Canad. Math. Bull. 67 (2024), 991–1000.
[3] M. S. Cihan, Digraph groups corresponding to digraphs with one more vertex than arcs, Eur. J. Sci. Technol. 41 (2022), 31–35.

Probability, Statistics and Data Science

Mateverse: A Comprehensive, Visualization-Driven Interactive Ecosystem for Calculus Education

Erkan Oğuzcan, Ceyda Şükufe Karahan, Cüneyt Çevik
Department of Mathematics, Faculty of Science, Gazi University, Ankara, Turkey
e-mail: erkan.oguzcan@gazi.edu.tr, csukufe.karahan@gazi.edu.tr, ccevik@gazi.edu.tr

ABSTRACT

The pedagogical transition from abstract mathematical formulation to intuitive spatial comprehension remains a core challenge in undergraduate STEM education. To address this, we present mateverse.net, a comprehensive, web-based digital ecosystem engineered to transform the teaching and learning of Calculus. Encompassing the entire Calculus curriculum, the platform seamlessly integrates rigorous theoretical lectures with an extensive repository of over 1400+ targeted practice problems. Crucially, traditional passive learning is eliminated by systematically pairing virtually every mathematical theorem with custom-built, dynamic visualization models.

A defining feature of the platform is the Mateverse Lab, a dedicated, highly interactive experimental environment where students can actively manipulate and stress-test mathematical structures. The technical architecture of Mateverse leverages a sophisticated modern web stack, combining the precise geometric capabilities of JSXGraph and SVG [1] with the advanced data and 3D rendering functionalities of Three.js [2], Plotly [3], and D3.js [4]. This multi-framework integration allows for real-time computational feedback, enabling students to explore complex concepts—such as Riemann sums, limits, and multidimensional coordinate transformations—not merely as static textbook graphics, but as responsive, interactive mathematical objects.

Optimized with a high-contrast, responsive dark-mode user interface to facilitate prolonged cognitive engagement and cross-platform accessibility, Mateverse.net effectively bridges the gap between analytical abstraction and geometric intuition. This presentation outlines the core architectural frameworks of the platform, demonstrates the interactive capabilities of the Mateverse Lab, and evaluates its potential to redefine modern mathematical pedagogy and independent student engagement.

Keywords: Mateverse, Mateverse Lab, dynamic visualization.

REFERENCES

[1] M. Gerhäuser, B. Valentin and A. Wassermann, JSXGraph-Dynamic Mathematics with JavaScript, International Journal for Technology in Mathematics Education 17(4) (2010).

[2] J. Dirksen, Learning Three. js: the JavaScript 3D library for WebGL (Vol. 3), Packt Publishing, Birmingham, UK, 2013.

[3] C. Sievert, Interactive web-based data visualization with R, plotly, and shiny, Chapman and Hall/CRC, 2020.

[4] M. Bostock, V. Ogievetsky and J. Heer, D^3 data-driven documents, IEEE transactions on visualization and computer graphics 17(12) (2011), 2301-2309.

Mathematics Education

Reflections on Preservice Mathematics Teachers' Interactions with Artificial Intelligence in Problem-Solving

Yasemin Kıymaz [1],
1 Kırşehir Ahi Evran University, Faculty of Education, Department of Mathematics Education
ykiymaz@ahievran.edu.tr

ABSTRACT

Problem solving in mathematics education is a fundamental process that requires students to explore different problem situations and employ a variety of approaches, ranging from algorithmic methods to intuitive strategies [3]. However, although problem solving is emphasized as an important goal in curricula, classroom practices often prioritize discourse about problem solving rather than student-centered and thinking-oriented activities that reflect the true nature of problem solving [3]. Addressing problem solving in classroom practices in accordance with this understanding is largely associated with teachers' knowledge, skills, and experiences related to problem solving. Therefore, in order to support the development of students' problem-solving skills, it is important for teachers to first develop their own problem-solving skills and awareness of their problem-solving processes [2]. Technological tools can be utilized to enable teachers to experience different solution methods, evaluate their own problem-solving processes, and explore alternative approaches. Indeed, rapid technological developments have led to significant transformations in teaching and learning processes, and the emergence of generative artificial intelligence (AI) tools has introduced new opportunities for problem solving in mathematics education. The ability of generative AI tools to produce different solution methods for mathematical problems not only supports the problem-solving process but also provides new opportunities to evaluate, verify, and question the solutions generated [1,4].

In this regard, it is important to examine how preservice teachers' experiences with AI-supported problem solving are shaped and how these experiences are reflected in their own problem-solving thinking. The purpose of this study is to examine preservice teachers' reflections on their experiences with AI in problem solving. The participants consist of 45 preservice teachers enrolled in the Department of Mathematics Education at a university in Central Anatolia who were taking a Problem Solving in Mathematics course. A qualitative research approach was adopted. The preservice teachers were asked to solve a given mathematical problem in as many different ways as possible, then ask AI to solve the same problem and generate different solution methods and engage in conversations with AI about the solutions. At the end of the process, the preservice teachers were asked to evaluate the problem-solving strategies used by AI and the different solutions it generated, explain how their interactions with AI affected their ways of thinking and which of their initial assumptions they subsequently questioned, and express their views on the use of AI in problem-solving instruction in their future teaching practices. The reflective writings obtained from the participants will be analysed through content analysis, and the findings will be discussed considering the relevant literature.

Keywords: Artificial intelligence, mathematics education, problem solving, preservice teachers.

REFERENCES

[1] McGalliard, W., & Otten, S. (2025). AI responses to challenging problems and educator responses to AI availability. Digital Experiences in Mathematics Education, 11(2), 319-332.

[2] Posamentier, A. S., & Krulik, S. (2008). Problem-solving strategies for efficient and elegant solutions, grades 6-12: a resource for the mathematics teacher. Corwin press.

[3] Schoenfeld, A. H. (2016). Learning to think mathematically: Problem-solving, metacognition, and sense making in mathematics. Journal of Education, 196(2), 1-38. https://doi.org/10.1177/002205741619600202

[4] Zhuang, Y. (2026). Empowering students to critically validate AI-generated mathematical solutions through the rational questioning approach. Educational Studies in Mathematics, 1-19.

Digital Game Design and Implementation for Four-Operation Achievement of Students at Risk of Dyscalculia

Tuğba KARGIN [1], Sabiha BAŞEĞMEZ [2], Eyüp ALSANCAK[3]
1 SİVAS BİLSEM, Sivas, Türkiye, tubakargan@gmail.com
2 SİVAS BİLSEM, Sivas, Türkiye, sabihakartalci @gmail.com
3 SİVAS BİLSEM, Sivas, Türkiye, alsancakeyup@gmail.com

ABSTRACT

The aim of this study is to examine the effects of digital game designs on the four-operation achievement of students at risk of dyscalculia. The study also aims to develop digital game designs for teaching the four operations to students at risk of dyscalculia. Dyscalculia is a specific learning difficulty generally used to describe difficulties in calculation skills and is also referred to as arithmetic learning difficulty, mathematical difficulties, calculation disorder, and mathematical disability (Emerson & Babtie, 2014).

A single-subject quasi-experimental research design with a pre-test and post-test was employed to evaluate the effects of digital game designs on the achievement of students at risk of dyscalculia. The participants consisted of five third-grade students attending a primary school. The students were identified based on teacher opinions, a dyscalculia checklist, and a four-operation preliminary assessment test. Digital games focusing on the four operations and aligned with third-grade mathematics curriculum learning outcomes were designed using Wordwall, LearningApps, Educandy, Kahoot, and artificial intelligence-supported Web 2.0 tools. The digital games were implemented through individualized instruction over a period of three weeks, with one session per week, resulting in three sessions. In addition, the students were provided with opportunities to play the games at home under parental supervision for three weeks.

Six achievement tests developed by the researchers in accordance with third-grade mathematics curriculum learning outcomes were used as data collection instruments. In the pre-tests and post-tests, students' numbers of correct answers, response times, and types of errors made in the four operations were examined. The findings showed that while no increase in achievement was observed in two students in addition and subtraction, the other three students demonstrated noticeable improvement. In multiplication, all students showed increases in achievement and speed, and a decrease in multiplication-table errors was observed. In division, increases in achievement and speed were observed in all but one student. The findings indicate that educational digital games can be used as a supportive tool for improving the four-operation skills of students at risk of dyscalculia. This result is also consistent with studies indicating that participation in digitally delivered instructional processes may produce positive outcomes for students experiencing mathematics learning difficulties (Higgins, Crawford, & Silvestri, 2016; Walcott & Romain, 2019).

Keywords: dyscalculia, risk of dyscalculia, digital game design, digital game-based instruction

REFERENCES

[1] Emerson, J., & Babtie, P. (2014). *The dyscalculia assessment*. Bloomsbury Publishing.

[2] Higgins, K., Crawford, L., & Silvestri, S. (2016). Student perceptions of an online mathematics curriculum designed for students with learning difficulties. *Social Welfare: Interdisciplinary Approach, 2*(6), 108–123. https://doi.org/10.21277/sw.v2i6.258

[3] Walcott, P., & Romain, N. (2019). Using digital games to enhance the mathematical skills of children with dyscalculia. *Journal of Education and Development in the Caribbean, 18*(2), 88–110. https://doi.org/10.46425/j418025822

Computer Sciences

An Explainable Deep Learning Approach Based on EfficientNetB3 and Grad-CAM for Enamel Caries Classification in Intraoral Images

Bilal İnan [1], Nurettin Doğan [2], Türkay Kölüş[3]
1 Tokat Gaziosmanpaşa University, Turkey. bilal.inan@gop.edu.tr
2 Selçuk University, Turkey. nurettin.dogan@selcuk.edu.tr
3 İzmir Demokrasi University, Turkey. turkay.kolus@idu.edu.tr

ABSTRACT

Early detection of enamel caries is critically important for preventive dental care [1]. In this study, an explainable deep learning approach based on EfficientNetB3 was developed for the automatic classification of enamel caries using open-access intraoral images [2]. Three classes from the Caries-Spectra dataset were considered: Early-Stage Enamel Caries, Advanced Enamel Caries, and Caries-Free. The dataset was stratified into training (70%), validation (15%), and test (15%) subsets. To improve model generalization and reduce overfitting, basic data augmentation techniques, including rotation, horizontal flipping, zooming, and contrast adjustment, were applied.

An ImageNet-pretrained EfficientNetB3 architecture was employed as the classification model, and the training process was conducted in two stages. In the first stage, only the classification layers were trained, whereas in the second stage, the upper layers of the network were further optimized through fine-tuning using a low learning rate. Model performance was evaluated using accuracy, precision, recall, F1-score, and ROC-AUC metrics. Experimental results showed that the model achieved 92.67% accuracy, 93.15% macro precision, 92.78% macro recall, 92.93% macro F1-score, and 92.69% weighted F1-score on the independent test set. In addition, the model achieved a macro ROC-AUC of 0.9829, indicating a high level of discriminative performance across the classes.

Within the scope of explainable artificial intelligence, the Grad-CAM method was used to visualize class-specific image regions that contributed to the model's classification decisions [3]. Grad-CAM results showed that the model particularly focused on fissure regions and image areas associated with enamel caries, thereby supporting the visual interpretability of its predictions. The findings demonstrate that the combined use of transfer learning and explainable artificial intelligence provides an effective and interpretable approach for enamel caries classification from intraoral images. The proposed system may provide a basis for future computer-aided diagnosis and teledentistry applications.

Keywords: Enamel caries, deep learning, explainable artificial intelligence, EfficientNetB3, Grad-CAM, intraoral image

REFERENCES

[1] Mohammad-Rahimi, H., Motamedian, S. R., Rohban, M. H., Krois, J., Uribe, S., Nia, E. M., Rokhshad, R., Nadimi, M., & Schwendicke, F. (2022). Deep learning for caries detection: A systematic review. *Journal of Dentistry, 122*, 104115.

[2] Kühnisch, J., Meyer, O., Hesenius, M., Hickel, R., & Gruhn, V. (2022). Caries detection on intraoral images using artificial intelligence. *Journal of Dental Research, 101*(2), 158–165.

[3] Selvaraju, R. R., Cogswell, M., Das, A., Vedantam, R., Parikh, D., & Batra, D. (2020). Grad-CAM: Visual explanations from deep networks via gradient-based localization. *International Journal of Computer Vision, 128*(2), 336–359.

Disentangling Persistent Excitation and Near-Critical Amplification in Cyber-Threat Indicator Streams: A Mittag-Leffler Hawkes Framework

Mehmet Sevri[1], Kevser Nur Çoğalmış[2], Tuğçem Partal[3]
[1] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, mehmet.sevri@erdogan.edu.tr
[2] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, kevser.cogalmis@erdogan.edu.tr
[3] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, tugcem.partal@erdogan.edu.tr

ABSTRACT

Threat-intelligence indicator streams arrive in bursts, their dependence decaying slowly across several time scales; Zhan and colleagues [1] observe the same in honeypot records. Over a finite window two quite different self-exciting mechanisms leave that signature. Excitation may decay slowly, so that distant events still carry weight [2]. Or a short-memory process may sustain comparable clustering because its branching ratio sits close to unity. The difference is not cosmetic. The branching ratio governs how strongly the system amplifies itself; the triggering kernel governs how long that amplification persists. An intervention removing one event therefore has a very different expected effect under each.

We estimate them jointly with a self-exciting point process whose triggering density is of Mittag-Leffler type [3]. At unit exponent the kernel collapses to the exponential, so the classical Hawkes process [4] is nested as a boundary case. The nesting is convenient for comparison but awkward for inference: a null on the boundary of the parameter space invalidates the usual chi-square approximation [5], so we calibrate the likelihood ratio by parametric bootstrap. Tractability comes from the spectral representation of the kernel. A finite quadrature of exponentials turns it into a mixture admitting a recursive likelihood, linear in the number of events for a fixed number of components. The branching ratio is constrained below unity throughout, keeping the fitted process subcritical.

The Bartlett spectrum [6] supplies a second, independent line of evidence. Within the near-critical crossover it relates the kernel exponent to the scaling of second-order fluctuations, which multifractal detrended fluctuation analysis [7] estimates without reference to the point-process model. We do not treat this relation as identification. It is a consistency criterion, and it can fail. Data come from the ThreatFox corpus, restricted to one malware family and reduced to deduplicated event times; pooling families would introduce heterogeneity the kernel would absorb as spurious self-excitation. Seasonal baselines, shuffled surrogates, higher-order detrending, sensitivity to aggregation scale and time-rescaling residuals [8] probe nonstationarity, resolution artefacts and model adequacy.

Against the exponential specification the fractional kernel is preferred decisively; the extension is not a marginal refinement of the classical model. The fitted kernel implies a second-order scaling exponent, and the measured one moves towards that value as aggregation coarsens, while shuffled surrogates stay flat throughout. The consistency criterion is therefore met in direction and trend rather than exactly, which we report as

agreement between two methods and not as proof of uniqueness. The answer is not the dichotomy the question invites. Both mechanisms operate: the estimated kernel lies far from the exponential boundary, and the branching ratio still sits close to criticality, closer still when the exponential specification, denied a heavy tail, has to carry the same persistence alone. Neither reading on its own would state the endogenous share correctly. Estimates, comparison statistics and stability diagnostics are reported in the full paper. The framework separates the two at linear cost, with the artefacts that could mimic either one examined rather than assumed away.

Keywords: Hawkes process, Mittag-Leffler kernel, near-criticality, branching ratio, multifractal detrended fluctuation analysis, cyber-threat intelligence.

REFERENCES

[1] Z. Zhan, M. Xu and S. Xu, Characterizing honeypot-captured cyber attacks: statistical framework and case study, IEEE Trans. Inf. Forensics Secur. 8 (2013), 1775-1789. https://doi.org/10.1109/TIFS.2013.2279800

[2] T. Jaisson and M. Rosenbaum, Rough fractional diffusions as scaling limits of nearly unstable heavy tailed Hawkes processes, Ann. Appl. Probab. 26 (2016), 2860-2882. https://doi.org/10.1214/15-AAP1164

[3] J. Chen, A.G. Hawkes and E. Scalas, A fractional Hawkes process, in: Nonlocal and Fractional Operators, SEMA SIMAI Springer Series 26, Springer, Cham, 2021, 121-131. https://doi.org/10.1007/978-3-030-69236-0_7

[4] A.G. Hawkes, Spectra of some self-exciting and mutually exciting point processes, Biometrika 58 (1971), 83-90. https://doi.org/10.1093/biomet/58.1.83

[5] S.G. Self and K.-Y. Liang, Asymptotic properties of maximum likelihood estimators and likelihood ratio tests under nonstandard conditions, J. Amer. Statist. Assoc. 82 (1987), 605-610. https://doi.org/10.1080/01621459.1987.10478472

[6] E. Bacry and J.-F. Muzy, First- and second-order statistics characterization of Hawkes processes and non-parametric estimation, IEEE Trans. Inform. Theory 62 (2016), 2184-2202. https://doi.org/10.1109/TIT.2016.2533397

[7] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde and H.E. Stanley, Multifractal detrended fluctuation analysis of nonstationary time series, Physica A 316 (2002), 87-114. https://doi.org/10.1016/S0378-4371(02)01383-3

[8] Y. Ogata, Statistical models for earthquake occurrences and residual analysis for point processes, J. Amer. Statist. Assoc. 83 (1988), 9-27. https://doi.org/10.1080/01621459.1988.10478560

Inverse Analysis of IoT Malware Spread under Partial Observation: A Physics-Informed Neural Network Approach

Mehmet Sevri[1], Tuğçem Partal[2], Kevser Nur Çoğalmış[3]

[1] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, mehmet.sevri@erdogan.edu.tr

[2] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, tugcem.partal@erdogan.edu.tr

[3] Department of Computer Engineering, Faculty of Engineering and Architecture, Recep Tayyip Erdoğan University, Rize, Türkiye, kevser.cogalmis@erdogan.edu.tr

ABSTRACT

Epidemiological models of computer contagion date from Kephart and White [1]; recent work fits them to observed campaigns [2]. Recovery is the harder problem. Telemetry gives a partial, censored projection of the latent infected population, and propagation intensity shifts across campaign phases, patching cycles and defensive interventions. Inverse estimation is therefore weakly identifiable and ill-conditioned under sparse, noisy data. Our formulation is a susceptible-exposed-infectious-quarantined-recovered-susceptible system carrying an explicit observation operator and a time-varying effective transmission rate.

The autonomous and non-autonomous regimes are treated separately. For constant transmission the next-generation matrix [3] gives the basic reproduction number, and the Routh-Hurwitz criterion the local stability of the malware-free equilibrium. When transmission varies the Jacobian is no longer constant and that criterion fails; the infected subsystem is bounded instead by a cooperative linear comparison system with Metzler generator. The result is a uniform sufficient eradication condition: if the reproduction threshold at an upper bound for transmission stays below unity, every admissible trajectory respecting that bound drives the infected compartments to extinction.

A physics-informed neural network [4] solves both problems. The transmission rate enters as an auxiliary subnetwork, not a prescribed parametric form, which is where least-squares calibration has no form to fit. Three devices handle the multiscale, sharply transient dynamics: positive state parameterisation in logarithmic coordinates, causality-aware weighting of collocation residuals, and cosine-similarity control of observational-versus-physics gradient conflict. Ablation separates their individual and joint effects.

Labelled IoT-23 traffic from the Muhstik, Hajime, Mirai and Okiru families supplies family-specific scanning and contact intensities. Per-host scanning runs from about 1.8x10^3 new targets an hour for Muhstik to 5.7x10^5 for Okiru, a spread of two and a half orders of magnitude. These are not direct estimates of the transmission coefficient; they constrain its contact component, and uncertainty in contact-to-compromise conversion is represented explicitly. Reference trajectories come from a positivity-preserving nonstandard finite-difference scheme, so recovery is graded against known ground truth.

Identifiability is central here, not incidental. Scale-invariance analysis shows that under frequency-dependent incidence the reporting process is confounded only with absolute population scale. Rate constants, the reproduction number, the growth rate and the remediation increase needed to reach the epidemic threshold therefore remain recoverable

and operationally usable. Structural analysis [5] determines which parameter combinations are theoretically identifiable from the observation model, and Fisher-information and profile-likelihood analyses then quantify practical identifiability under finite, noisy, sparse measurements [6], so the two layers of the identifiability question are answered separately. We benchmark against nonlinear least-squares calibration, Monte Carlo estimation [7], neural ordinary differential equations and a conventional physics-informed network [8]. The aim is not only to reconstruct trajectories but to establish which defence-relevant latent quantities can be inferred reliably from realistically incomplete observation.

Keywords: Physics-informed neural networks, malware propagation, compartmental epidemic model, inverse problem, parameter identifiability, partial observation.

REFERENCES

[1] J.O. Kephart and S.R. White, Directed-graph epidemiological models of computer viruses, Proceedings of the 1991 IEEE Computer Society Symposium on Research in Security and Privacy (1991), 343-359. https://doi.org/10.1109/RISP.1991.130801

[2] A. Chernikova, N. Gozzi, N. Perra, S. Boboila, T. Eliassi-Rad and A. Oprea, Modeling self-propagating malware with epidemiological models, Appl. Netw. Sci. 8 (2023), 52. https://doi.org/10.1007/s41109-023-00578-z

[3] P. van den Driessche and J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Math. Biosci. 180 (2002), 29-48. https://doi.org/10.1016/S0025-5564(02)00108-6

[4] M. Raissi, P. Perdikaris and G.E. Karniadakis, Physics-informed neural networks: a deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, J. Comput. Phys. 378 (2019), 686-707. https://doi.org/10.1016/j.jcp.2018.10.045

[5] H. Miao, X. Xia, A.S. Perelson and H. Wu, On identifiability of nonlinear ODE models and applications in viral dynamics, SIAM Rev. 53 (2011), 3-39. https://doi.org/10.1137/090757009

[6] E. Kharazmi, M. Cai, X. Zheng, Z. Zhang, G. Lin and G.E. Karniadakis, Identifiability and predictability of integer- and fractional-order epidemiological models using physics-informed neural networks, Nat. Comput. Sci. 1 (2021), 744-753. https://doi.org/10.1038/s43588-021-00158-0

[7] M. Severt, R. Casado-Vara and Á. Martín del Rey, A comparison of Monte Carlo-based and PINN parameter estimation methods for malware identification in IoT networks, Technologies 11 (2023), 133. https://doi.org/10.3390/technologies11050133

[8] C. Millevoi, D. Pasetto and M. Ferronato, A physics-informed neural network approach for compartmental epidemiological models, PLoS Comput. Biol. 20 (2024), e1012387. https://doi.org/10.1371/journal.pcbi.1012387

Improving Feature Representation in Underwater Acoustic Target Recognition via Nonlinear PSD Quantization and Custom Colormap Encoding

Gülsevin KODALOĞLU [1], Hasan Hüseyin SAYAN [2], Hamdi Tolga KAHRAMAN [3], Yusuf Sönmez
1 Gazi University, Faculty of Technology, Department of Electrical and Electronics Engineering, 06500 Yenimahalle/Ankara, Türkiye — gulsevin.kodaloglu@gazi.edu.tr
[2] Gazi University, Faculty of Technology, Department of Electrical and Electronics Engineering, 06500 Yenimahalle/Ankara, Türkiye — hsayan@gazi.edu.tr
[3] Karadeniz Technical University, Faculty of Engineering, Department of Industrial Engineering, 61080 Trabzon, Türkiye — htolgakahraman@ktu.edu.tr
[4] Gazi University, Faculty of Technology, Department of Computer Engineering, 06500 Yenimahalle/Ankara, Türkiye — ysonmez@gazi.edu.tr

ABSTRACT

In underwater acoustic target recognition (UATR), the time–frequency representation of ship-radiated noise is typically fed to convolutional neural networks (CNNs) using logarithmic (dB) scaling or the raw power spectral density (PSD). However, these conventional scaling approaches fail to adequately balance the dynamic range between the discriminative low-frequency line spectra and the broadband noise floor, thereby limiting inter-class separability. This study proposes a nonlinear quantization function applied to PSD representations obtained via LOFAR-TPSW, followed by a custom colormap encoding. The proposed transformation compresses the dynamic range so as to render weak spectral components visible relative to dominant ones, and maps the quantized values into a discriminative color space suitable as CNN input. The method is evaluated on the DeepShip and ShipsEar datasets through a controlled ablation study against linear and dB scaling, with the network architecture held fixed. Experimental results show that the proposed nonlinear quantization and colormap encoding improves classification accuracy by compared to conventional dB scaling, and yields a marked improvement in the confusion matrix particularly for spectrally similar vessel classes. The findings demonstrate that the choice of representation at the preprocessing level contributes substantially to recognition performance, independent of architectural modifications.

Keywords: Underwater acoustic target recognition, LOFAR, nonlinear quantization, power spectral density, colormap encoding, convolutional neural networks

ACKNOWLEDGEMENT

This work was supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK) under the 1001 Program, Project No. 125E210, entitled "Development of Deep Learning-Based Hydrophone-Integrated Data Recording and Evaluation Software."

REFERENCES

[1] M. Irfan, Z. Jiangbin, S. Ali, M. Iqbal, Z. Masood, and U. Hamid, "DeepShip: An underwater acoustic benchmark dataset and a separable convolution based autoencoder for classification," *Expert Systems with Applications*, vol. 183, art. no. 115270, Nov. 2021.

[2] D. Santos-Domínguez, S. Torres-Guijarro, A. Cardenal-López, and A. Pena-Gimenez, "ShipsEar: An underwater vessel noise database," *Applied Acoustics*, vol. 113, pp. 64–69, Dec. 2016. L. C. F. Domingos, P. E. Santos, P. S. M. Skelton, R. S. A. Brinkworth, and K. Sammut, "An investigation of preprocessing filters and deep learning methods for vessel type classification with underwater acoustic data," IEEE Access, vol. 10, 2022, doi: 10.1109/ACCESS.2022.3220265.

[3] W. Gao, Y. Liu, and D. Chen, "A dual-stream deep learning-based acoustic denoising model to enhance underwater information perception," Remote Sensing, vol. 16, art. no. 3325, 2024, doi: 10.3390/rs16173325.

[4] W. Xu, X. Han, Y. Zhao, L. Wang, C. Jia, S. Feng, J. Han, and L. Zhang, "Research on underwater acoustic target recognition based on a 3D fusion feature joint neural network," Journal of Marine Science and Engineering, vol. 12, art. no. 2063, 2024, doi: 10.3390/jmse12112063.

[5] N. Müller, J. Reermann, and T. Meisen, "Navigating the depths: A comprehensive survey of deep learning for passive underwater acoustic target recognition," IEEE Access, vol. 12, 2024, doi: 10.1109/ACCESS.2024.3480788.

Route and Resource Optimization in UAV-RIS Assisted 6G Wireless Networks: A Comparative Study on Population-Based Metaheuristic Algorithms

Furkan Üstünsoy[1], Sadık Yıldız[2] and H. Hüseyin Sayan[3]
1 National Defence University, Turkish Military Academy, Electronics and Communication Engineering Department, Ankara, Turkey, (e-mail: ustunsoy.furkan@gmail.com)
2 Ministry of National Education, Ankara, Turkey, (e-mail: sadikyildiz06@gmail.com)
3 Department of Electrical and Electronics Engineering, Gazi University, Ankara, Turkey, (e-mail: hsayan@gazi.edu.tr)

ABSTRACT

In recent years, researchers have been investigating the use of unmanned aerial vehicles (UAVs) and reconfigurable intelligent surface (RIS) technologies to meet the requirements of high speed, low data latency, energy efficiency, and coverage in sixth-generation (6G) wireless communication systems [1,2]. The three-dimensional movement capability of UAVs has the potential to enhance the performance of wireless networks to a considerable degree [3,4]. Furthermore, the ability of RISs to improve wireless channel conditions also contributes to performance enhancement [2,5]. The combination of these two features has been shown to result in significant performance gains, particularly in dense and dynamic communication environments [6,7]. The model presented in this paper is designed to optimise the UAV trajectory, the user-allocated transmission power, the bandwidth, and the RIS phase configurations simultaneously [7]. Consequently, the designed model has become a high-dimensional, nonlinear, and multivariable constrained engineering problem. However, it is important to note that numerous physical and communication requirements, including the speed limit of UAVs, the operational area, the minimum safety distance between UAVs, the total power and bandwidth, and the minimum user data rate, are also modelled as constraints in the problem [3-7]. In this study, a comprehensive problem model with a multidimensional (dimension=604) decision space has been developed for the optimisation of UAV-RIS supported 6G communication systems. The aforementioned decision vector encompasses three-dimensional UAV trajectories, user-based power allocation, bandwidth allocation, and four-level discrete RIS phase codes. The objective function is designed as a single weighted objective, with constraints incorporated into the objective function through a penalty mechanism. This enables the algorithms to search for both feasible and infeasible solutions within their lifecycle without directly interfering with the algorithm's natural search behaviour. In order to investigate the solution to the proposed constrained engineering problem, the Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), and L-SHADE algorithms were tested separately. In the experimental study, the performance of the algorithms in terms of solution quality, convergence performance, solution performance, constraint satisfaction rates, constraint violation levels, and computation times is examined in conjunction. In conclusion, this study provides a comparative framework that comprehensively evaluates the effectiveness of different population-based metaheuristic algorithms for the high-dimensional constrained UAV-RIS optimization problem and establishes a framework for determining the appropriate optimization strategy for 6G communication systems.

Keywords: UAV-RIS assisted communications, 6G communication, metaheuristic optimization.

REFERENCES

[1] N. A. El-Alfi, E. El-Sayed, H. M. Abdel-Atty and M. A. Mohamed, "Reconfigurable Intelligent Surfaces (RIS) and AI for Sustainable 6G Smart Cities: A Hybrid Framework," *2025 International Conference on Future Telecommunications and Artificial Intelligence (IC-FTAI)*, Alexandria, Egypt, 2025, pp. 1-7, doi: 10.1109/IC-FTAI67960.2025.11384579.

[2] X. Zhao, M. Jian, Y. Chen, Y. Zhao and L. Mu, "Reconfigurable Intelligent Surfaces for 6G: Engineering Challenges and the Road Ahead," in *Intelligent and Converged Networks*, vol. 6, no. 1, pp. 53-81, March 2025, doi: 10.23919/ICN.2025.0004.

[3] Y. Zeng, R. Zhang and T. J. Lim, "Wireless communications with unmanned aerial vehicles: opportunities and challenges," in *IEEE Communications Magazine*, vol. 54, no. 5, pp. 36-42, May 2016, doi: 10.1109/MCOM.2016.7470933.

[4] A. Al-Hourani, S. Kandeepan and S. Lardner, "Optimal LAP Altitude for Maximum Coverage," in *IEEE Wireless Communications Letters*, vol. 3, no. 6, pp. 569-572, Dec. 2014, doi: 10.1109/LWC.2014.2342736.

[5] Y. Zhu, B. Mao and N. Kato, "Intelligent Reflecting Surface in 6G Vehicular Communications: A Survey," in *IEEE Open Journal of Vehicular Technology*, vol. 3, pp. 266-277, 2022, doi: 10.1109/OJVT.2022.3177253.

[6] Mohsan SAH, Khan MA, Alsharif MH, Uthansakul P, Solyman AAA. Intelligent Reflecting Surfaces Assisted UAV Communications for Massive Networks: Current Trends, Challenges, and Research Directions. Sensors (Basel). 2022 Jul 14;22(14):5278. doi: 10.3390/s22145278.

[7] S. Li, B. Duo, X. Yuan, Y. -C. Liang and M. Di Renzo, "Reconfigurable Intelligent Surface Assisted UAV Communication: Joint Trajectory Design and Passive Beamforming," in *IEEE Wireless Communications Letters*, vol. 9, no. 5, pp. 716-720, May 2020, doi: 10.1109/LWC.2020.2966705.

ABSTRACTS OF ONLINE PRESENTATIONS

Analysis and Theory of Functions

Computer Sciences

Preserving the Completeness of Blockchain-Anchored Digital Evidence under Ledger Failures

Mehmet Meral [1], **Hasan Hüseyin Sayan** [2]
1 Canadian Institute of Technology, Tirana, Albania, mehmet.meral@cit.edu.al
2 Gazi University, Ankara, Türkiye, hsayan@gazi.edu.tr

ABSTRACT

The admissibility of digital evidence depends on a chain of custody whose completeness can be demonstrated to a third party: every evidence item must have a corresponding, unalterable record. A common way to obtain such records is anchoring: storing a cryptographic digest (hash) of each evidence file on a distributed ledger so that any later modification of the file can be detected by recomputing the digest and comparing it with the anchored value. However, the ledger may be temporarily unreachable while evidence continues to be collected, and an anchoring attempt may then fail without anyone noticing and completeness can be violated silently. We study this problem with a simple mathematical model. Every evidence record carries an anchor state from the set {pending, anchored, failed, disabled}, and the system evolves as a state-transition system. A reconciliation operator R runs periodically. It re-attempts every incomplete anchor, up to a bounded number of times. We make two natural assumptions. First, the ledger eventually becomes available again. Second, the operator R is scheduled fairly. Under these assumptions, every admissible record converges to the anchored state. Completeness is therefore eventually restored. A subtle case arises when an anchor was written to the ledger, but its acknowledgement was lost. Duplicate detection on the ledger makes the re-attempted anchoring action idempotent, so no record is ever anchored twice. Finally, an audit function compares the database and the ledger as two finite sets of records and returns their differences in both directions, providing a finite, checkable certificate of completeness. The model is implemented on a production incident-response platform using Hyperledger Fabric, and a controlled outage experiment illustrates convergence after recovery.

Keywords: digital forensics, chain of custody, eventual consistency, state-transition systems, distributed ledgers, fault tolerance.

REFERENCES

[1] E. Androulaki, A. Barger, V. Bortnikov et al., Hyperledger Fabric: a distributed operating system for permissioned blockchains, Proc. 13th EuroSys Conf. (2018), 1-15.

[2] P. Bailis and A. Ghodsi, Eventual consistency today: limitations, extensions, and beyond, Commun. ACM 56 (2013), 55-63.

[3] X. Burri, E. Casey, T. Bollé and D. Jaquet-Chiffelle, Chronological independently verifiable electronic chain of custody ledger using blockchain technology, Forensic Sci. Int. Digit. Investig. 33 (2020).

[4] M. Castro and B. Liskov, Practical Byzantine fault tolerance, Proc. 3rd Symp. Operating Systems Design and Implementation (OSDI) (1999), 173-186.

[5] S. Gilbert and N. Lynch, Brewer's conjecture and the feasibility of consistent, available, partition-tolerant web services, ACM SIGACT News 33 (2002), 51-59.

[6] S. Haber and W. S. Stornetta, How to time-stamp a digital document, J. Cryptology 3 (1991), 99-111.

[7] J. Jeong, D. Kim, B. Lee and Y. Son, Design and implementation of a digital evidence management model based on Hyperledger Fabric, J. Inf. Process. Syst. 16 (2020), 760-773.

[8] M. Li, C. Lal, M. Conti and D. Hu, LEChain: a blockchain-based lawful evidence management scheme for digital forensics, Future Gener. Comput. Syst. 115 (2021), 406-420.

[9] A. H. Lone and R. N. Mir, Forensic-chain: blockchain based digital forensics chain of custody with PoC in Hyperledger Composer, Digit. Investig. 28 (2019), 44-55.

[10] M. Meral and H. H. Sayan, A blockchain-based model proposal to enhance digital forensics readiness, Süleyman Demirel Üniv. Fen Bil. Enst. Derg. 29 (2025), 228-242.

[11] J. Sousa, A. Bessani and M. Vukolić, A Byzantine fault-tolerant ordering service for the Hyperledger Fabric blockchain platform, Proc. 48th IEEE/IFIP Int. Conf. Dependable Systems and Networks (DSN) (2018), 51-58.

[12] I. Weber, V. Gramoli, A. Ponomarev et al., On availability for blockchain-based systems, Proc. 36th IEEE Symp. Reliable Distributed Systems (SRDS) (2017), 64-73.

LIST OF PARTICIPANTS OF ICMME-2026

Name	Institution	Country
Abdulhamit Küçükaslan	Ankara Yıldırım Beyazıt University	Türkiye
Abdullah Akkurt	Kahramanmaraş Sütçü İmam University	Türkiye
Ali Dinler	İstanbul Medeniyet University	Türkiye
Ali Harun Daldallı	Kahramanmaraş Sütçü İmam University	Türkiye
Aslan Bahtiyar	META Technology Group	Türkiye
Beliz Özcan	Ankara Yıldırım Beyazıt University	Türkiye
Berrin Nur Abbasoğlu	Sivas Cumhuriyet University	Türkiye
Bilal İnan	Tokat Gaziosmanpaşa University	Türkiye
Carlo Cattani	University of Tuscia	Italy
Cenk Alp Arslan	Gazi University	Türkiye
Ceyda Şükufe Karahan	Gazi University	Türkiye
Cüneyt Çevik	Gazi University	Türkiye
Deniz Elmacı	Dokuz Eylül University	Türkiye
Derya Kahveci	Ankara University	Türkiye
Dilek Söylemez Özden	Selçuk University	Türkiye
Durdane Yilmaz	Beykoz University	Türkiye
Ece Güneş Güngör	Gazi University	Türkiye
Elif Özcan	Gazi University	Türkiye
Emre Baklacı	Sivas Cumhuriyet University	Türkiye
Ercan Tunç	Tokat Gaziosmanpaşa University	Türkiye
Erkan Oğuzcan	Gazi University	Türkiye
Eyüp Alsancak	Sivas BİLSEM	Türkiye
Fatih Polat	Sivas Cumhuriyet University	Türkiye
Fatma Yeşil Baran	Amasya University	Türkiye
Fatmagül Saklavcı	Kayseri University	Türkiye
Figen Kangalgil	Dokuz Eylül University	Türkiye
Fouzi Hathout	University of Saida	Algeria
Funda Türk	Bartın University	Türkiye
Furkan Üstünsoy	National Defence University	Türkiye
Gizem Şimşek	Gazi University	Türkiye
Gülay Koru Yücekaya	Gazi University	Türkiye
Gülsevin Kodaloğlu	Gazi University	Türkiye
Gülsüm Yüca	Aksaray University	Türkiye
Gülşen Orucova Büyüköz	Necmettin Erbakan University	Türkiye
Hamdi Tolga Kahraman	Karadeniz Technical University	Türkiye
Hasan Bulut	Fırat University	Türkiye
Hasan Hüseyin Sayan	Gazi University	Türkiye
Hatice Muti	Samsun University	Türkiye

Name	Institution	Country
Hazm Talab	TOBB University of Economics and Technology	Türkiye
Hünkar Acar	Samsun University	Türkiye
Hüseyin Budak	Kocaeli University	Türkiye
Hüseyin Demir	Samsun University	Türkiye
Hüseyin Yıldırım	Kahramanmaraş Sütçü İmam University	Türkiye
İbrahim Adalar	Sivas Cumhuriyet University	Türkiye
İbrahim Günaltılı	Eskişehir Osmangazi University	Türkiye
İbrahim Keleş	Samsun University	Türkiye
İlyas Haşimoğlu	Karabük University	Türkiye
İnci Çilingir Süngü	Ondokuz Mayıs University	Türkiye
Juan Luis García Guirao	Technical University of Cartagena	Spain
Kaan Kececi	İstanbul Medeniyet University	Türkiye
Kemal Aydın	Selçuk University	Türkiye
Kevser Nur Çoğalmış	Recep Tayyip Erdoğan University	Türkiye
Kiril St. Shterev	Bulgarian Academy of Sciences, Institute of Mechanics	Bulgaria
Maria Alessandra Ragusa	University of Catania	Italy
Mehmet Atçeken	Aksaray University	Türkiye
Mehmet Meral	Canadian Institute of Technology	Albania
Mehmet Sefa Cihan	Sivas Cumhuriyet University	Türkiye
Mehmet Sevri	Recep Tayyip Erdoğan University	Türkiye
Mert Çarboğa	Yüksek İhtisas University	Türkiye
Merve Esra Türkay	Sivas Cumhuriyet University	Türkiye
Mustafa Özkan	Gazi University	Türkiye
Mustafa Uçkun	Adıyaman University	Türkiye
Naim Tuğlu	Gazi University	Türkiye
Neslihan Yıldırım	Gazi University	Türkiye
Nida Nur Geneş	Adıyaman University	Türkiye
Nilay Akgönüllü Pirim	Başkent University	Türkiye
Nilüfer Topsakal	Sivas Cumhuriyet University	Türkiye
Nuh Durna	Sivas Cumhuriyet University	Türkiye
Nurettin Doğan	Selçuk University	Türkiye
Onur Alp Bahadır	Gazi University	Türkiye
Orhan Özdemir	Tokat Gaziosmanpaşa University	Türkiye
Osman Altıntaş	Başkent University	Türkiye
Ömer Akın	TOBB University of Economics and Technology	Türkiye
Özge Kazar	Hitit University	Türkiye
Özgür İnce	Sivas Cumhuriyet University	Türkiye
Pelin Ayşe Gökgöz	Gebze Technical University	Türkiye
Rabia Kılıç	Niğde Ömer Halisdemir University	Türkiye

Name	Institution	Country
Sabiha Başeğmez	Sivas BİLSEM	Türkiye
Sadık Yıldız	Ministry of National Education	Türkiye
Sare Şengül	Marmara University	Türkiye
Sedat Temel	Recep Tayyip Erdoğan University	Türkiye
Semra Kuş	Kırşehir Ahi Evran University	Türkiye
Semran Ipek	İstanbul Medeniyet University	Türkiye
Serkan Atmaca	Sivas Cumhuriyet University	Türkiye
Seval Işık	Sivas Cumhuriyet University	Türkiye
Stefan K. Stefanov	Bulgarian Academy of Sciences, Institute of Mechanics	Bulgaria
Sümeyye Ermeydan Çiriş	Kahramanmaraş Sütçü İmam University	Türkiye
Şerife Büyükköse	Gazi University	Türkiye
Şeyma Çelik	Selçuk University	Türkiye
Şeyma Firdevs Hızal	Fırat University	Türkiye
Şüheda Nerse	Ondokuz Mayıs University	Türkiye
Tahir Ceylan	Sinop University	Türkiye
Tuğba Kargın	Sivas BİLSEM	Türkiye
Tuğba Mert	Sivas Cumhuriyet University	Türkiye
Tuğçem Partal	Recep Tayyip Erdoğan University	Türkiye
Türkay Kölüş	İzmir Demokrasi University	Türkiye
Umut Baş	Kahramanmaraş Sütçü İmam University	Türkiye
Ümmügülsüm Dal	Tokat Gaziosmanpaşa University	Türkiye
Vincenzo Ciancio	Messina University	Italy
Yasemin Kıymaz	Kırşehir Ahi Evran University	Türkiye
Yusuf Gürsoy	Sivas Cumhuriyet University	Türkiye
Yusuf Sönmez	Gazi University	Türkiye
Yusuf Yaylı	Ankara University	Türkiye
Zakia Hammouch	ENS Moulay Ismaïl University	Morocco
Zeynep Adıgüzel	Sivas Cumhuriyet University	Türkiye

PARTNERS AND SPONSORS

Partners

Sivas Cumhuriyet University

Matematikçiler Derneği

Sponsors

Sivas Valiliği

Sivas Belediyesi

Sivas Ticaret ve Sanayi Odası

Sivas Esnaf ve Sanatkarlar Odaları Birliği